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Analyzing the cross section effect of hypersonic flow past a conical body via perturbation method

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Abstract: For hypersonic flow past a triangular cone the flow's parameters analysis is major subject in this research that has performed analytically and numerically. In analytic study of hypersonic flow the boundary layer is very thin and viscous effects are negligible and we assume the flow, outside viscous boundary layer, is invisid, adiabatic, and steady, but in numerical solution we have considered this and also viscous effects and For modeling of Reynolds stress in momentum equation, RNG K-ε turbulence model is used. For analytic solution perturbation expansion is considered for flow variables and is used in ε terms. The zeroth-orther approximation of hypersonic flow obtained by nonlinear asymptotic theory is chosen as the basic cone solution. To validate the analytical solution, a numerical investigation by Fluent software is also applied. C_p/δ^2 Is one of parameters in calculating lift to drag ratio as a major quantity in design of aircrafts and space vehicles. The graphs show that this parameter depends on δ and M_∞. Also there is good agreement between numerical and analytical solution.

Key words: Hypersonic flow, Conical body, Triangular cone, Numerical solution, Perturbation method.

1 Introduction

Different conical body configurations are of current interest as a means for providing high-performance hypersonic space vehicle, missile and rocket characteristics. These configurations provide suitable flow fields and aerodynamic properties. The axisymmetric supersonic flow past a circular cone is a suitable basis for constructing conical bodies. The flow past these shapes has been studied for many different cases. Taylor-Maccoll [1] used spherical coordinates to derive the governing equation from supersonic flow around a right circular cone that is in zero angle of attack. Perturbation method is mostly applied to study analytically of flow on conical bodies. Stone [2,3] applied the power series expansion for a small angle of attack and provided first and second order of perturbation solution. Hypersonic flow over slender pointed nose elliptic cone at zero incidence is studied [4]. Doty and Rasmussen [5] and Rasmussen [6] for obtaining solutions for flow past circular cones at small angle of attack perform analysis. Also Van Dyke [7] found a closed-form solution of the second approximation in the slender body theory for an elliptic cone at zero angle of attack.

In this paper considering the Stone's perturbation expansions and applying them for two conical bodies with different cross sections as circle and triangle at small angle of attack, the solution is obtained analytically. The purpose of the present work is to obtain pressure coefficient analytically and compare it with analytical solution. The results will be useful in increasing the lift to drag ratio for aircrafts, satellites, missiles and space vehicles.

2 Problem Formulation

Spherical coordinate system r, θ and φ are used in this study, θ is the polar angle and φ is the azimuthal angle. In this flow, the following statement denotes the velocity vector

$$\vec{V} = u\hat{e}_r + v\hat{e}_\theta + w\hat{e}_\Phi \tag{1}$$

As shown in Fig. 1, the triangular cone is represented

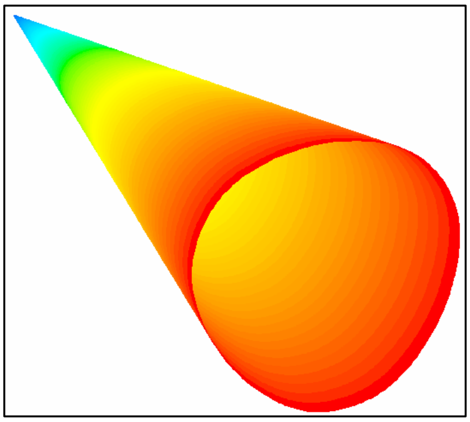


Fig.1. body produced by perturbing basic cone

The basic cone body is perturbed by the relation:

$$\theta_c = \delta[1 - \varepsilon\cos 3\varphi + o(\varepsilon^2)] \tag{2}$$

Where δ is the semi-vertex angle of basic cone, ε is a small perturbation parameter and $\cos(3\varphi)$ represents the triangular shape of cross section. The shape of the corresponding shock wave is expressed in a similar way,

$$\theta_s = \delta[1 - \varepsilon G\cos 3\varphi + o(\varepsilon^2)] \tag{3}$$

Where G represents the shock-perturbation factor, and $\sigma = \beta/\delta$ is the shock relation of basic conical flow.

