Accurate Fault Location Algorithm for Power Transmission Lines

J. Sadeh, A. M. Ranjbar, N. Hadjsaid, R. Feuillet

Abstract

In this paper we propose a new fault location algorithm for power transmission lines based on one terminal voltage and current data. A distributed time domain model of the line is used as a basis for algorithm development. The suggested technique only takes advantage of post-fault voltage and current samples taken at one end of the line and does not require filtering of DC offset and high-frequency components of the recorded signals, which are present during transient conditions. Another advantage of the proposed method is the application of a very narrow window of data i.e., less than 1/4 of a cycle. The paper also proposes two different algorithms for lossless and lossy line models. Computer simulations approved the accuracy of the proposed methods.

1 Introduction

Rapid detection, location and clearance of faults are essential factors of satisfactory operation of power supply networks. When a fault occurs on the transmission line of an electric power system, it is very important to find the exact location of the fault. This can result in reducing the time required for repairing the damage caused by the fault and consequently improving reliability and continuity of energy supply.

Fault location of transmission lines has always been a very well-known subject, being studied for a long time. Many techniques have been proposed and applied for locating the exact point of fault on transmission lines [1–14]. The main differences between these various algorithms are due to different transmission line models (lumped [1–3] or distributed [4–6]), and the data required (of one end [1, 4, 7] or two ends of the line [2, 8, 9]).

Sant and Paithankar [10] proposed a fault location algorithm technique that uses fundamental frequency voltage and current measured at one of the line terminals. This technique assumed that the line is connected to a source at one end only. The estimate of fault locations is not accurate if the fault current is supplied from both line terminals and some fault resistance is present. Takagi et al. [7], Wiszniewski [3], Eriksson et al. [1] and Cook [2] proposed methods which use fault current distribution factors, pre-fault and post-fault current and post-fault voltage from one line terminal. Impedances of equivalent sources connected to the line terminals are required. In practice, the system configuration changes from time to time modifying distribution factors. Richards and Tan [11] present a dynamic parameter estimation algorithm for locating faults, based on locally available currents and voltages. The differential equations are based on a lumped parameter line model. For modelling of the rest of the system at both ends of the line, Thevenin equivalents are utilised. The fault location problem is treated as a parameter estimation problem of a dynamic system, in which the response of the physical system is compared with the one of a lumped parameter model. The model parameters (location and resistance of the fault) are varied until an adequate match is obtained with the physical system response. The travelling wave approach was also studied [4–6, 12, 13]. This approach is based either on the travel time measurements using correlation technique [12] or on the calculation of voltages and/or currents profiles along the transmission line [4–6, 13]. The travelling wave techniques offer some advantages but the computational complexity is increased. Time domain representation of a transmission line model has also been considered [14]. The model is obtained using the Laplace and Z transforms. Data samples are considered as being available from one end only. The voltage at the other end is estimated using pre-fault data.

In calculating the distance to a fault point, using voltage and current signals, it is advantageous to use terminal data of both sides of the faulty transmission line. However, from the practical point of view, it is more appropriate that the algorithm uses only one terminal voltage and current data, which leads to simplicity of the equipment used.

This paper introduces a new fault location algorithm that needs one terminal post-fault data and uses distributed time-domain model of transmission line to achieve required accuracy. The idea of the introduced
algorithm is that it considers fault location, as an optimization problem that can be solved by an appropriate mathematical method.

2 Distributed Model of Transmission Line

A single-phase model of a three-phase transmission line with distributed parameters is shown in Fig. 1. In this figure S represents the sending end and F is taken as an arbitrary point at a distance $x$ (x ≤ $l_s$) from S along the line. The distributed model [15] of the SF segment is shown in Fig. 2.

The following equations can be derived according to this figure:

\[
i_S(t) = \frac{1}{Z_c} u_s(t) + I_S(t - \tau),
\]

\[
i_x(t) = \frac{1}{Z_c} u_s(t) + I_x(t - \tau).
\]

The distributed model of the SF segment is shown in Fig. 2.

