

A New Regressor for Bandwidth Calculation of a Rectangular Microstrip Antenna

Hadi Sadoghi Yazdi¹, Mehri Sadoghi Yazdi², Abedin Vahedian³

1-Computer Department, Ferdowsi University of Mashhad, IRAN, h-sadoghi@um.ac.ir

2-Electrical and Computer Engineering Department, Shahid Beheshti University of Tehran, IRAN
me.sadooghi@mail.sbu.ac.ir

3-Computer Department, Ferdowsi University of Mashhad, IRAN, vahedian@um.ac.ir

Abstract- Microstrip antennas (MSAs) offer a number of unique advantages over other types of antennas. In MSA design, it is important to determine the bandwidth of the antenna accurately because it is a critical parameter of a MSA. To calculate the bandwidth of the rectangular microstrip antennas with thin and thick substrates, we present a new method based on the support vector regression (SVR) and Fuzzy C-Mean (FCM). The support vector regression (SVR) is a statistical learning method that generates input-output mapping functions from a set of training data. The bandwidth results obtained using SVR and new proposed SVR are in excellent compliance with the experimental results available in the literature.

Index Terms- Microstrip antennas; support vector regression; FCM.

I. INTRODUCTION

In recent years, developing low cost, minimal weight, low profile planar configuration microstrip (patch) antennas, capable of maintaining high performance over a wide spectrum of frequencies has been a major trend [1, 2 and 3].

A microstrip device in its simplest form is a sandwich of two parallel conducting layers separated by a single thin dielectric substrate. The patch can assume any shape. They are used where compatibility with microwave and millimeter wave integrated circuits (MMICs), robustness, ability to conform to planar and non-planar surfaces are required [4, 5 and 6].

In MSA design, it is important to accurately determine the bandwidth of the antenna as a

critical parameter. Several techniques varying in accuracy and computational effort have been proposed [7, 8 and 9]. Analytical and mathematical solutions are used to understand the physical aspects and for computer-aided design, but they suffer from limitations.

Design of MSA elements having wider bandwidth using a simple method to calculate the bandwidth of electrically thin and thick rectangular MSAs are then required provided that the theoretical results are in fair agreement with the experimental results.

In this work, a new method based on the support vector regression (SVR) and FCM is presented which efficiently addresses this problem. Once the antenna parameters are determined, the bandwidth is calculated using the proposed SVR.

This paper is organized as follows: In section II, the bandwidth of a MSA is described followed by SVR explained in section III. Section IV includes the application of SVR to the computation of the bandwidth of rectangular MSAs and its simulation results for existing data. In section V new version of SVR is introduced followed by the results of applying the method in section VI. Finally, section VII draws conclusion of this work.

II. BANDWIDTH OF RECTANGULAR MICROSTRIP ANTENNAS

The rectangular microstrip antennas are made of a rectangular patch with dimensions width, W

and length, L , over a ground plane with a substrate thickness h and relative dielectric constants ϵ_r , as indicated in Fig. 1.

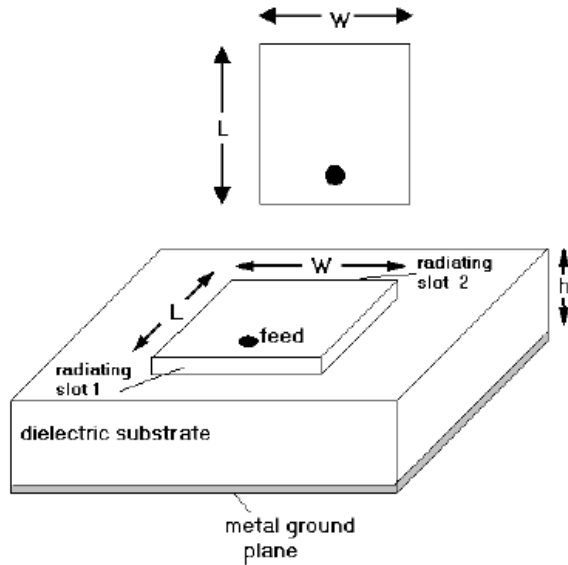


Fig. 1 A rectangular microstrip antenna.

