Tuned Mass Dampers for Earthquake Oscillations of High-rise Structures Using Ant Colony Optimization Technique

Saeed Soheili 1, Mahdi Abachizadeh 2, Anoushirvan Farshidianfar 3

1 PhD Candidate, Mechanical Engineering Department, Ferdowsi University of Mashhad; soheili78@yahoo.com
2 PhD Candidate, Mechanical Engineering Department, University of Tehran; m_abachizadeh@yahoo.com
3 Associate Professor, Mechanical Engineering Department, Ferdowsi University of Mashhad; farshid@um.ac.ir

Abstract
This paper investigates the optimized parameters for the tuned mass dampers to decrease the earthquake vibrations of high-rise buildings. Considering soil effects, the soil-structure interaction (SSI) is involved in this model. The Tuned Mass Damper (TMD) is also utilized on the roof of the building. The time domain analysis based on Newmark method is employed to obtain the displacement, velocity and acceleration of different stories and TMD. To illustrate the results, Tabas earthquake data is applied to the model.

In order to obtain the best parameters for TMD, Ant Colony Optimization (ACO) method is utilized. Mass, damping and spring stiffness quantities of TMD are assumed as the design variables for the optimization. The objective is to reduce the maximum displacement as well as Root Mean Square (RMS) of the total displacement of stories during earthquake vibration. The results show that the TMDs are very effective and beneficial devices for decreasing the oscillations of high-rise buildings. It is indicated that the soil type highly affects the time response of structures subjected to the earthquake oscillations. It is also shown that how the ant colony optimization technique can be effectively applied to design the optimum TMD device. This study helps the researchers to the better understanding of earthquake vibration of the structures, and leads the designers to achieve the optimized TMD for the high-rise buildings.

Keywords: High-rise Structures, Earthquake Oscillations, Tuned Mass Dampers, Ant Colony Optimization, Soil-Structure Interaction.

Introduction
In the last decades, high-rise buildings are widely developed and employed in most countries. These structures are generally flexible and possess low damping properties. They are usually subjected to the earthquake vibrations. Therefore, the study of tall buildings vibration mitigation and various absorbers has attracted the interest of many researchers. Moreover, the soil characteristics and the interaction between soil and structure greatly influence the structural responses.

A tuned mass damper (TMD) is a kind of vibration absorber consisting mass, spring and viscous damper attached to the vibrating system to mitigate oscillations. It passively dissipates energy through the interaction of inertial force produced by mass movement and damping effects induced by damper.

As Ormondroyd and Den Hartog [1] mentioned, the application of TMD was firstly proposed in 1909. Since then, many theoretical and experimental researches have been performed to study the TMDs mechanism of vibration mitigation and their application for the structures. The TMDs are usually installed on the top floor, and several researches have been conducted to study their effectiveness for earthquake [2] and wind [3,4] excitations.

Gupta et al. [5] investigated the effects of several TMDs with elastic-plastic properties on the response of single degree of freedom structures subjected to Kern County earthquake (1952). To investigate the effect of TMDs on the fundamental mode response, Kaynia et al. [6] studied the optimum reduction of structures response subjected to 48 earthquake spectra. They figured out that the TMDs are less effective in decreasing the response of structures than previously thought. Sladek and Klingner [7] investigated the best parameters of a TMD placed on the top floor of a 25 storey building, based on the minimization of response to sinusoidal loading.

An optimization method is employed by Wirsching and Campbell [8] to calculate the TMD parameters for 1-, 5- and 10-storey buildings. According to their study, TMDs are effective devices in reducing response. Ohno et al. [9] presented the optimized TMD parameters based on the minimization of mean square acceleration response to earthquake excitations. Several studies on the application of TMD and its best values are performed by other researchers such as Villaverde et al. [10]. Later, Sadek et al. [11] presented some formulations for computing the optimal parameters of TMD device based on the equal damping of the first two modes of system.

Considering soil effects, the structure response differs from the fixed base model. The oscillation energy is actually transferred to the soil through the foundation. Therefore, the soil and structure influence each other, which is called the soil-structure interaction (SSI). Various investigations are performed to study the SSI effects. For example, frequency domain analysis was performed by Xu and Kwok [12] to obtain the wind induced vibrations of soil-structure-damper system. Moreover, the frequency independent expressions are proposed by wolf [13] to determine the swaying and rocking dashpots, and the related springs of a rigid
circular foundation. Recently, Liu et al. [14] developed a mathematical model for time domain analysis of wind induced oscillations of a tall building with TMD considering soil effects. Although numerous works are performed concerning SSI effects, few investigations are carried out on the time response of high-rise buildings due to earthquake excitations. In fact, the earthquake time response of tall buildings has usually been calculated employing fixed base models. These analyzes cannot reasonably predict the structural responses. Moreover, the optimal parameters of TMD are extremely related to the soil type. Therefore, the time domain analysis of structures consisting SSI effects is an advantageous process for the better understanding of earthquake oscillations and TMD devices. Furthermore, few works have considered and employed heuristic algorithms, while the heuristic techniques such as ant colony optimization (ACO), can be effectively employed for the design optimization of TMDs.