3 Fault Location Algorithms

3.1 Mathematical Definition

Let $u_s(t)$ and $i_s(t)$ represent the voltage and current at point $x$ on a transmission line. $u_s(t)$ and $i_s(t)$ satisfy the hyperbolic system of partial differential equations:

\[
\frac{\partial u}{\partial x} + L' \frac{\partial i}{\partial t} = -R'i,
\]

\[
C' \frac{\partial u}{\partial t} + \frac{\partial i}{\partial x} = 0,
\]

where $R'$, $L'$ and $C'$ are the resistance, inductance and capacitance per unit length of the line, respectively. Let the voltage at a fault point F on the line be given by:

\[
u_s(t_f) = u_F, t \geq t_0,
\]

where $t_0$ represents the instant when the fault occurs.

The voltage and current records at the local end of the line during the fault can be represented over time $t$ by:

\[
\begin{bmatrix}
  u_s(t) \\
  i_s(t)
\end{bmatrix}, \quad t_0 \leq t \leq T + t_0,
\]

where $T$ is the window length of recording. The fault location problem can then be defined as follows:

- Determine the co-ordinate $x$ by using the line model of eqs. (7) and (8) and the data set of eq. (10).

In order to solve this problem, two distinct cases are considered. One with a lossless line model and the other by taking the line resistance into consideration.

3.2 Lossless Transmission Line

Suppose that a solid short circuit ($u_F = 0$) has occurred at point F, at distance $x$ from the sending end of the transmission line. According to the distributed time domain model of transmission line and considering eqs. (5) and (6), we obtain:
Without loss of generality, we suppose that the short circuit has occurred at \( t_0 = 0 \). In order to cancel the undetermined current \( i_s \) in eqs. (11) and (12), we shift eq. (11) by \( \tau \) seconds. According to these equations and eq. (9) one can write:
\[
\begin{align*}
\sum_{k=1}^{N} && \left[ u_s(k + m) + u_s(k - m) \right] \\
&+ &Z_c \left[ i_s(k + m) - i_s(k - m) \right] - Z_c \left[ u_s(t + \tau) + u_s(t - \tau) \right] &= 0. 
\end{align*}
\]
(14)

The distance to the fault point \( x \) does not appear explicitly in eq. (13), and is hidden in the surge travelling time \( \tau \). Furthermore, \( \tau \) does not appear as a variable in eq. (13), but as the value on which the voltage and current depend. Eq. (13) can be written in discrete form as
\[
\begin{align*}
\sum_{k=1}^{N} && \left[ u_s(k + m) + u_s(k - m) \right] \\
&+ &Z_c \left[ i_s(k + m) - i_s(k - m) \right] - Z_c \left[ u_s(t + \tau) + u_s(t - \tau) \right] &= 0. 
\end{align*}
\]
(14)

In this case, the unknown parameter \( x \) not only implicitly appears in the voltage and current argument, but also explicitly appears in the variables \( R_c, Z_c \) and \( Z_c^* \). By discretizing the above equation we introduced:
\[
F(m, k) = 0. 
\]
(19)

The \( F \) function is given in the Appendix. Eq. (19) is again over-specified since it has just one unknown variable distance \( x \) to the fault point. Therefore the best estimate may be found based on the minimum least-square estimate technique. In order to determine the location of the fault, the following optimization problem must be solved:
\[
\min J_1(m) = \min \sum_{m=1}^{N} F(m, k). 
\]
(20)

4 Three-Phase Unbalanced Faults

The location of unbalanced faults can also be determined by using this method. The procedure is similar to the case of balanced faults.

In the case of unbalanced faults, firstly the coupled equations in the phase domain must be transformed into the decoupled equations in the modal domain. In regard to balanced lines, there are a number of simple transformation matrices, which decouple the line equations. For instance, such a matrix and its inverse for three-phase lines are [15]:
\[
M = \begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2
\end{bmatrix}, 
\quad M^{-1} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 0 \\
1 & 0 & -1
\end{bmatrix}. 
\]
(21)

Using the inverse transformation matrix \( M^{-1} \), the sending end voltages in modal domain can be written as:
\[
\begin{align*}
u_0^0(t) &= \frac{1}{3} \left[ u_0^0(t) + u_0^2(t) + u_0^3(t) \right], \\
u_0^1(t) &= \frac{1}{3} \left[ u_0^1(t) + u_0^2(t) + u_0^3(t) \right], \\
u_0^2(t) &= \frac{1}{3} \left[ u_0^1(t) + u_0^2(t) + u_0^3(t) \right]. 
\end{align*}
\]
(22)

After decoupling the system equations, we are able to apply the same procedure for each of the three modes, leading to Bergeron’s equations for the three phase line.