The bandwidth of this MSA can be determined from the frequency response of its equivalent circuit. For a parallel-type resonance, the bandwidth is expressed as [10]:

$$BW = \frac{2G}{\omega_r \left. \frac{dB}{d\omega} \right|_{\omega_r}} \quad (1)$$

Where $Y=G+jB$ is the input admittance at the angular resonant frequency ω_r . For a series-type resonance, G and B are replaced by R and X , respectively, where $Z=R+jX$ is the input impedance at resonance. The bandwidth of a MSA can also be expressed as [11]:

$$BW = \frac{s-1}{Q_T \sqrt{s}} \quad (2)$$

Where s is the voltage standing wave ratio (VSWR) and Q_T is the total quality factor which can be written as:

$$\frac{1}{Q_T} = \frac{P_d + P_c + P_r + P_s}{\omega_r W_T} \quad (3)$$

P_d is the power lost in the lossy dielectric substrate, P_c is the power lost in the imperfect conductor, P_r is the power radiated in the space

waves, P_s is the power radiated in the surface waves, and W_T is the total energy stored in the patch at resonance.

It can be shown that only three parameters, h/λ_d , W , and the dielectric loss tangent, $\tan \delta$, are required to describe the bandwidth. The wavelength in the dielectric substrate, λ_d , is then given as:

$$\lambda_d = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{c}{f_r \sqrt{\epsilon_r}} \quad (4)$$

λ_0 is the free space wavelength at the resonant frequency, f_r and c is the velocity of electromagnetic waves in the free space. The method introduced in this paper calculates the bandwidth of rectangular MSAs based on only these three parameters, i.e. h/λ_d , W , and $\tan \delta$.

III. SUPPORT VECTOR REGRESSION

Support vector machines (SVMs) were originally introduced by Vapnik within the area of statistical learning theory and structural risk minimization aiming at creating a classifier with minimized VC dimension [12].

SVR is considered as a supervised learning method which generates input-output mapping functions from a set of labeled training data. The mapping function can be either a classification function, i.e., the category of the input data, or a regression function. Initially developed for solving classification problems, support vector techniques can be successfully applied to regression.

Suppose the training data $\{(X_1, Y_1), (X_2, Y_2), \dots, (X_l, Y_l)\} \subset X \times R$, is given where X denotes the space of the input patterns (e.g. $X = R^D$). In ϵ -SV regression, the goal is to find a function $f(x)$ that at most has a deviation of ϵ from the actually obtained targets y_i for all the training data [12]. The regressor must not only fit the given data well, but also make minimal errors in predicting values at any

other arbitrary point in R^D . Nonlinear regression is accomplished by fitting a linear regressor in a higher dimensional feature space. A nonlinear transformation ϕ is used to transform data points from the input space (with dimension D) into a feature space having a higher dimension $L (L > D)$. The nonlinear mapping is denoted by $\phi: R^D \rightarrow R^L$.

This problem can be stated as a convex optimization problem; hence, we arrive at the formula stated in [12]:

$$\begin{aligned} \text{Min } & \frac{1}{2} \|W\|^2 + C \left(\sum_{i=1}^l (\xi_i + \xi_i^*) \right) \\ \text{s.t. } & y_i - W^T \phi(X_i) - b \leq \varepsilon + \xi_i \\ & -y_i + W^T \phi(X_i) + b \leq \varepsilon + \xi_i^* \\ & \xi_i, \xi_i^* \geq 0 \end{aligned} \quad (5)$$

Where $C > 0$ is a constant, ξ_i, ξ_i^* are slack variables for soft margin SVR, which allow some deviation larger than ε as precision. It turns out that in most cases the optimization problem in (5) can be solved more easily in its dual formulation:

$$\begin{aligned} \text{Max } & \left(-\frac{1}{2} \sum_{i,j=1}^l (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) K(X_i, X_j) \right) \\ & \left(-\varepsilon \sum_{i=1}^l (\alpha_i + \alpha_i^*) + \sum_{i=1}^l y_i (\alpha_i - \alpha_i^*) \right) \\ \text{s.t } & \sum_{i=1}^l (\alpha_i - \alpha_i^*) = 0, \quad \alpha_i, \alpha_i^* \in [0, C] \end{aligned} \quad (6)$$

Where α_i, α_i^* are Lagrange coefficients and matrix K is termed as a kernel matrix such that

its elements are given by:
 $K(X_i, X_j) = \phi(X_i)^T \phi(X_j), \quad i, j = 1, 2, \dots, M$.

