Effects of slip and Marangoni convection on single fuel droplet heat-up in the presence of thermal radiation

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Abstract

Slip and Marangoni convection effects on an isolated fuel droplet heat up process are numerically studied in the presence of thermal radiation. For small droplets, when the droplet size becomes comparable to the mean free paths of the surrounding gas molecules, the continuum hypothesis breaks down and it is important to account for the gas rarefaction effects including velocity slip and temperature jump at the gas-liquid interface. Gas phase velocity slip at the liquid interface reduces the momentum transfer to the liquid phase, while temperature jump at the interface acts as a thermal contact resistance, and therefore, both effects reduce the heat transfer to the fuel drop. In addition, the variation of surface tension along the surface of the fuel droplet, which causes the well-known Marangoni convection, is taken into account in the presence of thermal radiation which can play an important role in the heat up process of fuel drops. It is shown that the presence of thermal radiation in some fuels leads to a more uniform drop surface temperature and therefore, opposing the Marangoni convection effects.

Keyword: Slip, Marangoni convection, radiation, droplet heating, temperature jump

1. Introduction

Accurate modeling of fuel droplets heating-up and vaporization is an essential element to many practical engineering applications including all the combusting systems, diesel engines, spray driers, spray cooling and so on. The complexity of considering all of the involving effects related to the evaporation and combustion of liquid drops at the same time makes it desirable to perform a parametric study in more simple yet realistic cases to be able to identify the true effects of each relevant parameter. Therefore, in this paper the rarefaction, thermal radiation and Marangoni effects have been examined on the heat-up process of a single fuel droplet, while the evaporation and combustion processes have been remained for future studies.

In 1877 the basis of the theory of evaporation of droplets in a gaseous medium was laid by Maxwell [1]. Since then, according to the widespread use of injecting processes, extensive work on this field of science has been done by many researchers [2]. The extensive literature reviews by Dwyer [3] and Sirignano [4] demonstrate the abundance of investigations analyzing droplet evaporation and combustion phenomena. Many different effects on isolated droplet heat-up and evaporation [5], such as thermo capillary convection [6], variation of free stream velocity [7], transients and variable properties [8], thermal radiation [9,10] have been studied. However, despite the rich literature in this field, rarefaction effects, which become important when the droplet diameter becomes comparable to the mean free path of the surrounding gas molecules have not been considered yet. Furthermore, previous studies related to the Marangoni convection [11, 12] is not accompanied with the thermal radiation effects. Since Marangoni effects are interfacial phenomena provoked by surface tension gradients that might be induced by interface gradients in temperature, concentration and surface charge, thermal radiation absorption in a semitransparent medium such as fuel droplet may affect the interface temperature gradients and thus the Marangoni effects.

One of the main difficulties in predicting fluid transport in micron-sized devices lies in the fact that the continuum hypothesis breaks down because of the very small length scales involved. The mean free path of air molecules at standard temperature and pressure is approximately 70 nm [13]. Typical diameter of the droplets injected in many engineering applications is in the range of 1 to100 microns. Consequently, the ratio of the mean free path of the molecules to the characteristic length scale of a micron sized droplet can be appreciable. This ratio is referred to as the Knudsen number and as it increases, the momentum transfer starts to be affected by the discrete molecular composition of the fluid; in other words the gas begins to exhibit non-continuum flow effects. For Knudsen numbers in the range $0.001 < Kn < 0.1$, which is called slip flow regime, it is well established that the non-continuum effects can be taken into account by employing standard Navier-stokes equations along with modified boundary conditions as will be discussed later.

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2. Governing equations

Governing equations in the integral form and in a cylindrical coordinates system are as follow:

$$\frac{\partial}{\partial t}\iiint_{\forall} r\, d\forall + \iint_{A} r\vec{V}.\vec{n}dA = 0 \tag{1}$$

$$\iiint_{\forall} r\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]d\forall = -\iint_{A} pdA_x + \iint_{A} m\vec{\nabla}u.dA \tag{2}$$

$$\iiint_{\forall} r\left[\frac{\partial u}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]d\forall = -\iint_{A} pdA_y + \iint_{A} m\vec{\nabla}v.dA - \iiint_{\forall}\frac{t_{qg}}{y}d\forall \tag{3}$$

$$\frac{\partial}{\partial t}\iiint_{\forall} rc_pT\, d\forall + \iint_{A} rc_p(\vec{V}.\vec{\nabla}T)d\forall = -\iint_{A} k\vec{\nabla}T.\vec{n}dA \tag{4}$$

3. Numerical method

Governing equations are integrated over the corresponding control volumes upon transformation into a generalized non orthogonal coordinate system. One of the main difficulties in dealing with low Mach number incompressible flows is the lack of a time independent equation for the pressure. The numerical method used here is based on the calculation of velocity field using a guessed or old pressure field. The velocity field obtained from momentum equations does not necessarily satisfy the continuity equation so a correction which is the form of the gradient of a scalar potential in introduced. The numerical scheme is second order in space and time.

4. Interface and Boundary Condition

Boundary conditions and the grid distribution are shown in Figure 1. Numerical experimentations indicate that a mesh with 61x71 grid points in the circumferential and radial direction respectively produce reasonable grid independent results. Interface condition is as follows:

4.1. Interface condition

In slip flow regime, gas velocity at the interface is no longer equal to the liquid velocity. The slip boundary condition proposed by Maxwell which is the first order slip velocity for a single atomic ideal gas is given by [14]:

$$u_{qg} = u_{ql} + \left(\frac{2-s_v}{s_v}\right)\left(\frac{Kn}{D}\right)\left(\frac{\partial u_q}{\partial y}\right) \tag{5}$$

Where u_{qg} and u_{ql} are gas and liquid tangential velocity at the interface, *D* is the droplet diameter, *y* is the direction normal to the interface and σ is the tangential momentum accommodation coefficient (TMAC) to account for the momentum retained by the reflected gas molecules [15]. Similarly, there exists a temperature difference between gas and liquid temperatures at the interface, which is proportional to the local temperature gradient normal to the interface. Smoluchowski [16] defined slip temperature as:

$$T_g = T_l + \left(\frac{2g}{g+1}\right)\left(\frac{2-s_T}{s_T}\right)\left(\frac{Kn}{D\,Pr}\right)\left(\frac{\partial T}{\partial y}\right) \tag{6}$$

Where T_g and T_l are gas and liquid temperature at the interface, respectively.

4.2 Marangoni Convection

Surface tension of liquids varies with temperature, and therefore, any temperature gradients along the interface lead to surface tension gradients, which in turn affect the internal circulation of the liquid drop known as Marangoni convection. The tangential stress balance equation at the interface is:

$$(\bar{\bar{t}}_l.\vec{n}).\vec{t} - (\bar{\bar{t}}_g.\vec{n}).\vec{t} = \frac{\Delta s}{\Delta s} = \frac{\partial s}{\partial T}.\frac{\Delta T}{\Delta s} \tag{7}$$

where Δs is the distance between surface nodes, $\vec{n}$ the surface normal vector, $\vec{t}$ the surface tangential vector, T surface temperature, s surface tension, and subscripts *l* and *g* refer to the liquid and gas phases respectively.

5. Thermal radiation model

The model used for calculating thermal radiation absorption is an efficient simplified model suggested in [10]. It is based on the assumptions that droplets are spherical and semi-transparent, the irradiation is spherically symmetric and the geometric optics approximation is valid, i.e. the size parameter is considerably greater than 1. Optical properties are chosen similar to those used in [10] for n-dodecane. The simplification is performed by introducing a normalized function w_l as:

$$w_l(r) = \frac{p_l(r)}{\frac{3}{r_d^3}\int_0^{r_d} p_l(r) r^2 dr} \tag{8}$$

where r_d is the droplet radius and $p_l(r)$ is the spectral power of radiation absorbed per unit volume inside the droplet. Based on Dombrovsky's model [10], radiation power absorbed per unit volume inside the droplet is as follows:

$$P(r) = \frac{0.75}{r_d}\int_0^{\infty} Q_a w_l(r) I_l^{0(ext)} d\, l \tag{9}$$

where Q_a is the efficiency factor of absorption [10]. If the external thermal radiation is that of a black body at temperature T_{ext} then $I_l^{0(ext)} = 4p B_l(T_{ext})$ and B_l is the Planck function. As follows from eq. (9) the problem of the approximate calculation of the radiation power absorbed per unit volume inside droplets reduces to the problem of finding an approximation for $w_l(r)$. Following [10] $w_l(r)$ can be approximated as:

$$w_l(\bar{r}) = \frac{[1 - m^* \Theta(\bar{r} - 1/n)](\bar{r}^2 + \bar{g})}{[0.6(1 - m_c^5) - m_c^3/n^2] + \bar{g}(1 - m_c^3)} \tag{10}$$

where $\bar{g} = 1.5/t_0^2 - 0.6/n^2$, $t_0 = a_l r_d$, $\bar{r} = r/r_d$, n is the refractive index, a_l is the absorption coefficient and Θ is the Heaviside unit step function. If $t_0 > \sqrt{2.5n}$ the following approximation is used [10]:

$$w_l(\bar{r}) = \frac{x^2 t_0^3}{3} \frac{\exp[-x(t_0 - t)]}{t_0(x t_0 - 2) + (2/x)[1 - \exp(-x t_0)]} \tag{11}$$

Where $x = 2/(1 + m_c)$ and $m_c = \sqrt{1 - (1/n^2)}$.

It must be emphasized that the effect of thermal radiation heat transfer is only important for fuels with high absorption coefficients [9].

Results and discussion

The effects of velocity slip and temperature jump are considered separately to identify their relative importance regarding to the no-slip case. Droplet diameter is 100 microns with the initial temperature of 300 K injected into a hot stream of air with the temperature of 1000 K. The fuel is n-Dodecane ($C_{12}H_{26}$). The fuel and air conductivities are 0.1405 and 26.3e-3 (W/m.K) respectively. All the results are presented for the viscosity ratio of 55 (i.e. $m_l / m_g = 55$) for validation purposes. Fuel and air densities are taken as 744.11 and 1.29 (Kg/m^3), respectively.

Drag coefficient (Cd) for flow over a solid sphere at three moderate Reynolds numbers and for a wide range of Knudsen numbers are shown in figure (2). The results compare well with the available results in literature, when no-slip condition (Kn = 0) is applied. Slip in velocity decreases the velocity gradient normal to the interface of liquid and gas phase, and therefore decreases the shear stress at the interface, which in turn decreases the drag coefficient. An important effect associated with slip velocity is the reduction in droplet convection since the internal circulation of the droplet is induced by the shear stresses at the interface, which can decrease dramatically in the presence of high velocity slip.

One of the important parameters in predicting the life time of an evaporating droplet is its surface temperature which determines the vapor pressure at the interface related to the evaporating rate of the droplet. Therefore, in the present study the effects of slip velocity, temperature jump, thermal radiation, and Marangoni convection on the temperature distribution inside the droplet and along the surface will be considered.

In figure (3) the temperature distribution inside the droplet for the no-slip condition and in the presence of velocity slip with Kn=0.01 for the initial Reynolds number of 50 for a moving droplet is plotted. It must be emphasized that for this case temperature jump has not been applied and the droplet slows down due to the drag applied by the surrounding gas phase. Clearly, the heat penetration into the droplet in the presence of slip is much less than that in the no slip condition as can be seen in figure (3). The heat transfer to the drop is reduced further if temperature jump is also applied at the interface, since temperature jump acts similar to a contact resistance as will be discussed later. Results are calculated for the initial Reynolds number of 50, which is common in many applications including diesel engines [17].

Now the effects of thermal radiation on the droplet heat-up process is studied in the absence of all other effects. Results of calculations of the normalized radiation power absorbed in a unit volume inside droplets for n-dodecane for external radiation temperature of 1500K are shown in figure (4). As can be seen in this figure, by increasing droplet radius, surface absorption increases. The surface temperature variation in the presence of thermal radiation is plotted in figure (5). As expected, due to relatively high absorption coefficient of the specified diesel fuel, thermal radiation has increased the surface temperature and also the mean temperature.

The effect of Marangoni convection on droplet surface temperature and the temperature distribution inside the droplet, when other effects are neglected is shown in figure (6). In general Marangoni convection opposes strong surface temperature variations and tries to generate a more uniform surface temperature. At the front section of the droplet, between 0-115 degrees, the temperature increases along the surface and thus the Marangoni convection enforces the internal circulation, while in the rear section there is a negative temperature gradient along the surface which opposes the internal circulation produced and maintained by the imposed gas phase shear stresses. When the radiation heat transfer is added surface temperature increases slightly and compensates for the reduction temperature in the front section due to the Marangoni effects as shown in figure (7).

Finally, slip velocity and temperature Jump, which acts as a thermal resistance, decreases the rate of heat transfer most dramatically as shown in figure (8). Clearly, the rarefaction effects as compared to other effects studied here can more significantly change the heating rate of the droplet and also the temperature distribution inside it.

Conclusions

Rarefaction has dramatic effects on the heating process of a micron sized droplet. Slip velocity reduces the internal circulation and changes the thermal distribution inside the droplet, while temperature jump weakens the rate of heat transfer to the droplet. Marangoni convection which affects the internal circulation, also changes the surface temperature but the presence of thermal radiation can compensate this effect.

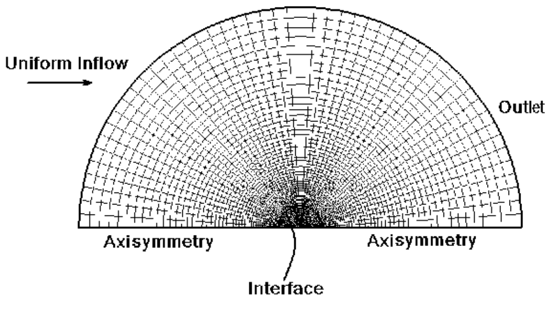

Figure 1- Geometry, grid distribution and applied boundary conditions.

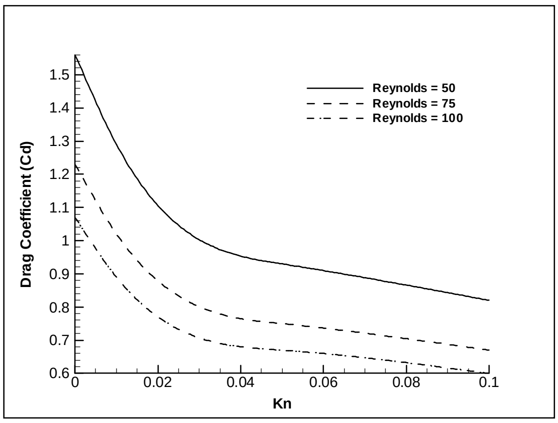

Figure 2- Drag coefficient variations as a function of Knudsen number at different Reynolds numbers.

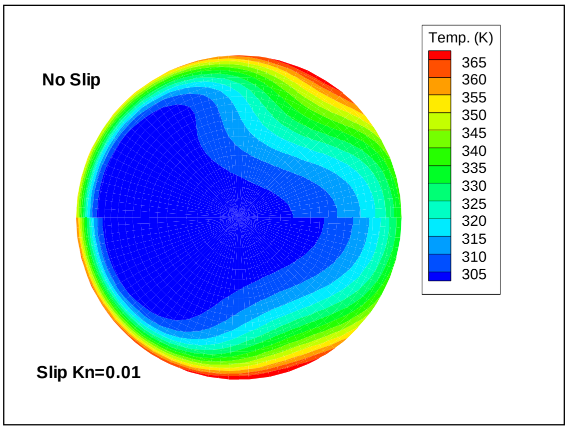

Figure 3- Temperature distribution inside the droplet, top half no-slip, down half Kn=0.01.

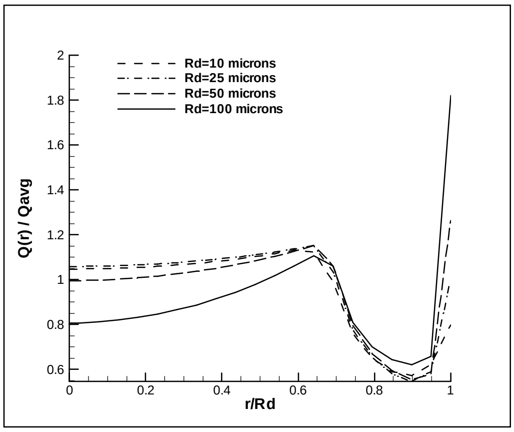

Figure 4- Normalized absorbed radiative power in a n-dodecane droplet

$(R_d = 10m, Q_{avg} = 13.74*10^9)(R_d = 25m, Q_{avg} = 9.9*10^9)$

$(R_d = 50m, Q_{avg} = 7.26*10^9)(R_d = 100m, Q_{avg} = 4.81*10^9)$

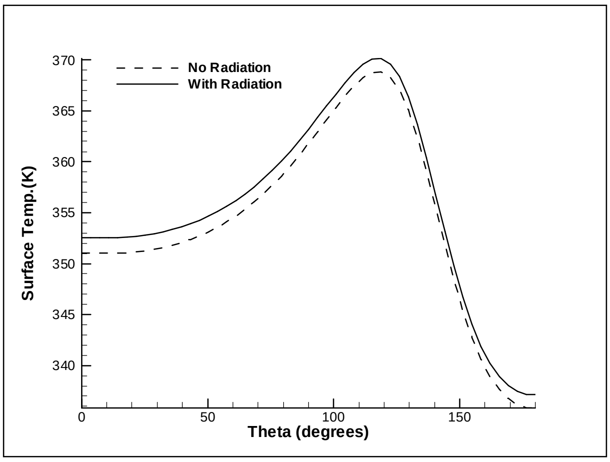

Figure 5- Droplet surface temperature variations with and without thermal radiation.

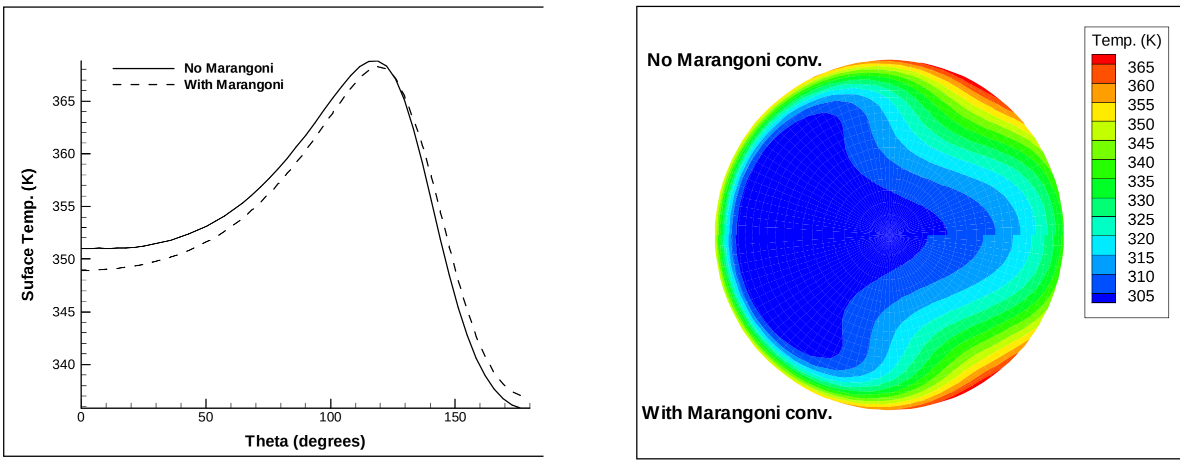

Figure 6- Droplet surface temperature variations and temperature distribution inside the droplet for cases with (down half) and without Marangoni convection (top half).

