Takagi–Sugeno fuzzy modelling and parallel distributed compensation control of conducting polymer actuators

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Abstract: Conducting polymer actuators are used in a diverse range of applications including biomimetic robots and biomedical devices. In comparison to robotic joints, they do not have friction or backlash, but on the other hand, they have complicated electro-chemo-mechanical dynamics which makes modelling and control of the actuator really difficult. In addition they also have the disadvantages of creep, hysteresis, and highly uncertain and time-varying dynamics. In this paper a Takagi–Sugeno (T–S) fuzzy model is used to represent the uncertain dynamics of the actuator, and the resulted fuzzy model is validated using experimental data. A system that consists of fuzzy state feedback to a PI controller is designed on the basis of the obtained T–S fuzzy model using the parallel distributed compensation scheme. The sufficient conditions for the existence of such a controller are derived in terms of linear matrix inequalities. The obtained results show that the designed controller can achieve a good control performance despite the existence of uncertain actuator dynamics.

Keywords: conducting polymer actuators, polypyrrole, Takagi–Sugeno fuzzy modelling, proportional-integral controller, parallel distributed compensation

1 INTRODUCTION

There is an increasing requirement for a new generation of actuators for use in devices such as artificial organs, micro-robots, human-like robots, and medical applications. Considerable effort has been expended on developing new actuators such as shape-memory-alloys-based actuators, piezoelectric actuators, magnetostrictive actuators, contractile polymer actuators, and electrostatic actuators [1, 2]. A comparison of these actuators indicates that conducting polymer actuators have the best overall characteristics [2, 3].

The main process which is responsible for volumetric change and the resulted actuation ability of the conducting polymer actuators is reduction/oxidation (RedOx). By changing the fabrication process, various actuator configurations can be obtained including: linear extenders, bilayer benders, and trilayer benders [3–6]. By applying a voltage to the actuator, the polypyrrole (PPy) layer on the anode side is oxidized while that on the cathode side is reduced. Ions can transfer inside the conducting polymer actuator via one of two mechanisms: diffusion or drift [7]. The diffusive elastic metal (DEM) model which was first proposed in 2000 the main model is still used to describe the actuation process in conducting polymer actuators [7]. Several assumptions are needed to achieve the DEM model including:

(a) the electrical and mechanical parameters of the model are time-invariant;
(b) there is no coupling between the mechanical and electrical models;
(c) the charge-to-strain ratio is linear and unidirectional;
(d) there is no degradation in the electrical or mechanical models;
(e) the actuator is isothermal.

The dynamics of an actuator are, however, highly uncertain, and both electrical and mechanical degradation is inevitable during the actuator’s lifecycle.

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Also, the continuum structure of the DEM model is not suitable from a control perspective. The reticulated diffusion (RD) model was proposed by Bowers in 2002 [8]. This model uses a reticulated network of linear circuit elements. The main advantage of the RD model over the DEM model is that it can be represented in a state space format and it is suitable for linear system analysis techniques; however, it cannot take into account system uncertainties based on its linear time-invariant (LTI) structure. In previous work, the present authors used the Golubev method [9] to build a suitable model for control of the actuator [3]. By taking into account the effects on uncertainties such as variation of the resistance and diffusion coefficient in the modelling, it was possible to represent the dynamics of the actuator with a family of third-order LTI systems. However, interactions between these linear systems was not considered in [3] and this is the starting point of the current paper. In order to solve this problem a Takagi–Sugeno (T–S) fuzzy model is proposed which can define the relation between local linear systems, and therefore accurately predict the actuator’s behaviour under variation of the actuator’s parameters. The application of a proportional–integral–derivative (PID) controller for a PPy actuator based on a first-order model was presented in [10]. PID and adaptive control approaches based on a first-order empirical model were demonstrated in [8]. Robust control qualitative robust feedback theory was used to control a PPy actuator using a third-order model in [3]. In this paper fuzzy state feedback combined with a PI controller is used for the obtained T–S fuzzy model by using parallel distributed compensation (PDC).

The reminder of the paper is organized as follows. First, the classical model of the actuator will be reviewed briefly. Then, the experimental data will be presented. A suitable T–S fuzzy model which can take into account variation of the actuator’s parameter will be obtained. Finally, fuzzy state feedback combined with a PI controller based on the PDC method will be designed.