By solving (6), we can find Lagrange coefficients and by replacing them, we have:

$$\begin{aligned} W &= \sum_{i=1}^l (\alpha_i - \alpha_i^*) \phi(X_i), \quad \text{thus} \\ f(x) &= \sum_{i=1}^l (\alpha_i - \alpha_i^*) K(X_i, X_j) + b \end{aligned} \quad (7)$$

IV. SVR BASED BANDWIDTH CALCULATION

For the SVR, the inputs are $h/\lambda_d, W$ and $\tan \delta$, while the output is the measured bandwidths BW_{me} . The training and test data sets used in this work have been obtained from previous experimental works [13, 14], and are given in Table 1 were used to train the SVR. The 6 data sets, marked with an asterisk in Table 1, were used for testing. The training and test data sets used are also the same as those used for ANNs [15, 7] and FISs [16]. The electrical thickness of antennas given in Table 1 vary from 0.0065 to 0.2284, in physical thickness from 0.17 to 12.81 mm, and operate over the frequency range 2.980–8.000 GHz. Some kernel functions have been also used for SVR like polynomial with different degrees, radial basis function and linear functions. Three evaluation methods were used, namely apparent, hold out and leave-one-out to compare average error of the proposed method with ANFIS (Adaptive Neuro Fuzzy Inference System appeared in the appendix).

Table 1. The measured bandwidths for electrically thin and thick rectangular microstrip antennas [13, 14].

Patch no	h (mm)	F_r (GHZ)	h/λ_d	W (mm)	$\tan \delta$	Measured [13, 14] BWme (%)
1	0.17	7.740	0.0065	8.50	0.001	1.070
2	0.79	3.970	0.0155	20.00	0.001	2.200
3	0.79	7.730	0.0326	10.63	0.001	3.850
4	0.79	3.545	0.0149	20.74	0.002	1.950
5	1.27	4.600	0.0622	9.10	0.001	2.050
6	1.57	5.060	0.0404	17.20	0.001	5.100
7*	1.57	4.805	0.0384	18.10	0.001	4.900
8	1.63	6.560	0.0569	12.70	0.002	6.800
9	1.63	5.600	0.0486	15.00	0.002	5.700
10*	2.00	6.200	0.0660	13.37	0.002	7.700
11	2.42	7.050	0.0908	11.20	0.002	10.900
12	2.52	5.800	0.0778	14.03	0.002	9.300
13	3.00	5.270	0.0833	15.30	0.002	10.000
14*	3.00	7.990	0.1263	9.05	0.002	16.000
15	3.00	6.570	0.1039	11.70	0.002	13.600
16	4.76	5.100	0.1292	13.75	0.002	15.900
17	3.30	8.000	0.1405	7.76	0.002	17.500
18*	4.00	7.134	0.1519	7.90	0.002	18.200
19	4.50	6.070	0.1454	9.87	0.002	17.900
20	4.76	5.820	0.1475	10.00	0.002	18.000
21	4.76	6.380	0.1617	8.14	0.002	19.000
22	5.50	5.990	0.1754	7.90	0.002	20.000
23	6.26	4.660	0.1553	12.00	0.002	18.700
24	8.54	4.600	0.2091	7.83	0.002	20.900
25	9.52	3.580	0.1814	12.56	0.002	20.000
26	9.52	3.980	0.2017	9.74	0.002	20.600
27*	9.52	3.900	0.1976	10.20	0.002	20.300
28	10.00	3.980	0.2119	8.83	0.002	20.900
29	11.00	3.900	0.2284	7.77	0.002	21.960
30	12.00	3.470	0.2216	9.20	0.002	21.500
31	12.81	3.200	0.2182	10.30	0.002	21.600
32	12.81	2.980	0.2032	12.65	0.002	20.400
33*	12.81	3.150	0.2148	10.80	0.002	21.200

*Test data set

Starting with leave-one-out method for computing error of SVR against ANFIS, average error for both methods is shown in Table 2, using different kernel functions for SVR.

