COMPARING OF DIFFERENT METHODS OF PERFORMANCE PREDICTION OF GAS TURBINE POWER PLANTS

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ABSTRACT
Different models are being used to predict the performance of a gas turbine in steady state operation. This paper compares three different procedures that are normally used to model the performance of gas turbines. The compared methods are trial and error method, Newton-Raphson method and inverse algorithm. Mathematical model for each method is developed for an arbitrary free power gas turbine and is solved. Results represent that the inverse algorithm is the best numerical method from the stability and convergence point of view.

INTRODUCTION
Gas turbine is one of the most sophisticated thermal power engines, even in its simplest form, consisted of compressor, combustion chamber, turbine and other auxiliary devices. Although each of these components has its own characteristics, interaction of the components dictates the overall performance of the gas turbine. The elementary evaluation of the gas turbine performance can be conducted at design state conditions. Due to fluctuations of ambient temperature or load, gas turbine can rarely work under their design conditions. Therefore, it is necessary to predict the performance of a gas turbine under off-design conditions.

Different methodologies for predicting the performance of gas turbine cycle have been presented by Cohen et al. [1], Ismail et al. [2], Kim et al. [3], for instance, but most of these experiments are numerically unstable algorithms and computational simulations sometimes fail to converge. It is important to find the best method that has good accuracy and stability and rapid convergence.

ANALYSIS
Although modeling of the design operations of gas turbine components, for sake of the simplicity, is not more than a synthesis of thermodynamic principles, yet, it is also a basis for the off-design modeling. The alternation of mass flows, pressure ratios, and various losses beyond the design conditions merely determine the transition of the equilibrium from design operation to off-design.

Mathematical modeling of thermodynamic performance of gas turbine helps the user to predict fault matrices that can be used for fault identification. A major problem, however, is the lack of information required to construct such a model. This work represents a form of reverse engineering using publicly available data. The first step required is to define thermodynamic design point, specifying temperatures and pressures through the engine, knowing the over-all performance specifications. The next step is to estimate compressor

and turbine characteristics to provide a model capable of operating over the entire range of loads. An alternative method that does not need compressor characteristics is also developed based on gas dynamic relationships.

A free power turbine is considered (Fig 1). In this twin shaft arrangement, the high-pressure turbine drives the compressor and the low-pressure power turbine drives the generator. These arrangements, is also used for large scale electricity generating units with the power turbine designed to run at the alternator speed without the need for an expensive reduction gearbox.

Design point calculation
The initial step in developing the computational engine model is to re-establish the design point conditions from the available data. Since this information will be incorporated into the off-design model, it is very important to ensure that an accurate design point is obtained.

Compressor and turbine characteristic maps
The performance of a compressor can be described by means of one or numbers of either dimensionless or normalized parameters. The major impediment in developing a component based on gas turbine model is the lack of component characteristic maps. For the most of commercially available engines, the full-scale component maps are not available due to the manufacturer policy. So, a generalized speed-independent turbine characteristic curve is used to model the turbine performance. The curve can be expressed either by a polynomial or a modified nozzle equation [4].

Fig 2 shows the maps that have been used for axial compressor model.

According to the compressor maps, pressure ratio is usually plotted against the mass flow parameter

\[ \frac{\gamma_m}{N \sqrt{\frac{T_4}{P_4}}} \]

for lines of the constant speed parameter

\[ \left( N \sqrt{\frac{T_4}{P_4}} \right) \]

The subscript 4 here denotes turbine inlet conditions. Increasing the pressure ratio will increase the mass flow up to a certain limit. The maximum value of \[ \frac{\gamma_m}{N \sqrt{\frac{T_4}{P_4}}} \] will be reached at a pressure ratio that produces choking conditions at the same point in the turbine. Usually choking occurs in the stator nozzle blades or inlet casing. In this case the constant speed lines converge to a single vertical line as indicated in Fig 3 [5].
Fig. 1: Free power turbine

Fig. 2: Compressor mass flow map

Fig. 3: Mass flow map for turbine model

Fig. 4: The computational effort for matching the overall airflow rate with compressor and turbine flow characteristics.
OFF-DESIGNED PERFORMANCE CALCULATION METHODS

Three different methodologies and implementation for predication of gas turbine cycle have been presented. These methods are:
- Trial and Error method based on iteration procedures
- Newton-Raphson method to solve non-linear set of equation
- Inverse Algorithm

**Trial and Error**

The trial and error method has been developed and introduced by Cohen et al [1]. In general terms, the procedure for obtaining an equilibrium working points are in the following:

a) To select a constant speed line on the compressor characteristic and choose any point on this line. If there is not any information, general map can be used. The value of \( q, T_c, P_c, P_o, \eta_c \) and \( N, T_0 \) are then determined.

b) To obtain the corresponding point on the turbine characteristic from consideration of compatibility of rotational speed and flow.

c) Having matched the compressor and turbine characteristic, it is necessary to ascertain whether the work output corresponding to select operating point is compatible with that required by the driven load.

Fig 4 demonstrates the computational routines needed for matching the overall operation of a twin shaft engine with a power turbine.

**Newton-Raphson Method**

A set of non-linear equations in \( n \) dimensions with the convention that the right-hand side of each equation is zero, consist of \( n \) non-linear functional relations to be solved simultaneously [3]. The vector notation of this equation is shown by \( F \) and variable is shown by \( x \).

\[
F(x) = 0
\]

The expansion of the functions in the neighborhood of \( x \) in Taylor series can be presented correspondingly in matrix notation as

\[
F(x + \Delta x) = F(x) + J(x) \Delta x + O(\Delta x^2)
\]

In the above equation, \( O(\Delta x^2) \) denotes the terms of order \( \Delta x^2 \) and higher. \( J(x) \) is the Jacobian matrix at \( x \).

\[
J(x) = \begin{bmatrix}
\frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_n(x)}{\partial x_1} & \cdots & \frac{\partial f_n(x)}{\partial x_n}
\end{bmatrix}
\]

Considering \( F(x + \Delta x) = 0 \) and neglecting the higher order terms in Taylor series, a set of non-linear equation remains that determines the appropriate correction \( \Delta x \) to be added to the solution vector to conform the basic Newton method for solving sets of non-linear equation.

The set of equation can be solved, by using LU decomposition [7], for instance. Table 1 shows the equation for modeling gas turbine performance after division into subsystems.

<table>
<thead>
<tr>
<th>Component</th>
<th>Mathematical Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet duct</td>
<td>( P_b = (1 - K_s)P_{amb} )</td>
</tr>
<tr>
<td>Compressor</td>
<td>( \eta_m = f(\eta_m, \Phi, \beta_c) )</td>
</tr>
<tr>
<td>Turbine</td>
<td>( h_{m} = \frac{P_{m}}{\eta_{m0}} h_{m0} )</td>
</tr>
<tr>
<td>Combustion Chamber</td>
<td>( \beta = 1 - k \alpha^{\beta} )</td>
</tr>
<tr>
<td>Fuel</td>
<td>( \eta_{in} = f(\eta_{in}, \Phi, \beta_c) )</td>
</tr>
</tbody>
</table>

**Inverse Algorithm Method**

This method presented by Youngnong[8] for a marine gas turbine engine. Following is the derivation of mathematical inverse algorithm model for a twin shaft free power turbine.

a) Continuity equation

\[
\eta_c \frac{q_m}{\rho_m} \frac{T_m}{P_m} \frac{T_{in}}{C_{in}} \frac{1}{1 + f} = \frac{q_m}{\rho_m} \frac{T_m}{P_m} \frac{T_{in}}{C_{in}} \frac{1}{1 + f}
\]

b) Coupling equation

\[
\frac{\rho_c}{\rho_m} \frac{T_c}{C_{in}} \frac{1}{1 + f} = \frac{T_c}{C_{in}} \frac{1}{1 + f}
\]

c) Compressor and turbine pressure ratio relation
\[ \lambda_i = \frac{\gamma_{ij} \tau_{ij}}{\sigma_{ij} \sigma_{jk} \sigma_{kl} \sigma_{km} \sigma_{ln}} = C_i \]

Also compressor efficiency is a function of \( \frac{q_{e} \sqrt{T_{e1}}}{P_{o1}} \)

And \( \eta_{\text{c}} \). So we can consider

\[ X = \pi_{\text{c}} \quad X = \frac{q_{e} \sqrt{T_{e1}}}{P_{o1}} \]

\[ x^{(i+1)} = x^{(i)} - \lambda \left[ f(x^{(i)}) \right] / F(x^{(i)}) \quad 0 \leq \lambda \leq 1 \]

**CASE STUDY**

In this study a twin shaft gas turbine with a gas generator and a power turbine according to Fig 1 has been considered. The fix values as applied to all the cases in this study are presented in Table 2. The design point calculation has been shown in Table 3. To summarize these constitutive design parameters for the off-design study, the turbine inlet temperature is selected to be 1500 K and the heat exchanger effectiveness for regenerating cycles to be 0.90. Design pressure ratio, for correspond with the optimum efficiency values for the selected degree of regeneration is 7.

<table>
<thead>
<tr>
<th>Table 2. Fix value for case study</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ambient Condition</strong> 15 °C 101.3 kPa 0%</td>
</tr>
<tr>
<td><strong>Compressor intake mass flow</strong> 100 kg/s</td>
</tr>
<tr>
<td><strong>Fuel Specification</strong></td>
</tr>
<tr>
<td>Natural gas 15 °C</td>
</tr>
</tbody>
</table>

**RESULTS AND DISCUSSION**

Based on the results obtained from the model of each method, Fig 5 to 10 present the influence of ambient pressure and temperature, turbine inlet temperature as well as, on the net efficiency and net power. The computational time for the models was in the range of 1 to 3 second on PC Pentium III. Inverse algorithm and Newton--Raphson methods nearly have the same results. Only in some cases Newton--Raphson method is unstable during computational processing.