2 ELECTRO-CHEMO-MECHANICAL MODELLING

The electro-chemo-mechanical model is comprised of two parts, namely an electrochemical model and an electromechanical model.

2.1 Electrochemical modelling

The electrochemical model relates the input voltage and chemical RedOx reaction inside the PPy actuators. Figure 1 depicts the electrical admittance model. Based on the DEM model, transport of ions within the polymer is only caused by diffusion [7]. Using Fig. 1 and Kirchhoff’s voltage law it is possible to write

\[ I(s) = I_c(s) + I_D(s) \]  
\[ V(s) = I(s)R + \frac{1}{sC} I_c(s) \]

where \( Z_D \) is the diffusion impedance, \( C \) denotes the capacitance of the double-layer capacitor, and \( R \) is the electrolyte and contact resistance. Then, based on Fig. 2 and Fick’s law of diffusion, the diffusion current is

\[ i_D(t) = -FAD\frac{\partial c(x, t)}{\partial x} \bigg|_{x=0} \]

where \( A \) is the surface area of the polymer, \( F \) is the Faraday constant, \( D \) is the diffusion coefficient, \( h \) is

![Fig. 1](image1.png) Description of diffusion and double-layer charging and its equivalent electrical circuit

![Fig. 2](image2.png) Description of frame assignment for diffusion
the thickness of the PPy layer, and \( c \) is the concentration of the ions.

The current flowing in the double-layer capacitor is

\[
i_c(t) = FA_0 \frac{\partial c(x, t)}{\partial t}\bigg|_{x=0} \tag{4}
\]

where \( \delta \) is the thickness of the double-layer capacitor. The diffusion equation is

\[
\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}, \text{ for } 0 < x < h \tag{5}
\]

Finally, the boundary condition is

\[
\frac{\partial c(x, t)}{\partial x}\bigg|_{x=h} = 0 \tag{6}
\]

Now based on equations (1), (2), (3), (4), (5), and (6), it can be shown that the admittance model \((Y(s) = I(s)/V(s))\) of the conducting polymer [7] is

\[
Y(s) = \frac{s[(\sqrt{D}/\delta) \tanh(h\sqrt{s/D}) + \sqrt{s}]}{(\sqrt{s/C} + Rs^3/2 + R(\sqrt{D}/\delta)s \tanh(h\sqrt{s/D}))} \tag{7}
\]

### 2.2 Electromechanical modelling

The electromechanical model relates the input voltage and displacement of the PPy actuators. The relationship between the induced in-plane electrochemical strain \((\varepsilon_c)\) and the density of the transferred charges \((\rho)\) is [7, 11]

\[
\frac{\varepsilon_c}{V(s)} = \frac{1}{sR + \{1/C[1+(\sqrt{D}/\delta)\tanh(h\sqrt{s/D})]\}} \sum_{n=0}^{\infty} \frac{1}{(s + \pi^2(2n+1)^2D(2h)^{-2})} \tag{14}
\]

where \( \varepsilon_c = \varepsilon_R \rho \)\)

\[
\sigma_c = \varepsilon_R E_{PPy} \rho \tag{9}
\]

where \( E_{PPy} \) is the Young’s modulus of PPy, and \( \rho \) can be obtained in the Laplace domain as [7, 10]

\[
\rho(s) = \frac{I(s)}{sWLh} \tag{10}
\]

where \( W \) is the width and \( L \) the length of the PPy layer. It is experimentally shown in [12] that the relation between the electrochemical strain and charge stored in the PPy actuator is a unidirectional phenomenon, thus an applied mechanical load will not affect the electrical charge in the actuator. According to Fig. 3, for the case where a mechanical load is applied to the actuator the total displacement of the actuator is the summation of the electrochemical and mechanical displacements \((y_t = y_c + y_m)\).

The mechanical displacement \( y_m \) is caused by a load which can be obtained as

\[
y_m = \frac{mgL}{WhE_{PPy}} \tag{11}
\]

where \( m \) is the mass of the applied load.