Due to high Mach number, thin boundary layer and decrease of viscous effects, the flow field outside the viscose boundary layer is governed by mass, momentum and energy equations.

$$div(\rho\vec{V}) = 0 \tag{4}$$

$$\rho[\nabla(\frac{V^2}{2}) - \vec{V}\times curl\vec{V}] = -\nabla p \tag{5}$$

$$\vec{V}.\nabla s = 0 \tag{6}$$

$$\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{u^2+v^2+w^2}{2} = cons. \tag{7}$$

$$\frac{s-s_r}{C_v} = Ln(\frac{p}{p_r}) - \gamma Ln(\frac{\rho}{\rho_r}) \tag{8}$$

Where p is static pressure, ρ density of the fluid and s is the entropy.

It can be used of superposition effect for writing the perturbation expansion for flow variables [2, 3].

Substituting the perturbation expansions in the governing equations and separating zero-order and first-order terms in ε and α, three systems of equations are obtained. Zeroth-order functions u_0, v_0, p_0, ρ_0, and s_0 are the solutions of basic cone problem. In this investigation, the zeroth-order solution from [8] is opted as the basic solution.

The cone surface $\theta = \delta$ boundary condition can be yielded by the tangency of inviscid flow across the cone surface as

$$\vec{V}.\hat{n}_C = 0 \tag{9}$$

$$v_0(\delta) = 0 \text{ and } v_{mn}(\delta) = -mV_\infty\delta \tag{10}$$

Where n_C is normal unit vector from the cone surface.With equations of mass conservation and conservation of tangential velocity, two velocity components can be calculated at the shock, $\theta = \beta$.

$$\rho_\infty V_\infty.\hat{n}_s = \rho V_s.\hat{n}_s \tag{11}$$

$$V_\infty \times \hat{n}_s = V_s \times \hat{n}_s \tag{12}$$

$$u_1(\beta) = \delta\sin\beta\left(1 - G(1-\xi_0)\right) \tag{13}$$

$$v_1(\beta) = \delta G\cos\beta\left(-\xi_0 + 2\frac{\gamma-1}{\gamma+1}\right) + \delta G v_0'(\beta) \tag{14}$$

The second-order perturbation expansion of α system yields:

$$2(\rho_0 u_2 + u_0\rho_2) + (\rho_2 v_0 + v_2\rho_0)' + \cot\theta(\rho_0 v_2 + v_0\rho_2) + \frac{\rho_0 w_2}{\sin\theta} = 0 \tag{15}$$

$$v_n u_0' + v_0 u_n' - 2v_0 v_n = 0 \tag{16}$$

$$\rho_0(v_0 v_2)' + \rho_2 v_0 v_0' + v_0(\rho_2 u_0 + u_2\rho_0) + \rho_0 u_0 v_2 + p_2' = 0 \tag{17}$$

$$w_2' + \frac{u_0}{v_0}w_2 + w_2\cot\theta - \frac{1}{\sin\theta}\frac{p_2}{\rho_0 v_0} = 0 \tag{18}$$

$$v_0 s_2' + v_2 s_0' = 0 \tag{19}$$

$$s_n = \frac{p_2}{p_0} - \gamma \frac{\rho_2}{\rho_0} \tag{20}$$

$$\frac{1}{2}\left(u_0^2 + v_0^2\right) + \left(u_2 u_0 + v_2 v_0\right)\frac{\rho_0}{\rho_2} + \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2} \tag{21}$$

$$-\frac{1}{2} - \frac{1}{(\gamma - 1)M_\infty^2} = 0$$

To find pressure coefficient it can be shown that

$$C_p = C_{p_0} + \varepsilon C_{p1}\cos 3\varphi + \alpha C_{p2}\cos\varphi \tag{22}$$