Table 1. Average error of SVR and ANFIS for bandwidths for electrically thin and thick rectangular microstrip antennas with leave-one-out method.

kernel	Average Error SVR	Average Error ANFIS
Poly, p=2	37.8586	37.2162
Poly, p=3	19.6580	37.2162
Poly, p=4	37.5095	37.2162
erbf	28.8429	37.2162
rbf	44.8668	37.2162
Linear	20.4241	37.2162

Using separate train and test data sets (hold out method) determined in Table 1 results in

measured error indicated in Table 3 for both ANFIS and SVR.

Table 2. Error of SVR and ANFIS for bandwidths for electrically thin and thick rectangular microstrip antennas with hold out method

kernel	Error of SVR	Error of ANFIS
Poly, p=2	0.0787	0.1757
Poly, p=3	0.1431	0.1757
Poly, p=4	0.1998	0.1757
erbf	0.0833	0.1757
rbf	0.2530	0.1757
Linear	0.0612	0.1757

Finally, same train and test data sets were used in apparent method with all data represented in Table 1. The measured error of our proposed method against ANFIS is shown in Table 4Table 3.

Table 3. Error of SVR and ANFIS for bandwidths of electrically thin and thick rectangular microstrip antennas with apparent method.

kernel	Error of SVR	Error of ANFIS
Poly, p=2	0.1113	5.4071e-016
Poly, p=3	0.1282	5.4071e-016
Poly, p=4	0.1142	5.4071e-016
erbf	6.5168e-004	5.4071e-016
rbf	0.2741	5.4071e-016
Linear	0.0744	5.4071e-016

As we can see from Table 2 to Table 4, when training data set is used to test ANFIS, it can compute bandwidth of MSAs with minimum error; however when test data set is different from training data set, ANFIS cannot outperform SVR for some kernel functions.

Some of the key notes about the proposed SVR as opposed to ANFIS are:

- 1- ANFIS uses a linear piecewise technique, the proposed SVR, however, is a nonlinear piecewise approach.
- 2- Swapping kernels to obtain better results is more possible in the proposed SVR.
- 3- All properties of SVR can be incorporated in the proposed SVR such as kernel tricks, high dimensional space, and employing margin in the regression.
- 4- SVR is more general due to utilizing margin and permeate aspects.

V. NEW SUPPORT VECTOR REGRESSION

As mentioned earlier, the support vector machine is an approximate implementation of the method of structural risk minimization. This is based on the fact that the error rate of a learning machine on test data is bounded by the sum of the training-error and a term which depends on the Vapnik-Chervonenkis (VC) dimension. In this

method, optimal hyper plane is determined to guarantee the minimum error for test samples, whereas, neural networks fail to guarantee to find optimum hyper plane for test samples. Therefore, SVR yields better results compared to ANFIS as indicated in section 4. There are, however, the following problems in the SVR:

- a) Since each sample appears as one constraint in support vector, increasing training samples is equivalent to increasing the number of constraints. Solving equations to find optimal hyper plane then becomes fairly hard.
- b) Finding suitable kernel for modeling of nonlinear space is not straightforward.

We, therefore, propose a new version of SVR which works based on divide and conquer principle which can solve two aforementioned problems. Input space is divided to several subspaces so that in each subspace, a SVR models the data. This causes that the new generated space incorporate the properties of high dimensional space. A weighting procedure is then performed using probability density function of each subspace and gives the portion of each SVR according to generated rules. Results of weighted SVRs are, then, combined to perform the fitting task.

The proposed SVR method is reviewed in detail in the following steps.