By combining equations (7), (8), and (10) it is possible to obtain a model that relates the input voltage \((V)\) and the output chemical displacement \((y_c)\) [7, 10]

\[
y_c(s) = \frac{\varepsilon_R}{V(s)} \left(sR + \{1/C[1+(\sqrt{D}/\delta)\tanh(h\sqrt{s/D})]\} \sum_{n=0}^{\infty} \frac{1}{(s + \pi^2(2n+1)^2D(2h)^{-2})}\right) \tag{12}
\]

where

\[
\varepsilon_R = \frac{1}{WLh} \tag{13}
\]

By replacing the tanh term by its equivalent series in equation (12) the actuator model becomes [7]

![Fig. 3 Description of the frame assignment for displacement of the actuator](image-url)
3 EXPERIMENTAL STUDIES

PPy was tested using an electrolyte that was a 0.1 M solution of tetraethylammonium hexafluorophosphate in propylene carbonate. The polymer film was held in the test fixture with clamps at both ends. The reference electrode used in the experiment was Ag/AgClO4.

Mechanical loading was created using a voice coil actuator (Bruel & Kjaer Minishaker 4810). For the purpose of isotonic testing, a force transducer feedback control was used. The position sensor was a photodiode (PPS-DL700-7PCBA) with a resolution of 250 nm. Figure 4 depicts the testing equipment. Typical values of physical parameters are presented in Table 1 [8].

3.1 Isotonic testing based on voltage input

The voltage was increased in steps of 0.1 V starting from about ~0.5 V versus Ag/AgClO4 which is the potential of the zero charge. Subsequent to each potential step the current was permitted to drop down to 30 μA before the next step was applied. This value has been reported to allow capture of a considerable section of the time response of the polymer electrical domain [8]. Figure 5 shows the potential input and Fig. 6 depicts the current output of the actuator.

4 T–S FUZZY MODELLING

Fuzzy logic was first proposed by Zadeh [13] in 1965. Nowadays, it is widely used in industrial applications. Fuzzy logic can model the non-linear relationship between inputs and outputs. It can simulate the operator's behaviour without use of a mathematical model [14]. It is a method that transfers human knowledge into mathematics. Incomplete, vague and/or inaccurate expert knowledge is formulated with the aid of if–then rules. Each rule explains a non-linear relationship between inputs and outputs.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Values of physical parameters</th>
</tr>
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<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>$D$</td>
<td>$1 \times 10^{-12}$ m$^2$/s</td>
</tr>
<tr>
<td>$h$</td>
<td>19 μm</td>
</tr>
<tr>
<td>$R$</td>
<td>800 Ω</td>
</tr>
<tr>
<td>$δ$</td>
<td>25 nm</td>
</tr>
<tr>
<td>$C$</td>
<td>$5.33 \times 10^{-5}$ F</td>
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</tbody>
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Fig. 4 Schematic of the experimental setup

![Fig. 4 Schematic of the experimental setup](image)

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Fig. 5 Input voltage applied to the PPy actuator

![Fig. 5 Input voltage applied to the PPy actuator](image)

Fig. 6 Experimental current output

![Fig. 6 Experimental current output](image)
All rules together define a linguistic model [15, 16]. The T–S fuzzy system is one of the most popular systems in model-based fuzzy control. It is described by fuzzy if-then rules that represent local linear input–output relations of a non-linear system. The T–S model is capable of approximating many real non-linear systems, e.g. mechanical systems, electrical systems, chemical systems, etc. Since it uses linear models in the consequent part, linear control theory can be applied for system analysis and design subsequently, based on the PDC approach [17]. The basic feature of T–S fuzzy modelling is to represent the local dynamics of a system with a linear model, and the overall fuzzy model is a combination of this linear model. One can represent the local linear systems as follows

\[
\begin{align*}
\dot{x}_i(t) &= Ax_i(t) + Bu(t) \\
y &= C_i x(t)
\end{align*}
\tag{15}
\]

where \( r \) is the number of selected points for linearization. We consider the following T–S fuzzy system with \( r \) plant rules that can be represented as

Plant rule \( i \):