And it can be calculated that

$$\frac{c_{p_0}}{\delta^2} = 1 + \frac{\sigma^2 \ln\sigma^2}{\sigma^2 - 1} \tag{23}$$

$$\frac{c_{p2}}{\delta} = \frac{2N}{k_\delta^2}\left(1 + \frac{\gamma}{2}k_\delta^2\left(1 + \frac{\sigma^2 \ln\sigma^2}{\sigma^2 - 1}\right)\right) \times \tag{24}$$

$$\left(\frac{1 - g_1}{\sigma^3} - \frac{a_0^2(\beta)}{a_0^2(\delta)}\frac{u_1(\delta)}{V_\infty \delta}\right)$$

where (25)

$$\frac{u_1(\delta)}{V_\infty \delta} = -2 + \frac{(1 - g_1)}{\sigma^3}\left[\frac{4\sigma^2}{\gamma + 1} + \sigma^3 - \frac{\sigma^2}{2}\right.$$

$$\left. + 1 - \frac{\ln\left(\sigma + \sqrt{\sigma^2 - 1}\right)}{2\sqrt{\sigma^2 - 1}}\right]$$

$$\frac{a_0^2(\beta)}{a_0^2(\delta)} = 1 + \frac{(\gamma - 1)\sigma^2\left[\ln\sigma^2 + \frac{1}{\sigma^2} - 1\right]}{(\sigma^2 - 1)(2\sigma^2 + \gamma - 1)} \tag{26}$$

and (27)

$$N = \frac{2\sigma^2}{(\sigma^2 - 1)(2\sigma^2 + \gamma - 1)}$$

C_{p0}/δ^2 and $-C_{p2}/\delta$ fare shown in figures (2) and (3) as the following,

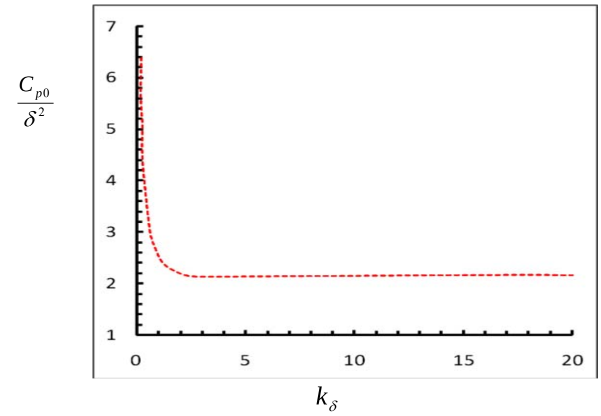


Fig. 2. Cp_0/δ^2 versus k_δ.

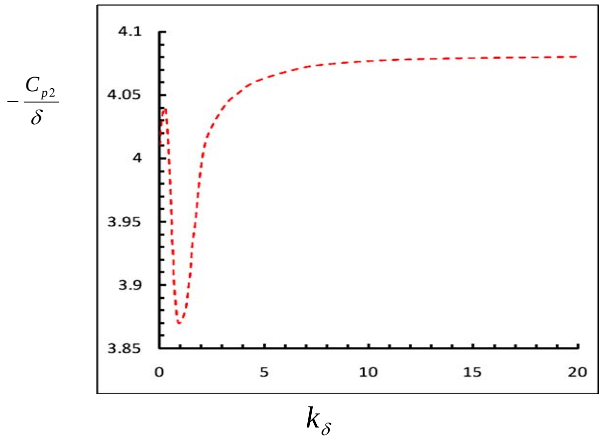


Fig. 3. – Cp_2/δ versus k_δ

2.1 Numerical solution

Fluent CFD software has been used to model the triangular cone employing in this problem to solve the 3D problem. The 3D volume grid is represented in Fig. 4. A grid dependence study was conducted to arrive at the appropriate size of the grid for optimal accuracy and efficiency. The conservation equations for mass, momentum, energy and state equation are solved by finite-volume analysis, using a second-order upwind scheme for discretisation of the convective terms in the transport equations. Mass, momentum and energy equations are in the form

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{v}) = 0 \tag{28}$$

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \nabla.(\rho\vec{v}\vec{v}) = -\nabla p + \nabla.(\overset{=}{\tau}) + \rho\vec{g} \tag{29}$$

