Dynamic simulation of multi-effect falling-film evaporator: Milk powder production plant

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Abstract: In this paper, two types of dynamic models, lumped and distributed, were developed for an industrial four-effect falling-film evaporator which is used to concentrating whole milk. The predictions of each model are compared to industrial data. The results showed that the predictions of distributed model have better agreement with industrial data rather than lumped model. Although in predictions of some variables, these models have the same performances.

Key words: falling-film evaporator, dynamic simulation, lumped model, distributed model

Introduction
Falling-film evaporators are commonly used as part of the powder production process to make whole milk, skim milk and whey powder, in fact a very wide range of products. Despite their common usage falling-film evaporators are still not well understood. Thus a comprehensive study of dynamics and control of this process is well motivated [1]. Analytical models can be beneficial in assisting engineers to better understand and control the dynamic behavior of processes. Analytical can be complex but the best of these models may not be the one which describes all the phenomena of the process behavior. The ultimate application of an analytical model largely determines the method of analysis of the system and complexity of the final model [2].

One of the recent lumped models for industrial evaporation plant is Miranda and Simpson's work. Miranda and Simpson (2005) developed a lumped model based on energy and mass balances of the evaporator system [3]. This model was implemented in a tomato concentration plant and includes semi-empirical equilibrium functions. Recently, Stefanov and Hoo (2004) developed a distributed model of a multiple-effect falling-film evaporator plant [4, 5]. The model describes the important phenomena of evaporation, heating and condensation for different hydrodynamic regimes as well as the pressure dynamics of the plant. This model precision was validated using steady-state real plant data but there is no real dynamic data to comparing with simulation results.

In the current work, lumped and distributed models are developed for four-effect falling film evaporators to investigate the performance of each model in predictions the dynamic behavior of plant. Existence of real dynamic data helps us to compare the simulation results of each model and this comparison is beneficial to understanding their advantages and disadvantages.

Process description
The industrial equipment, as schematically shown in figure (1) includes a forward feed flow, four-effect evaporator for concentrating the whole milk. It consists of four bodies in which the whole milk is concentrated and equipped with thermo compressor that raises the pressure of a part of second effect vapor to that of first one with live steam. Second body is divided in to two parts. Each effect consists of one pre heater. At the end of the forth effect there are a pre heater and condenser to accumulate vapor and produce vacuum for the plant. Other specifications of the simulated industrial plant are shown in Fig.1.

Mathematical model
In each tube milk flows as falling film and outside of the tube film condensation occurs. It is assumed that, in the energy balance equations, there is no accumulation of energy and that only phase change occurs. This assumption is reasonable because temperature of feed entering to each evaporator is higher than its saturated temperature and after flashing in distributor liquid feed achieves its saturated temperature. Therefore heat to the falling film will spend to phase change. In practice there is a small accumulation of energy due to the boiling point rise, however, this term can be neglected for milk.

Table 1. Specifications of the simulated industrial evaporator effects

<table>
<thead>
<tr>
<th>Evaporator effects</th>
<th>Number of</th>
<th>Diameter of tubes</th>
<th>Length of</th>
<th>Diameter of</th>
</tr>
</thead>
</table>

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### Table

<table>
<thead>
<tr>
<th>Effect</th>
<th>tubes (mm)</th>
<th></th>
<th>tubes (m)</th>
<th>effect (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First effect</td>
<td>249</td>
<td>49</td>
<td>51</td>
<td>14</td>
</tr>
<tr>
<td>First part of second effect</td>
<td>139</td>
<td>49</td>
<td>51</td>
<td>14</td>
</tr>
<tr>
<td>Second part of second effect</td>
<td>110</td>
<td>49</td>
<td>51</td>
<td>14</td>
</tr>
<tr>
<td>Third effect</td>
<td>64</td>
<td>49</td>
<td>51</td>
<td>14</td>
</tr>
<tr>
<td>Fourth effect</td>
<td>64</td>
<td>49</td>
<td>51</td>
<td>14</td>
</tr>
</tbody>
</table>

### Figure 1

![Process flow diagram of the industrial evaporator plant](image)

**Lumped model**

In this section the differential and algebraic equations which are used in lumped model are described briefly. These equations are derived by using material and energy balances for falling film inside and condensing film outside the tubes.