Step 1: Input training data is divided into n subsets using a clustering algorithm such as fuzzy c-means (FCM) which assigns weights to any input data. Fig. 2 indicates as example 3 partitions clustered by FCM. The PDF (Probability Density Function) of each cluster is obtained which are shown. The corresponding weights of an input data are then calculated based on membership values to each partition (i.e. clusters)

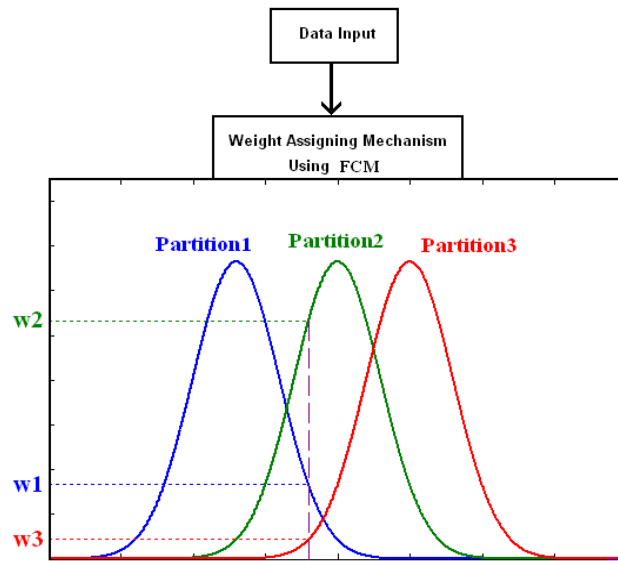


Fig. 2. Assigning weights to input data

Step 2: Each available subset for any partition is applied for training of each Support Vector Regressor (SVR) as depicted in

Fig. 3. Therefore, for training samples of partition 1 in Fig. 2, SVR1 is trained (as shown in Fig.3).

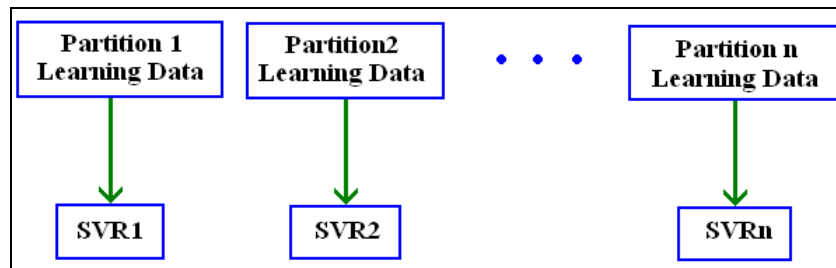


Fig. 3. Applying SVR in each partition

Some kernel functions have been also used in this work for SVR such as polynomial with different degrees, radial basis and linear functions. Results were examined for best state.

Step 3: This step involves testing procedure. We used leave-one-out method for computing average error for our proposed method and compared it with ANFIS. In order to calculate the output of the proposed system, membership values were computed for each test sample (w_i).

$$w_i = \frac{1}{2\pi|\tilde{\Sigma}|^{0.5}} \exp(-(x_t - \mu)\tilde{\Sigma}^{-1}(x_t - \mu)^T) \quad (8)$$

Where $\tilde{\Sigma} = \alpha\Sigma$ and Σ is covariance of training samples and μ is the mean of training data.

x_t is a test sample and $|\cdot|$ denotes the determinant.

In equation (8), α is a variable to control spreading of the Gaussian distribution considered for samples of each training set.

By applying equation (8) to test samples, corresponding weights are obtained so one can normalize these weights by dividing any weight to the sum of them. Finally, these normalized weights are multiplied by each test sample to generate final values.

Fig. 4 depicts this procedure.

In Fig. 4 output value is the computed value for bandwidth of MSA of test sample. In order to compute the error of leave-one-out method for evaluation of performance, we can compute difference between computed value and measured value of bandwidth of MSA of any test sample and then obtain average value for all of data, as given in Table 1, for instance.