If \( z_1 \) is \( A_1 \) and \( z_2 \) is \( A_2 \) and, ..., and \( z_p \) is \( A_p \), Then,

\[
\begin{align*}
\dot{x}_i(t) &= A_i x(t) + B_i u(t) \\
y &= C_i x(t)
\end{align*}
\tag{16}
\]

where \( r \) is the total number of rules, \( \tilde{Z} \) is the premise input vector, and \( A_p \) is a fuzzy set, then the fuzzy system can be given as

\[
\dot{x}(t) = \frac{\sum_{i=1}^{r} (A_i x(t) + B_i u(t)) \mu_i(z(t))}{\sum_{i=1}^{r} \mu_i(z(t))}
\tag{17}
\]

or

\[
\dot{x}(t) = \left( \sum_{i=1}^{r} A_i h_i(z(t)) \right) x(t) + \left( \sum_{i=1}^{r} B_i h_i(z(t)) \right) u(t)
\tag{18}
\]

where \( \mu_i(z(t)) \) is the fuzzy membership function and \( h \) can be defined as

\[
h^T = [h_1, \ldots, h_r] = \left[ \frac{1}{\sum_{i=1}^{r} h_i(z(t))} \right] [\mu_1, \ldots, \mu_r]
\tag{19}
\]

Since the term \( \tanh \) in equation (12) is not suitable for real-time control of the actuator it cannot take into account system uncertainties. In this paper a T–S fuzzy model is used for purpose of modelling the actuator. As shown in [3] a third-order model can accurately describe the actuation process. The experimental data shows that a LTI model based on the initial physical parameter of the actuator cannot accurately predict the behaviour of the actuator, thus based on observation of the experimental data three zones are considered for the actuation process. These zones which somehow indicate the variation of the physical parameter of the actuator are chosen as the premise of the proposed T–S fuzzy model. These zones are labelled the initial, middle, and final zone. A genetic algorithm (GA) is used to optimize the performance of the fuzzy system [18]. In particular the GA is used to tune the membership functions based on minimization of the mean least square error between the experimental data and the output of the fuzzy system. Corresponding membership functions for these zones are depicted in Fig. 7. For example the linear system in the initial zone is

\[
A_1 = \begin{bmatrix}
-0.918 & -0.087 & 0 \\
0.125 & 0 & 0 \\
0 & 0.125 & 0
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
0.0039 \\
0 \\
0
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
0 & 0.0061 & 6.68 \times 10^{-6}
\end{bmatrix}
\]

Since the proposed model is going to be used as a multi-purpose model and is therefore able to satisfy the rules needed to implement the PDC control approach, the PPy actuator dynamic can be controlled. This can be checked using the controllability test matrix \( \Phi_c \)

![Fig. 7 Membership function for the fuzzy zones](image-url)
Clearly the rank of $\Phi_c$ is three, thus the system is controllable.

A comparison of experimental data with the T–S fuzzy model and DEM model is shown in Fig. 8. This figure shows notable deviations of the DEM model from experimental results which increases as the time increases. This could be due to the time-varying nature of the conducting polymer actuator that is compensated using the proposed fuzzy model.

5 THE COMBINATION OF FUZZY STATE FEEDBACK AND A PI CONTROLLER USING THE PDC METHOD

Based on the PDC concept, the output of a fuzzy controller is used as the feedback of state variables.

In this paper a PI controller is added to the control structure in order to eliminate steady-state error.

Figure 9 shows a block diagram of the fuzzy state feedback combined with a PI controller. Equation (20) indicates the integral state of $q$ where $r$ is the desired input

$$\dot{q}(t) = r - y(t) = r - Cx(t)$$

By taking into account the integral state, the state space equation can be written as equations (21) and (22)

$$\begin{cases} \dot{x}(t) = [A & 0] x(t) + [B & 0] u(t) + [O & 1] r \\ \dot{q}(t) = [C & 0] [x(t) & q(t)] \end{cases}$$

$$y(t) = [C & 0] [x(t) & q(t)]$$

In order to check the controllability of the new system with an integral state it is possible to build the controllability matrix based on $A$ and $B$

$$A = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

Thus, it is possible to write

$$\begin{bmatrix} B & AB & \cdots & (A)^nB \\ 0 & -CB & -CAB & \cdots & -CA^{n-1}B \end{bmatrix}$$