**Material balance of the falling film inside the tube:**

\[
\frac{dM}{dt} = G_{Li} - G_{Lo} - W
\]

Where \(G_{Li}\), \(G_{Lo}\), and \(W\) are inlet mass flow rate of milk, outlet mass flow rate of milk and mass flow rate of produced vapor, respectively. \(M\) is the mass of liquid film inside the tube and can be defined as

\[
M = \tau_{\text{avg}} \cdot G_{\text{avg}}
\]

\(\tau_{\text{avg}}\) is the average residence time and assumed constant. It can be calculated as

\[
\tau_{\text{avg}} = \frac{L}{v_{\text{avg}}}
\]
$L$ is the length of tube. The average values of liquid film velocity ($v_{zav}$) and liquid film mass flow rate ($G_{zav}$) can be defined as:

$$v_{zav} = \alpha_1 v + (1 - \alpha_1) v_w$$

$$G_{zav} = \alpha_2 G + (1 - \alpha_2) G_{L0}$$

$\alpha_1$ and $\alpha_2$ are tuning parameters between zero and one.

Material balance of the falling film solid contents:

$$\frac{d(M \cdot c_w)}{dt} = G_{L0} c - G_{zav} c_w$$

The average values of solid mass fraction ($c_{zav}$) can be defined as:

$$c_{zav} = \alpha_3 c + (1 - \alpha_3) c_w$$

$\alpha_3$ is a tuning parameter between zero and one.

Energy balance of the falling film inside the tube:

$$\frac{dE}{dt} \equiv 0 \Rightarrow Q_{L0} = W\lambda$$

From the above equation, the mass of evaporated water can be calculated as:

$$W = \frac{Q_{L0}}{\lambda}$$

Where $\lambda$ is the latent heat of water and $Q_{L0}$ is the rate of heat transferred to the falling film.

Material balance of the condensate film outside the tube:

$$\frac{dM_c}{dt} = G_{cz} - G_{czw} + W_c$$

In the same as the falling film, the mass of condensate outside the tube ($M_c$) can be calculated as follows:

$$M_c = \tau_{zav} \cdot G_{zav}$$

$$\tau_{zav} = \frac{L}{v_{zav}}$$

Where

$$v_{zav} = \alpha_4 v_{zav} + (1 - \alpha_4) v_w$$

$$G_{zav} = \alpha_5 G_{zav} + (1 - \alpha_5) G_{cz}$$

$\alpha_4$ and $\alpha_5$ are tuning parameters between zero and one.

Energy balance of the condensate film outside the tube:

$$\frac{dE_c}{dt} \equiv 0 \Rightarrow Q_{L0} = W_c \lambda_c \Rightarrow W_c = \frac{Q_{L0}}{\lambda_c}$$

Where $W_c$ is the mass flow rate of condensate and $Q_{L0}$ is the rate of heat transfer from condensate.

Energy balance on the wall of the tube:

It is assumed that there is no heat loss. In the current study the tube wall is made of stainless steel with thickness of 2 mm. Thus, it is possible to solve energy balance of the wall assuming that equilibrium is achieved instantaneously. With the above consideration, all heat produced by vapor condensation outside the tube lead to evaporation of water inside the tube. Therefore

$$Q_{L0} = Q_{L0} = Q_i$$

Where

$$Q_i = U_i A (T_c - T)$$

$$U_i = \frac{1}{h_i} + \frac{d_i \ln(d_i/l_d)}{2k_w} + \frac{d_i}{h_d} \frac{1}{h_i}$$

Distributed model
In this paper the distributed model presented by Stefanov and Hoo [4, 5] was used for milk powder plant. Evaporators of the current study have tubes instead of plates. Thus some small changes are introduced in Stefanov and Hoo's model. Also in the current study the film thickness is used for calculating the film velocity.

Material balance of the falling film inside the tube:
\[
\frac{\partial G}{\partial t} + v_\tau \frac{\partial G}{\partial z} = -v_\tau W', \quad W' = \frac{W}{\Delta z} \tag{19}
\]

Material balance of the falling film solid contents:
\[
\frac{\partial c}{\partial t} + v_\tau \frac{\partial c}{\partial z} = eW'v_\tau c, \quad W' = \frac{W}{\Delta z} \tag{20}
\]

Energy balance of the falling film inside the tube:
\[
\frac{dE}{dt} \equiv 0 \Rightarrow Q = W\lambda \tag{21}
\]

Where
\[
Q = \pi d\Delta z h(T_{wi} - T) \Rightarrow W = \frac{\pi d\Delta z h(T_{wi} - T)}{\lambda} \tag{22}
\]

Material balance of the condensate film outside the tube:
\[
\frac{\partial G}{\partial t} + v_\tau \frac{\partial G}{\partial z} = -v_\tau W_c, \quad W' = \frac{W}{\Delta z} \tag{23}
\]

Energy balance of the condensate film outside the tube:
\[
\frac{dE_c}{dt} \equiv 0 \Rightarrow Q_c = W_c\lambda \tag{24}
\]

Where
\[
Q_c = \pi d_c\Delta z h_c(T_{wi} - T_{wi}) \Rightarrow W_c = \frac{\pi d_c\Delta z h_c(T_c - T_{wi})}{\lambda} \tag{25}
\]

Energy balance on the wall of the tube:
In the same as lumped model we have
\[
h_d(T_{wi} - T) = h_d(T_c - T_{wi}) = 2k_r(T_{wi} - T_{wi}) \ln(d_r/d) \tag{26}
\]

In addition to the above equations, modeling the evaporation plant requires other algebraic and differential equations for pressure dynamics, flashing process, thermo compressor and control valve. These equations will be explained in the full paper version.

Numerical solution of models
Ordinary differential equations (ODEs) in the lumped model are solved using the variable time step size method defined by the function ode113s in MATLAB 7.1.
To solve the distributed model, the partial derivatives of location (z dimension) were approximated by backward finite difference method using equal space grids in the length of the tube. The resulted set of ordinary differential equations is solved by the function ode113s in MATLAB 7.1.

Results and discussion
In this section the dynamic performance of lumped and distributed models are compared to industrial plant data. There are specific difficulties related to experimentation on the industrial plant, also the number of sensors are restricted. However, in this study we were able to measure the required variables in the evaporation plant such as pressure of live steam to the thermo compressor, feed volume flow rate, product mass flow rate, shell side temperature of each effect,
and product density. These variables recorded at 30 seconds intervals. Thus we can introduce changes of input variables to the models and also compare dynamic behavior of the plant with output data of lumped and distributed models. Figures (17) to (20) show the actual steam temperature of each effect and outputs of lumped and distributed models. These results are pretty coincident when the responses of the two models are compared qualitatively with industrial data, but in precise look, predicted results of distributed models after second load are slightly better than lumped model. It can be due to omitting some details in calculating condensate and vapor production in lumped model. Figure (21) shows the density of the outlet flow. Dynamic behaviors of the two models are slightly similar and there are no significant differences between the two models.

In the case of time needed for calculating, with a given data processing system, the time of calculations for the distributed model is twice as much as that of calculations for lumped model. It is reasonable because the equations of the distributed model are complicated and solving them needs more time.

![Figure 17. Steam temperature of the first effect](image1)

![Figure 18. Steam temperature of the second effect](image2)
Conclusion
Lumped and distributed models were developed for multi-effect falling-film evaporation plant. Performance of distributed model in prediction temperature of each effect is better than the lumped model. But results of these models in the case of product density have similarity to dynamic behavior. Although the distributed model needs more time than the lumped model for calculation, it is more reliable for dynamic simulation and also the multi-variable control analysis.

References