A new modeling algorithm – Normalized Kernel Least Mean Square

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Abstract

In this paper Normalized Kernel Least Mean Square (NKLMS) algorithm is presented which has applications in system modeling and pattern recognition. In 2007 a similar algorithm was proposed Named Kernel Least Mean Square (KLMS), and a modified version of KLMS was introduced in 2008. Although KLMS has good results in prediction of some time series, high sensitivity to step-size and signal amplitude stability, still remain as problems. In this paper NKLMS and its ability in prediction and identification of time series is presented and is compared to KLMS method. A variable named step-size that was used in the algorithm has made NKLMS more efficient in prediction of time-series which have inconsistency in amplitude. Thus, convergence speed and system tracking are improved. Furthermore the proposed algorithm is applied to channel modeling.

Keywords— Least Mean Squar, System modeling, Pattern recognition, Time-Series Prediction

1. Introduction

Kernel Methods (Kernel Tricks) have been presented to map non-linear data to high dimensional linear ones. These methods employ some special functions called kernel functions. Kernel machine has found wide range applications in pattern recognition, artificial intelligence and signal processing [1-2].

The Least Mean Square (LMS) algorithm introduced by Widrow and Hoff in 1959 [3], is an adaptive algorithm widely applied in machine learning, system identification and prediction topics. LMS is an iterative procedure that applied to correct the weight vector of system model based on a gradient-based method of steepest decent and leads to the minimum mean square error. LMS algorithm is simple; calculations do not require correlation function and matrix inversions.

In the field of signal processing LMS were widely used in neural network and training algorithms. In [4] a new version of LMS algorithm is presented as KERNEL LMS (KLMS) and is used to predict some sample time series. The basic idea in KLMS algorithm is to perform the linear LMS algorithm in the kernel feature space.

Behind the KLMS there may be different types of kernel functions such as radial basis function (RBF), sigmoid and polynomial [1-2]. In this paper we show that convergence in KLMS is not absolute, due to its sensitivity to step-size and signal amplitude, but normalized KLMS (NKLMS) reduces the problem, effectively.

LMS, NLMS, Kernel Method and KLMS are introduced in section 2. NKLMS formulation is presented and compared to the previous methods in section 3. Experimental results presented in section 4 will help us to demonstrate the claim more clearly.

2. Background

2.1. Least Mean Square Algorithm

In 1959 the LMS algorithm was introduced as a simple way of training a linear adaptive system with mean square error minimization. An unknown system - $y(n)$ - is to be identified and the LMS algorithm attempts to adapt the filter $\hat{y}(n)$ to make it as close as possible to $y(n)$. The algorithm uses *u(n)* as the input, *d(n)* as desired output and *e(n)* as calculated error.

LMS uses steepest-descent algorithm to update the weight vector so that the weight vector converges to optimum Wiener solution. Updating weight vector based on the following rule is applied:

$$w(n+1) = w(n) + 2\mu \times e(n) \times u(n) \qquad (1)$$

Where *w(n)* is weight vector and μ is step size. The filter output y is calculated by: