An Analytical Study on Elastic Flange Wrinkling of Laminated Circular Plates in Deep Drawing Process

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Abstract

Based on two-dimensional plane stress wrinkling model of an elastic annular plate and a bifurcation functional from Hill's general theory of uniqueness for polar coordinate, the critical conditions for the elastic wrinkling of the flange of a two layered circular blank during the deep-drawing process are obtained analyticaly. In addition to critical conditions for onset of elastic wrinkling of the flange, the elastic limitation of the material is also be considered. Finally the critical number of wavs and the limitation of deep drawing of the laminated Steel-Aluminum is obtained and explaned.

Keywords: Elastic wrinkling, Bifurcational functional, Laminated plates, Deep Drawing Process.

Introduction

Wrinkling is one of the major modes of failure in automotive sheet pressing operations. Wrinkling is caused by excessive compressive stresses during forming. In a deep-drawing operation two initially flat round blank is drawn over a die by a cylindrical punch, as shown in Figure 1. The annular part of the blanks are subjected to a radial tensile stress, while in the circumferential direction compressive stress is generated during drawing, Figure 2. For particular drawing-tool dimensions and blank thicknesses, there is a critical blank diameter/thickness ratio as the critical stress causes plastic buckling of the annular part of the blanks so that an undesirable mode of deformation ensues with waves being produced in the flange as shown in Figure. 3. A bifurcation functional was proposed by Hutchinson [1] based on Hill general theory of uniqueness and bifurcation in elastic-plastic solids . This functional is given and explained by [2]:

$$F(u,v,w)=\frac{1}{2}\iint(M_{ij}\kappa_{ij}+N_{ij}\varepsilon_{ij}^{0}+t\,\sigma_{ij}w_{,i}w_{,j})\,ds. \qquad (1)$$

where s denotes the region of the shell middle surface over which the wrinkles appear, u and v in-plane displacements, w the buckling displacement, t the thickness of the plate, N_{ij} the stress resultants, M_{ij} the stress couples (per unit width), κ_{ij} the bending strain (or the change of the curvature) tensor and ε_{ij}^{0} the stretch strain tensor. This bifurcation functional is the total energy for wrinkling occurrence. $F=0$ corresponds to the critical conditions for wrinkles to occur for some non-zero displacement fields [1-3].

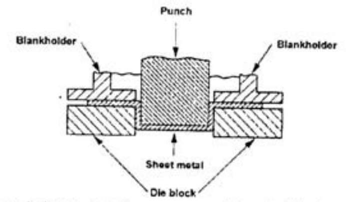


Figure 1: Deep drawing process with cylindrical punch

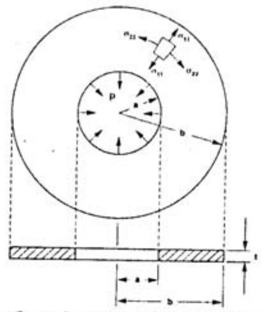
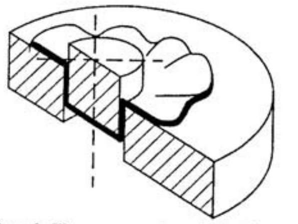


Figure 2: The flange is modeled as two annular plates with radial stress distribution in their inner edges.


Figue 3: The waves produce in the flange

The elastic wrinkling of two layered annular plates

From Eq. (1), we prove that the bifurcation functional for two layered annular plates is :

$$\begin{aligned}F(u,v,w)=&\frac{1}{2}\int_0^{2\pi}\int_a^b\frac{t^3}{3}L^1_{ijkl}\kappa_{ij}\kappa_{kl}\,r\,dr\,d\theta+\frac{1}{2}\int_0^{2\pi}\int_a^b t\,L^1_{ijkl}\varepsilon^0_{ij}\varepsilon^0_{kl}\,r\,dr\,d\theta-\frac{1}{2}\int_0^{2\pi}\int_a^b t^2L^1_{ijkl}\kappa_{ij}\varepsilon^0_{kl}\,r\,dr\,d\theta+\\&\frac{1}{2}\int_0^{2\pi}\int_a^b t\,\sigma^1_{ij}w_{,i}w_{,j}\,r\,dr\,d\theta+\frac{1}{2}\int_0^{2\pi}\int_a^b\frac{t^3}{3}L^2_{ijkl}\kappa_{ij}\kappa_{kl}\,r\,dr\,d\theta+\frac{1}{2}\int_0^{2\pi}\int_a^b t\,L^2_{ijkl}\varepsilon^0_{ij}\varepsilon^0_{kl}\,r\,dr\,d\theta+\\&\frac{1}{2}\int_0^{2\pi}\int_a^b t^2L^2_{ijkl}\kappa_{ij}\varepsilon^0_{kl}\,r\,dr\,d\theta+\frac{1}{2}\int_0^{2\pi}\int_a^b t\,\sigma^2_{ij}w_{,i}w_{,j}\,r\,dr\,d\theta.\end{aligned} \qquad (2)$$