where

$$\Phi_c = \begin{bmatrix} B & AB & \cdots & (A)^nB \end{bmatrix}$$

Therefore, the controllability matrix can be rewritten as
\[
\begin{bmatrix}
B & AB & \cdots & (A)^{n-1}B
\end{bmatrix} = \begin{bmatrix}
B & A\Phi_x
0 & -C\Phi_x
\end{bmatrix}
= \begin{bmatrix}
B & A
0 & -C
\end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix}
\]
\[
\begin{bmatrix}
\dot{x}(t) - \dot{x}(\infty)
\dot{q}(t) - \dot{q}(\infty)
\end{bmatrix} = \begin{bmatrix}
A & 0
-C & 0
\end{bmatrix} \begin{bmatrix} x(t) - x(\infty) \\
q(t) - q(\infty)
\end{bmatrix} + \begin{bmatrix}
B
0
\end{bmatrix} \begin{bmatrix} u(t) - u(\infty) 
\end{bmatrix}
\] (28)

By following the definitions it is possible to write
\[
x(t) - x(\infty) = x_e(t)
q(t) - q(\infty) = q_e(t)
u(t) - u(\infty) = u_e(t)
\]
\[
\begin{bmatrix}
\dot{x}_e(t) \\
\dot{q}_e(t)
\end{bmatrix} = \begin{bmatrix} A & 0 \\
-C & 0
\end{bmatrix} \begin{bmatrix} x_e(t) \\
q_e(t)
\end{bmatrix} + \begin{bmatrix} B \\
0
\end{bmatrix} u_e(t)
\] (29)

where
\[
u_e(t) = -Kx_e(t) + k_1q_e(t) = -K \begin{bmatrix} x_e(t) \\
q_e(t)
\end{bmatrix}
\] (30)

where
\[
\bar{K} = \begin{bmatrix} K & -k_1
\end{bmatrix}
\]

5.1 Stability analysis

As previously discussed the new system with integral state can be expressed as follows
\[
\dot{e}(t) = \left( \sum_{i=1}^{r} \bar{A}_i h_i \right) e(t) + \left( \sum_{i=1}^{r} \bar{B}_i h_i \right) u_e(t)
\] (31)

where
\[
e(t) = \begin{bmatrix} x_e(t) \\
q_e(t)
\end{bmatrix}
\]

By substituting equation (30) in equation (31) it is possible to write
\[
\dot{e}(t) = \left( \sum_{i=1}^{r} \bar{A}_i h_i + \left( \sum_{i=1}^{r} \bar{B}_i h_i \right) \left( \sum_{i=1}^{r} \bar{K}_i h_i \right) \right) e(t)
\] (32)

The Lyapunov direct method is used for the stability analysis [19]. In the first step a quadratic Lyapunov function is chosen
\[
V(e) = e^T P e
\]
where \( P \) is a ‘positive definite matrix’. If \( P \) is positive definite, then for all \( e \neq 0, e^TPe > 0 \). Consequently, \( V(e) > 0 \) and \( V(e) = 0 \) only if \( e = 0 \). Also, if \( |e| \to \infty \), then \( V(e) \to \infty \).

To show that the equilibrium \( e = 0 \) of the closed-loop system in equation (32) is globally asymptotically stable, it must be shown that \( V(e) < 0 \) for all \( e \). Observe that

\[
\dot{V}(e) = e^TPe + e^TPe
\]

Thus,

\[
\dot{V}(e) = e^TPe + e^TPe
\]

After some manipulations it is possible to write that

\[
\dot{V}(e) = e^T \left[ \sum_{i,j} \mu_i \mu_j \left( P(A_i + B_i K_j) + (A_i + B_i K_j)^TP \right) \right] e
\]

It is obvious that

\[
0 \leq \sum_{i,j} \mu_i \mu_j \leq 1
\]

Thus,

\[
\dot{V}(e) \leq \sum_{i,j} e^T \left( P(A_i + B_i K_j) + (A_i + B_i K_j)^TP \right) e
\]

Thus, to prove that the equilibrium \( e = 0 \) of equation (32) is globally asymptotically stable, we must find a single \( n \times n \) positive definite matrix \( P \) such that

\[
P(A_i + B_i K_j) + (A_i + B_i K_j)^TP < 0
\]

for all \( i = 1, 2, \ldots, r \) and \( j = 1, 2, \ldots, r \)