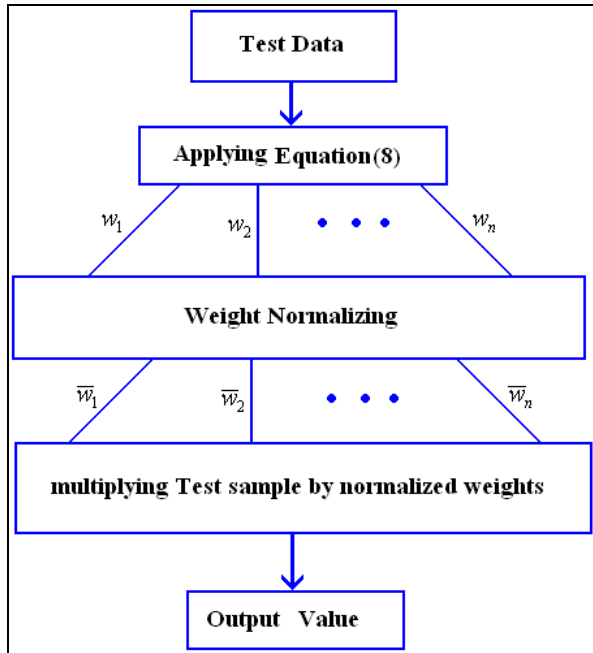


Fig. 4 Testing procedure

VI. EXPERIMENTAL RESULTS

We now compute the bandwidth for electrically thin and thick rectangular microstrip patch antennas by the proposed SVR. First we used leave-one-out method for testing our method against ANFIS with different kernel functions for SVR and different values for α . In this experiment we considered values 0.1, 0.2, ..., 1 for α with the number of clusters set to be 2. Average error for both methods is shown in Table 5.

As it can be seen from Table 5, with $\alpha=0.1$ or $\alpha=0.2$ our proposed method has the minimum error for the given kernel functions.

The proposed method was also compared with some conventional methods presented in [11, 17, 18 and 19]. Fig. 5 represents comparison between computed error in calculating the bandwidth of MSAs with a number of conventional methods, ANFIS and our proposed SVR method. We used simple criteria for computing error as absolute value of difference between measured BW [13 and 14] and computed value using each method. Since the proposed methods in [11, 17, 18 and 19] have used the data set in Table 1 for training and testing similar to our method, we too used the above mentioned criteria to perform the comparison. For each case (each row in Table 1), the obtained error is calculated and shown in Fig.5 for each method. It can be seen that the resulted error in our proposed method is less than the other methods. Mean value of computed error for each method appeared in Fig. 5 is also represented in Table 6.

Table 4. Average error of proposed SVR and ANFIS for bandwidths of electrically thin and thick rectangular microstrip antenna

kernel	Average Error New SVR										Average error for all α	Average Error ANFIS
	$\alpha=0.1$	$\alpha=0.2$	$\alpha=0.3$	$\alpha=0.4$	$\alpha=0.5$	$\alpha=0.6$	$\alpha=0.7$	$\alpha=0.8$	$\alpha=0.9$	$\alpha=1$		
erbf	1.6950	1.6950	2.9466	2.9466	2.9428	2.9466	2.9470	4.156	4.213	4.217	3.0707	37.2162
rbf	56.308	56.207	57.979	58.079	57.979	58.079	58.079	58.475	58.495	58.495	57.817	37.2162
poly (p=2)	15.803	15.803	16.641	16.641	16.641	16.641	16.535	18.105	18.181	18.181	16.917	37.2162
poly (p=3)	7.856	7.856	8.314	8.314	8.314	8.314	8.314	8.594	8.6089	8.599	8.3089	37.2162
poly (p=4)	16.232	16.232	17.328	17.416	17.416	17.416	17.329	20.731	20.898	20.898	18.190	37.2162
Linear	17.290	17.290	17.637	17.637	17.637	17.636	17.637	19.4233	19.513	19.514	18.121	37.2162