5 Test Results

In this section the performance of the algorithms were evaluated. Fig. 3 shows a 400-kV, 308-km long
three-phase transmission line with distributed parameters that are given in the Appendix. A three-phase fault occurs at an arbitrary point F at a distance x from sending end. For x equal to 100 km, the voltage and current of phase L1 at the sending end are shown in Fig. 4 and Fig. 5, respectively. For this case the fault inception angle has been assumed to be 90°. These curves are obtained from MATLAB, and resemble the results from EMTP to a great extent.

For this condition, the criterion functions $J_1$ and $J_2$ are shown in Fig. 6 versus distance d. The minima of these criterion functions determine the fault location in two cases, namely lossless and lossy transmission line. According to this figure, it is concluded that high accuracy in line model leads high accuracy in results. For calculation of criterion functions we applied only the post-fault voltage and current samples taken at the sending end of the line. These data do not require filtering of DC offset and high-frequency content. The data window used in the calculation contained 2.5 ms to 3.5 ms of post-fault data, starting from the fault occurrence.

In order to analyse the accuracy of the algorithm at different fault situations, we performed extensive tests by various inception angles. As an instance, for zero fault inception angle, the voltage and current at the sending end for fault location at 100 km are shown in Fig. 7 and Fig. 8, respectively. The criterion functions for this case are depicted in Fig. 9. The result was still satisfactory despite very large DC offset in current signal, as shown in Fig. 8.

To investigate the accuracy of the proposed methods for various fault locations, the percentage of error is plot-
6 Conclusions

In this paper we proposed a new fault location algorithm using post-fault voltage and current samples taken at one end of the transmission line. The advantages claimed for this new algorithm are:

- No requirement for elimination of DC offset and high-frequency components of the data.
- Low sensitivity to fault inception angle and location of fault.
- High accuracy.
- Very narrow window of data (2.5 ms to 3.5 ms of post-fault data).

The algorithms were tested under a variety of simulated fault conditions. In the case of balanced faults, the estimated fault location errors were within 0.25% line length for accurate model case while using the lossless (inaccurate) model lead to unacceptable errors specially when the fault occurs near the end of the line. But the results in the case unbalanced faults indicate that the estimated fault location errors were very small for all cases, as it never exceeds 1%.

7 List of Symbols and Abbreviations

- \( t \): time
- \( T \): window length of recording
- \( \tau \): time elapsed for the wave propagation from S to F
- \( x \): distance between F and S
- \( \Delta t \): sampling step
- \( i_s(t), u_s(t) \): sending end current and voltage
- \( i_r(t), u_r(t) \): receiving end current and voltage
- \( i_f(t), u_f(t) \): current and voltage of fault point
- \( Z_c \): characteristic (surge) impedance
- \( R \): line resistance from S to F
- \( L \): line length
- \( R' \): line resistance per unit length
- \( L' \): line inductance per unit length
- \( C' \): line capacitance per unit length
- \( N \): total number of samples
- \( J_1, J_2 \): criterion functions
- \( M \): transformation matrix
- \( x_{\text{est}} \): calculated location of fault
- \( x_{\text{real}} \): actual location of fault

Appendix

A1 Function \( F \)

\[
F(m, k) = u_s(k + m) - Z_c' i_s(k + m)
- \frac{R^*/4}{Z_c} \left[ \frac{R^*/2}{Z_c} u_s(k) + 2 Z_c'' i_s(k) \right]
+ \frac{Z_c'''}{Z_c} \left[ u_s(k - m) + Z_c'' i_s(k - m) \right] = 0. (A1)
\]
**A2 Transmission-Line Data**

<table>
<thead>
<tr>
<th>Positive sequence</th>
<th>Zero sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'' = 0.01537 , \Omega/km$</td>
<td>$R^0 = 0.04612 , \Omega/km$</td>
</tr>
<tr>
<td>$L'' = 0.8858 , mH/km$</td>
<td>$L^0 = 2.6574 , mH/km$</td>
</tr>
<tr>
<td>$C'' = 13.065 , nF/km$</td>
<td>$C^0 = 4.355 , nF/km$</td>
</tr>
</tbody>
</table>

**References**


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