After assuming displacement fields u,v,w in terms of r,θ which satisfing boundary conditions and finding plastic stresss distribution in each of two layers, with substituting them into Eq. (2) it is found that:

$$F = \{c \quad d \quad e\} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{Bmatrix} c \\ d \\ e \end{Bmatrix} = u^T M u, \tag{3}$$

$$\begin{cases} M_{11} = \frac{\pi t^3}{6}[\frac{E_1}{1-\nu_1^2} G_1(m,n,\nu_1) + \frac{E_2}{1-\nu_2^2} G_2(m,n,\nu_2)] + \pi t b^2 p[H_1(m,n,E_1,E_2,\nu_1,\nu_2) + \\ H_2(m,n,E_1,E_2,\nu_1,\nu_2)], \\ M_{12} = M_{21} = \frac{1}{2}\frac{\pi t^2 b}{2}[\frac{E_2}{1-\nu_2^2} T_2(m,n,\nu_2) - \frac{E_1}{1-\nu_1^2} T_1(m,n,\nu_1)], \\ M_{13} = M_{31} = \frac{1}{2}\frac{\pi t^2 b}{2}(\frac{E_2}{1-\nu_2^2} - \frac{E_1}{1-\nu_1^2})U(m,n), \\ M_{22} = \frac{\pi t b^2}{8}[\frac{E_1}{1-\nu_1^2} Q_1(m,n,\nu_1) + \frac{E_2}{1-\nu_2^2} Q_2(m,n,\nu_2)], \\ M_{23} = M_{32} = \frac{1}{2}\frac{\pi t b^2}{8}[\frac{E_1}{1-\nu_1^2} R_1(m,n,\nu_1) + \frac{E_2}{1-\nu_2^2} R_2(m,n,\nu_2)], \\ M_{33} = \frac{\pi t b^2}{8}(\frac{E_1}{1-\nu_1^2} + \frac{E_2}{1-\nu_2^2})S(m,n). \end{cases} \tag{4}$$

In this case, the critical condition for onset of wrinkling($F=0$) becomes $Det(M_{ij})=0$, Finally it is found that,

$$\frac{b^2}{t^2} p > \frac{K(m,n,E_1,E_2,\nu_1,\nu_2)}{H(m,n,E_1,E_2,\nu_1,\nu_2)}. \tag{5}$$

we have another limitations for elastic wrinkling that is:

$$(\sigma_r^1 - \sigma_\theta^1)|_{r=a} < Y_1 , \tag{6}$$

$$p < \frac{[k_1^2 - k_2^2 + k_3^2 - k_4^2 + 2(k_1 k_3 - k_2 k_4)](1-m^2)}{4(k_1^2 - k_2^2 + k_1 k_3 - k_2 k_4 + k_1 k_4 - k_2 k_3)} Y_1 . \tag{7}$$

and

$$(\sigma_r^2 - \sigma_\theta^2)|_{r=a} < Y_2 , \tag{8}$$

$$p < \frac{[k_2^2 - k_1^2 + k_4^2 - k_3^2 + 2(k_2 k_4 - k_1 k_3)](1-m^2)}{4(k_4^2 - k_3^2 - k_1 k_3 + k_2 k_4 + k_1 k_4 - k_2 k_3)} Y_2 . \tag{9}$$

In these conditions elastic Eqs. (5,7,9) wrinkling will occur before plastic wrinkling.

Results and Discussion

By considering Steel($E = 200Gpa, \nu = 0.3$) and Aluminum ($E = 70Gpa, \nu = 0.25$) and substituting these values in Eq. (7) and using least squre method for approximation we can find $n_{critical}$ and $p_{critical}$, Figures 4-6.

Conclusions

The elastic wrinkling of flange in deep drawing have been studied analytically using a bifurcation functional. To improve our method we simplify our relations for to symmetric layer that we obtain the same conclusion as one layer [4].

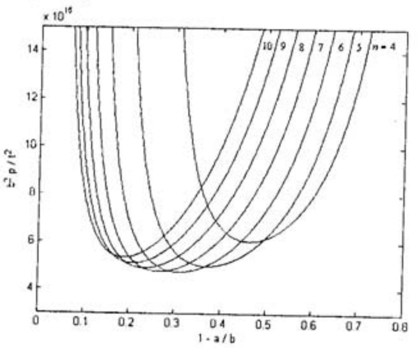


Figure 4: wrinkling load for different n

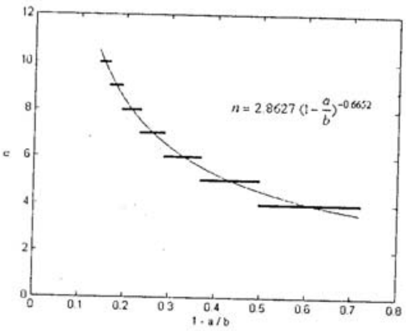


Figure 5: number of waves produced in the flange.

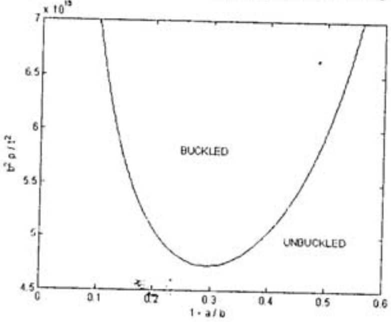


Figure 6: limitation of elastic wrinkling

List of Symbols

a	The inner radius of annular plates
b	The outer radius of annular plates
E	Modulus of elasticity
ν	Poisson ratio
p	Radial stress distribution in the inner edge
Y	Yield point
m	Dimensionless ratio $\frac{a}{b}$
n	Number of waves
c,d,e	Constants

Greek symbols

σ_{ij}	Stress distribution

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