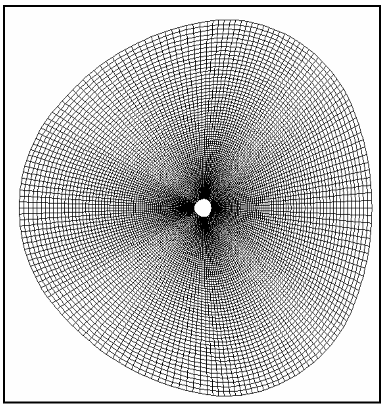
$$\frac{\partial}{\partial t}(\rho E)+\nabla.(\vec{v}(\rho E+p))=\nabla.(k_{eff}\nabla T- \sum_j h_j \vec{J}_j+(\overline{\overline{\tau}}_{eff}.\vec{v})) \qquad (30)$$

Where p is the static pressure, $\overline{\overline{\tau}}$ is the stress tensor. For invisid flow $\overline{\overline{\tau}}$ is zero. k_{eff} Is the effective conductivity ($k+k_t$, where k_t is the turbulent thermal conductivity, defined according to the turbulence model being used), and $\vec{J}_j$ is the diffusion flux of species j. The first term and third term of the right hand side is zero if flow is considered invisid. Used turbulence model is RNG and it's equations are

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}(\alpha_k \mu_{eff}\frac{\partial k}{\partial x_j}) + G_k + G_b - \rho\varepsilon - Y_M + S_K \qquad (31)$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_i}(\rho\varepsilon u_i)=\frac{\partial}{\partial x_j}(\alpha_\varepsilon \mu_{eff}\frac{\partial \varepsilon}{\partial x_j})+C_{1\varepsilon}\frac{\varepsilon}{k}(G_k+C_{3\varepsilon}G_b)-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}-R_\varepsilon+S_\varepsilon \qquad (32)$$

In these equations, G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients. G_b Is the generation of turbulence kinetic energy due to buoyancy. Y_M Represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate.



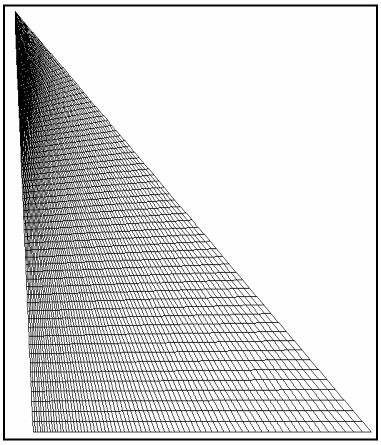


Fig. 4. Three-dimensional tetrahedral grid

3 Problem Solution

Numerical and analytical calculations were performed on the hypersonic flow for the triangular cone as shown in Fig. 1**.** This problem has solved for cones with different angles, Mach numbers and attack angles that we represent some results of them. In Fig. 5 there are three cones which inner cone is body cone and supersonic flow with Mach number 5 and no angle of attack passes on it. The next cone that surrounds body cone is shock cone and outer cone is surrounding of region of solution that pressure far field as one of boundary conditions has imposed on it. In Fig. 6 pressure contour has been shown in different views that shock wave is seen clearly. Inside the shock region when flow is approaching to body's surface, the pressure increases and out of this region relative pressure is zero.

In Figs. 7a-d, the effect of free stream Mach number on shock angle and pressure distribution is shown. It can be seen that shock angle decreases and pressure on cone surface increases, with increasing of Mach number. In Fig. 8 the numerical result of C_p/δ^2 has been compared with perturbation result for basic cone. Numerical solution has been performed for invisid flow and turbulent flow. For modeling of Reynolds stress in momentum equation, RNG K-ε turbulence model is used. The results show good agreement between turbulent and invisid model for hypersonic flow.

In Fig. 9 the perturbation of C_p/δ^2 for basic and triangular cone is compared with numerical solution of triangular cone.

In analytical solutions it can be seen that C_p/δ^2 is infinitive for small amounts of k_δ, because perturbation method is considered for hypersonic flow and it yield illogical result for small k_δ. In Fig. 10 the numerical result of C_p/δ^2 for different cone angle is considered.