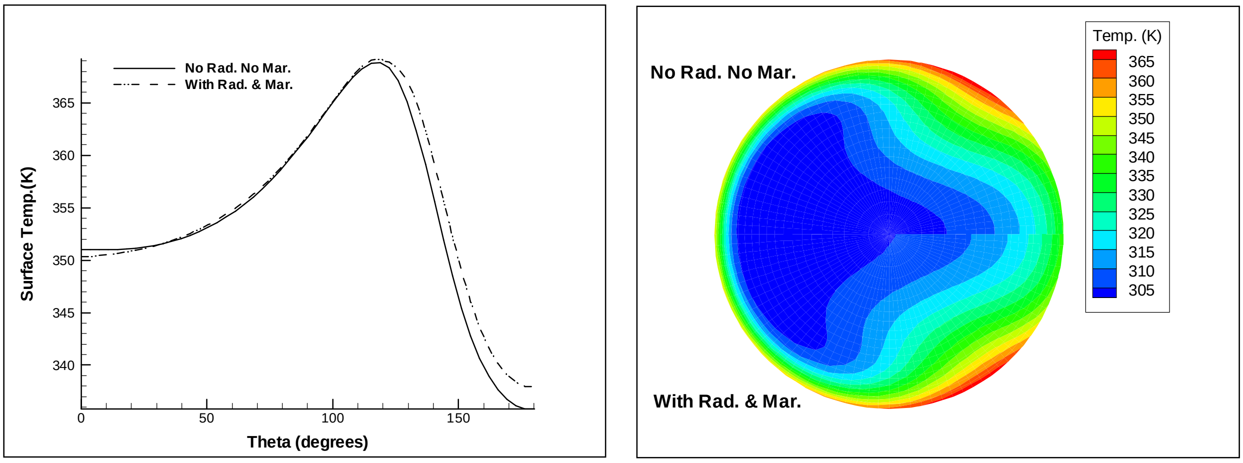

Figure 7- Droplet surface temperature variations and temperature contours inside the droplet for cases with (down half) and without Marangoni convection and thermal radiation (top half).

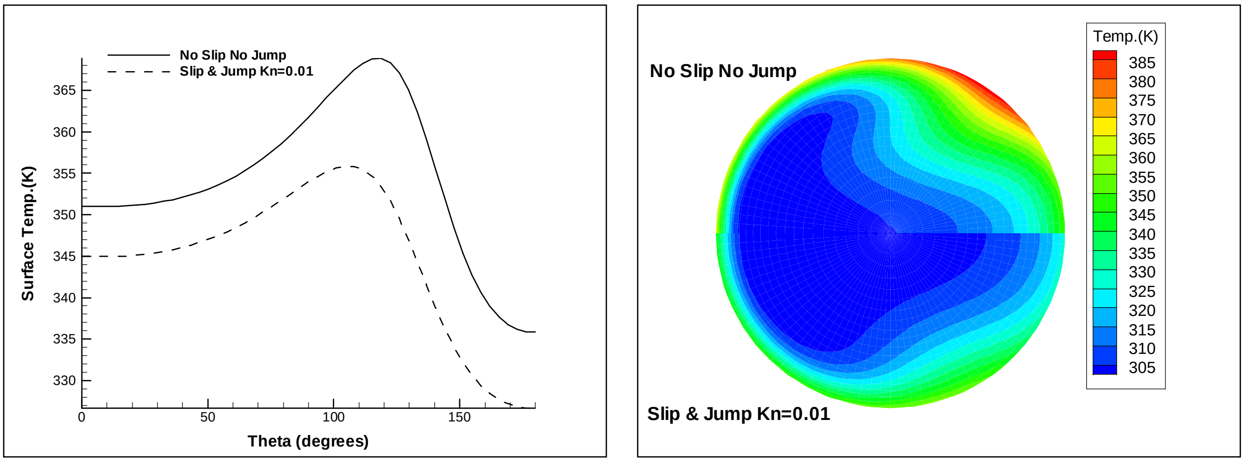

Figure 8-Droplet surface temperature variations and temperature distribution inside the droplet for cases with Kn = 0 and Kn=0.01

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