**CONCLUSION**

This study considers the modeling of gas turbine and off-design performance under steady state conditions, and computing the operating values by solving different methods of non-linear sets of equation formed by the models. Experience from computation shows that when the dimensions of the Jacobian matrix in Newton-Raphson iteration are greater than or equal to 5, its inverted matrix is liable to become ill conditioned. Also when a gas turbine works at part load, the surge of compressor and the over temperature have great influences upon the gas turbine engine. It can greatly decrease the work range of the engine and make the local convergence region narrower. It follows that the initial approximate solution has to be chosen close to the correct solution, but it is hard to do it. Thus, the iteration in the computational process sometimes fails to converge.

Trial and Error method result in significant errors and turbine performance and overall performance under certain conditions. The inverse algorithm has good numerical stability. It has also has a good effect on reducing the accumulation and propagation of round-off errors and interpolation errors. Using this algorithm to solve the performance of gas turbine engines, the accuracy and stability of computational can be greatly improved.

**NOMENCLATURE**

- **T** temperature
- **P** pressure
- **N** rotational speed
- **q** mass flow rate

**Greek Letters**

- \( \pi \) pressure ratio
- \( \eta \) efficiency
- \( \sigma \) recovery factor of total pressure
- \( \tau(\lambda) \) temperature ratio

**Subscripts**

- **B** combustion chamber
- **C** compressor
- **c** critical value
- **e** effective
- **in** inlet
- **ex** exit
- **H** high pressure
- **L** low pressure
- \( \Theta \) design condition

**Superscripts**

- \( k \) \((k)\)th iteration
- \( k+1 \) \((k+1)\)th iteration

**REFERENCES**

Table 3. Design Point Results for Case Study

<table>
<thead>
<tr>
<th>Ambient conditions</th>
<th>T(_{\text{amb}}) 288.15 K</th>
<th>P(_{\text{amb}}) 101.3 kpa</th>
<th>Humidity</th>
</tr>
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<tbody>
<tr>
<td>ISO - turbine inlet temp</td>
<td>1452.3 K</td>
<td>900</td>
<td></td>
</tr>
<tr>
<td>Fuel</td>
<td>Natural gas</td>
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</tr>
<tr>
<td></td>
<td>49.06 MJ/kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cycle points (k, kpa, kJ/kg)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>T</td>
<td>288.1</td>
<td>536.6</td>
<td>920.8</td>
</tr>
<tr>
<td>P</td>
<td>100.3</td>
<td>702.0</td>
<td>688.0</td>
</tr>
<tr>
<td>h</td>
<td>15.0</td>
<td>268.8</td>
<td>683.6</td>
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</table>

<table>
<thead>
<tr>
<th>Power (MW)</th>
<th>P(_{\text{in}})</th>
<th>Pr1</th>
<th>Pt1</th>
<th>Pr2</th>
<th>P(_{\text{out}})</th>
<th>P(_{\text{out}})</th>
<th>P(_{\text{net}})</th>
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<tbody>
<tr>
<td></td>
<td>68.497</td>
<td>536.6</td>
<td>26.003</td>
<td>30.659</td>
<td>21.285</td>
<td>0.626</td>
<td>0.626</td>
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<table>
<thead>
<tr>
<th>Mass rate (kg/s)</th>
<th>ecpp</th>
<th>ecpp1</th>
<th>ecpp2</th>
<th>estl</th>
<th>est2</th>
<th>omec1</th>
<th>etr</th>
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</table>

<table>
<thead>
<tr>
<th>Component Efficiency</th>
<th>ecpp</th>
<th>ecpp1</th>
<th>ecpp2</th>
<th>estl</th>
<th>est2</th>
<th>omec1</th>
<th>etr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ecpp</td>
<td>ecpp1</td>
<td>ecpp2</td>
<td>estl</td>
<td>est2</td>
<td>omec1</td>
<td>etr</td>
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</table>

<table>
<thead>
<tr>
<th>Pressure Ratio and Loss</th>
<th>compr</th>
<th>turb1</th>
<th>turb2</th>
<th>k0</th>
<th>k2</th>
<th>k3</th>
<th>k6</th>
<th>k7</th>
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<tbody>
<tr>
<td></td>
<td>7.00</td>
<td>2.14</td>
<td>2.98</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.20</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Fig 5. The influence of ambient pressure on net efficiency

Fig 6. The influence of ambient pressure on net power

Fig 7. The influence of ambient temperature on net efficiency

Fig 8. The influence of ambient temperature on net power

Fig 9. The influence of turbine inlet temperature on net efficiency

Fig 10. The influence of turbine inlet temperature on net power