Define

\[
G_{ij} = A_i - B_i K_j
\]

Based on equation (37) the controlling problem is to find suitable gain matrix \( \hat{K} \) such that the closed-loop system remains stable. One can convert the above problem to a linear matrix inequality (LMI) problem. The Matlab LMI toolbox [20] can be used to solve this problem. Multiplying both sides of equation (37) by \( P^{-1} \), and by defining \( X = P^{-1} \) then the stability condition can be rewritten as

\[
-XA_i^T - A_iX + XG_{ij}^T B_i^T + B_i K_j X > 0
\]

By defining \( M_i = \hat{K}X \) for \( X > 0 \), the stability condition can be express as

\[
-XA_i^T - A_iX + M_i^T B_j^T + B_i M_j \geq 0
\]

These conditions illustrates a LMI problem based on variables \( M_i \) and \( X \). Hence, the feedback coefficient matrix \( \hat{K}_n \) and positive definite matrix \( P \) can be obtained as

\[
P = X^{-1}, \quad \hat{K}_n = M_i X^{-1}
\]

### 5.2 Simulation studies

Based on the control approach developed in the previous section the feedback coefficient matrix can be obtained as

\[
\hat{K}_1 = 10^8 \times \begin{bmatrix}
0.000102 & 0.011038 & 0.0000119 \\
-7.39885
\end{bmatrix}
\]

\[
\hat{K}_2 = 10^8 \times \begin{bmatrix}
0.000049 & 0.002759 & 0.000066 \\
-2.2520
\end{bmatrix}
\]
\[ \mathbf{K}_3 = 10^8 \times \begin{bmatrix} 0.000 & 101 & 0.005 & 519 & 0.001 & 5306 \\ -4.04632 \end{bmatrix} \]

As previously mentioned the designed system is stable only if there exist a common positive definite matrix \( \mathbf{P} \) that satisfies the conditions in equations (39a) and (39b).

\[
\mathbf{P} = \begin{bmatrix}
0.001 & 071 & -0.017 & 44 & -0.000 & 407 & 3.834 & 60 \\
-0.017 & 44 & 23.538 & 53 & -1.915 & 828 & 9405.34 \\
-0.000 & 407 & -1.915 & 828 & 35.053 & 27 & 325.391 \\
3.834 & 60 & 9405.34 & 325.391 & 6433032 \\
\end{bmatrix}
\]

Obviously, the \( \mathbf{P} \) is symmetric and positive definite.

Furthermore, it can be easily checked that the stability equations (39a) and (39b) are satisfied. In order to show the effectiveness of the designed fuzzy controller simulations were run on the tracking problem in all three fuzzy zones. Figures 10 to 12 show the tracking problem for the reference input \( R = 2 \times 10^{-5} \sin (0.1 \pi t) \) m. Figures 13 to 15 depict the tracking error, while Figs 16 to 18 illustrate the state variables in the three fuzzy zones.

6 CONCLUSIONS

The main contributions of this paper include the following points.

1. In the modelling part, based on the application of T–S fuzzy modelling, system uncertainties are incorporated into the model. Comparison of the experimental data with the proposed T–S fuzzy model indicates that, it can predict the actuator’s behaviour as its parameters are varied. The proposed T–S fuzzy model is able to overcome the obstacles experienced by the DEM and RD modelling approaches.

2. Fuzzy state feedback is combined with a PI controller to control the actuator. A state space
equation based on the addition of an integral state is derived. Sufficient conditions for stability of the system based on the Lyapunov direct method are presented. The stability of the proposed design is shown by introducing a common positive definite matrix $P$. Simulation results are presented on the tracking performance of the proposed control system.

**Fig. 13** Tracking error for the first fuzzy zone

**Fig. 14** Tracking error for the second fuzzy zone

**Fig. 15** Tracking error for the third fuzzy zone

**Fig. 16** State variables for the first fuzzy zone

**Fig. 17** State variables for the second fuzzy zone
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REFERENCES


16 Wang, L. X. A Course in fuzzy systems and control, 1997 (Prentice-Hall).