This figure verifies that C_p/δ^2 is a function of k_δ, as seen in perturbation results.

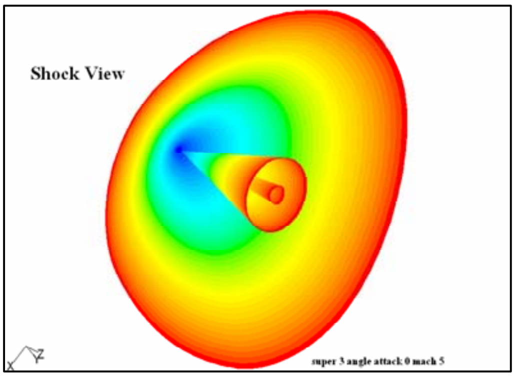


Fig.5. Shock view

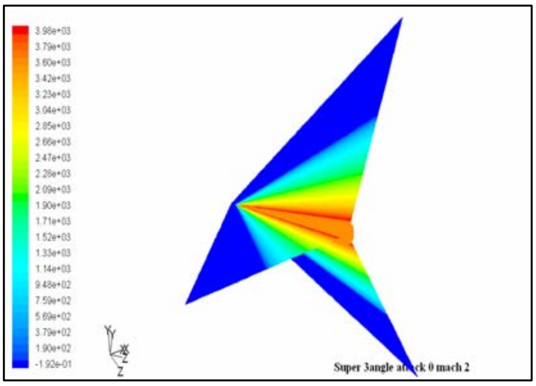
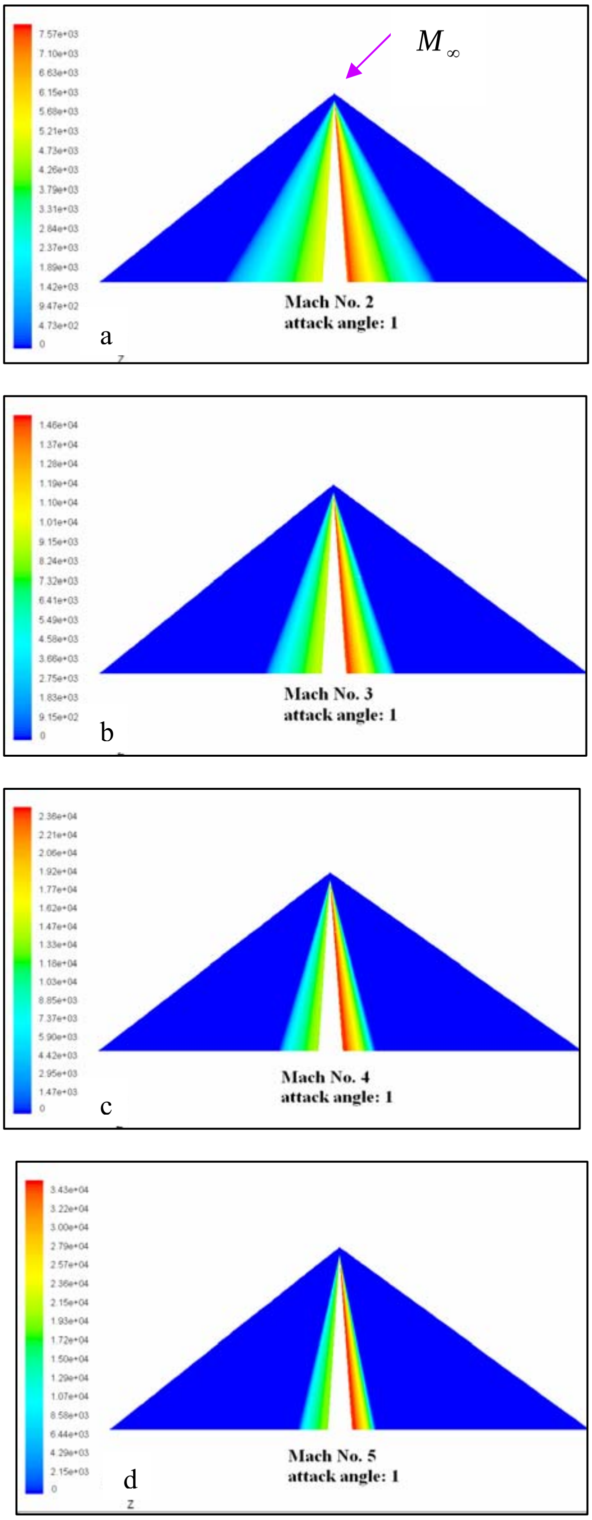


Fig.6. pressure contour in Mach number 2 and angle of attack 0



Figs. 7. Mach numbers effect on shock wave in triangular cone

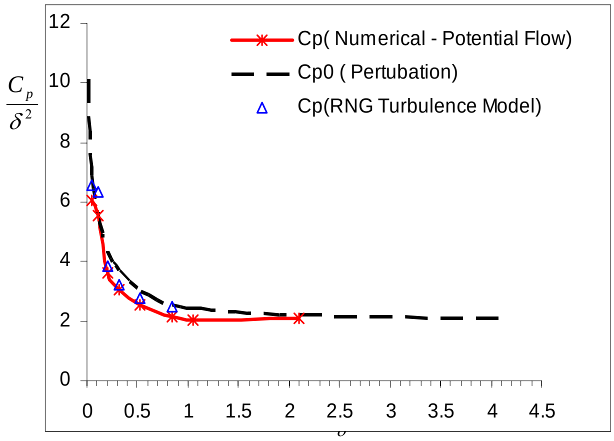
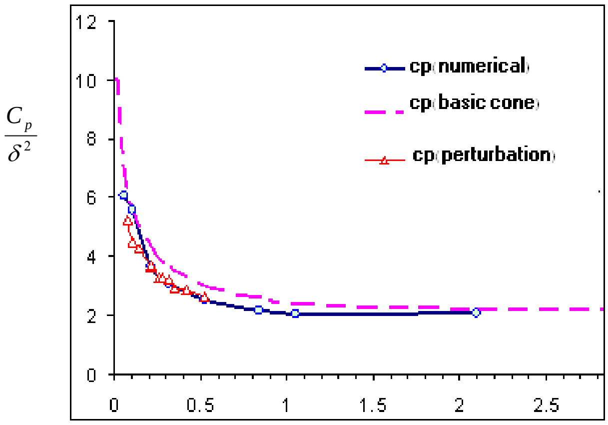
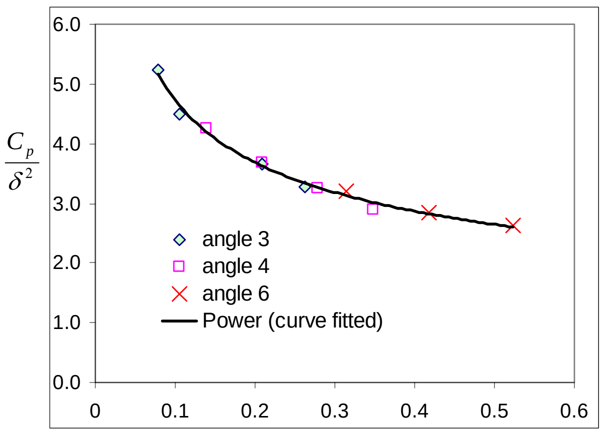


Fig. 8.analitic solution and numerical solution (basic cone)



k_δ

Fig. 9. C_p/δ^2 vs. k_δ in numerical and analytic solution



k_δ

Fig. 10. Effect of cone angle on C_p/δ^2 v.s k_δ

4 Conclusion

The perturbation method was applied analytically to obtain flow variables over conical body of triangular cone. The aim of the present work is to study flow's parameters in conical body. Also numerical calculation was performed to validate the analytical solution. Based on the presented results, the following conclusions may be drawn:

- Analytic solution is applicable in large range of variables, but in numerical solution there are limitations.
- In high velocity, boundary layer is thin thus viscous effects can be neglected. The result for turbulence model and invisid flow is in good agreement.

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