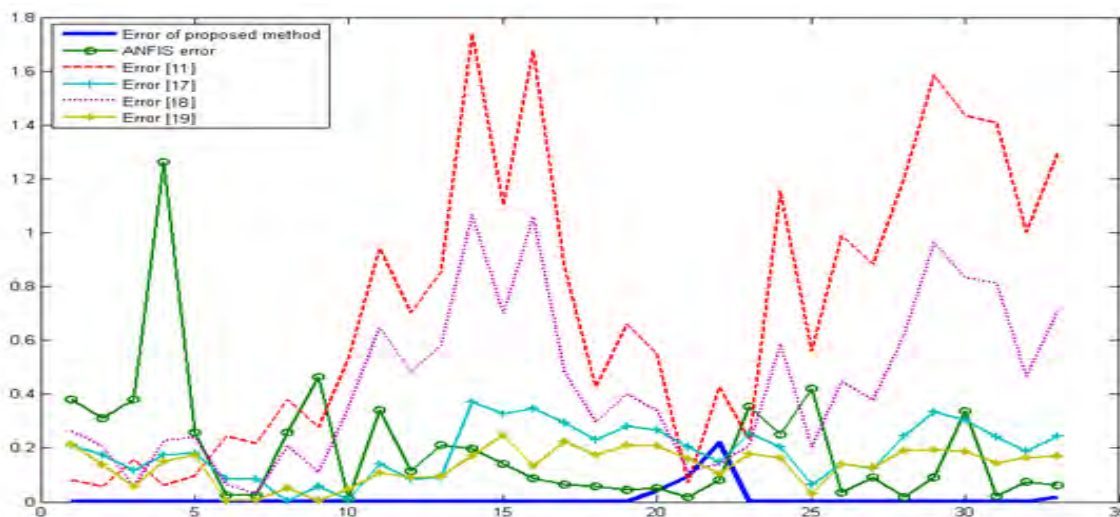


Fig. 5. Comparing error of some conventional methods and ANFIS with the proposed SVR-x-axis:samples, y-axis:resulted error

Table 5. Mean value of error of computed bandwidths obtained from conventional methods presented in [11, 17, 18 and 19], ANFIS and the proposed method for MSAs

Method	Error
[11]	0.7241
[17]	0.1891
[18]	0.4337
[19]	0.1359
ANFIS	0.1984
Proposed SVR	0.0117

VII. CONCLUSION

In this paper we used Support Vector Regression (SVR) method to calculate bandwidth of electrically thin and thick rectangular microstrip patch antennas. Resulted error from our method was compared to ANFIS which showed that the method results in lower error than ANFIS and other conventional methods.

A suitable method for calculation of optimum input parameters ($h/\lambda_d, W, \tan\delta$) is suggested to be carried out as the future work with desired

constraints over each parameters to obtain desired bandwidth. An artificial search method, therefore, is to be proposed for searching input parameters to converge to desire bandwidth as shown in Figure 6.

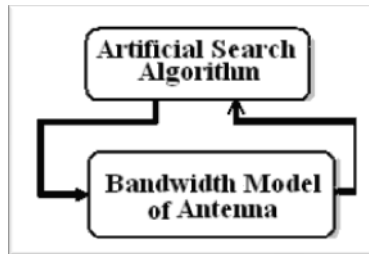


Fig.6. Searching problem of optimum input parameters for obtaining desired bandwidth which is appeared in the future work.

- Real-time processing of instantaneous system input and output data. This property helps using this technique for many operational research problems.
- Offline adaptation instead of online system-error minimization, thus easier to manage with no iterative algorithms involved.
- System performance is not limited by the order of the function since it is not represented in polynomial format.
- Fast learning time.
- System performance tuning is flexible as the number of membership functions and training epochs can be altered easily.
- The simple if-then rule declaration and the ANFIS structure are easy to understand and implement.

APPENDIX

ADAPTIVE NEURO FUZZY INFERENCE SYSTEM (ANFIS)

Recently, there has been a growing interest in combining neural network and fuzzy inference system. As a result, neuro-fuzzy computing techniques have been evolved. Neuro-fuzzy systems are fuzzy systems which use neural networks theory in order to determine their properties (fuzzy sets and fuzzy rules) by processing data samples. Neuro-fuzzy integrates to synthesize the merits of both neural networks and fuzzy systems in a complementary way to overcome their disadvantages.

ANFIS has been proved to have significant results in modeling nonlinear functions [20]. In an ANFIS, the membership functions (MFs) are extracted from a data set that describes the system behavior. The ANFIS learns features in the data set and adjusts the system parameters according to given error criterion. In the ANFIS architecture, NN learning algorithms are used to determine the parameters of fuzzy inference system. Below, the advantages of the ANFIS technique are summarized.

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