In this paper, a mathematical model is developed for calculating the earthquake response of a high-rise building with TMD. The model is employed to obtain the time response of 40 story building using TMD. The ant colony optimization (ACO) is applied on the model to obtain the best TMD parameters. The parameters are calculated with and without soil structure interaction (SSI) effects. This study may improve the researchers’ knowledge of earthquake oscillations for a building with TMD when SSI is considered.

### Modeling of Tall Buildings

Figure 1 shows the N-storey structure with a TMD and SSI effects. Mass and Moment of inertia for each floor are indicated as $M_i$ and $I_i$, and those of foundation are shown as $M_0$ and $I_0$, respectively. The stiffness and damping between floors are assumed as $K_j$ and $C_j$, respectively. $M_{TMD}$, $K_{TMD}$ and $C_{TMD}$ are the related parameters for TMD. Damping of the swaying and rocking dashpots are represented as $C_s$ and $C_r$, and the stiffness of corresponding springs are indicated as $K_s$ and $K_r$, respectively. Time histories of displacement and rotation of foundation are respectively defined as $X_0$ and $\theta_0$, and displacement of each storey is shown as $X_i$.

Using Lagrange’s equation, the equation of motion for a building shown in fig. 1 can be represented as follows [15]:

$$[m][\ddot{x}(t)] + [c][\dot{x}(t)] + [k][x(t)] = -[m^*][\ddot{u}_g]$$

(1)

where $[m]$, $[c]$ and $[k]$ denote mass, damping and stiffness of the oscillating system. $[m^*]$ indicates acceleration mass matrix for earthquake and $\ddot{u}_g$ is the earthquake acceleration. Considering SSI effects, the N-storey structure is a N+3 degree-of-freedom oscillatory system. For such building, the mass, damping and stiffness matrices are obtained by employing Lagrange’s equation in the following form [14, 15]:

$$[m] = \begin{bmatrix} [M]_{N\times N} & [0]_{N\times 1} & [M]_{N\times 1} \\ M_0 & M_{TMD} & M_{TMD} \\ \end{bmatrix}$$

$$[c] = \begin{bmatrix} [C]_{N\times N} & [0]_{N\times 1} & [0]_{N\times 1} \\ C_{TMD} & 0 & 0 \\ C_{TMD} & 0 & 0 \\ \end{bmatrix}$$

$$[k] = \begin{bmatrix} [K]_{N\times N} & [K_{TMD}]_{N\times 1} & [0]_{N\times 1} & [0]_{N\times 1} \\ K_{TMD} & 0 & 0 & K_s \\ K_{TMD} & 0 & 0 & K_r \\ \end{bmatrix}$$

Ignoring the SSI effects, rows and columns N+2 and N+3 are neglected, and the mentioned matrices are reduced to (N+1)×(N+1) dimensional matrices.
\[ [c]_{N×N} = A_0[m]_{N×N} + A_1[k]_{N×N} \] in which \( A_0 \) and \( A_1 \) are Rayleigh damping coefficients.

The displacement vector \( \{x(t)\} \) including both displacement and rotation of floors and foundation as well as TMD motion can be represented as follows:
\[
\{x(t)\} = \{X_1(t)\} X_2(t) \ldots X_N(t) X_{TMD}(t) X_0(t) \{θ_0(t)\}^T
\] 
(7)

The parameters \( C_s, C_r, K_s \) and \( K_r \) can be obtained from soil properties (i.e. poisson’s ratio \( ν_s \), density \( ρ_s \), shear wave velocity \( V_s \) and shear modulus \( G_s \)) and radius of foundation \( R_0 \) [14].

In this paper, Tabas earthquake acceleration spectrum is applied to the structure, and time response of TMD and building are calculated based on Newmark integration method [16].

**Ant Colony Optimization (ACO)**

In order to obtain the best parameters for TMD, the ant colony optimization (ACO) is employed. This algorithm firstly proposed by Dorigo and Gambardella [17] is based on the behavior of ants finding the shortest paths from a food source to their nest only by sensing the intensity of pheromone deposited by other ants. It is observed that they usually select the path with higher pheromone. This mechanism makes the shorter paths more desirable as it takes shorter time to march on them.

This behavior is simulated by three rules in ACO algorithm, which was originally applied on combinatorial problems with discrete variables such as Traveling Salesman or Quadratic Assignment. For engineering problems where the design variables are usually continuous, the method of discretization is an acceptable approach [18]. Once the continuous variables are divided into separated domains, the problem can be treated as a problem with discrete variables.

Regarding the discretization procedure, the design variables are presented by \( i \) and their divided search domains are shown by \( j \). The sections of total solution are chosen in a constructive approach named as “state transition rule”:
\[
S = \begin{cases} 
\text{argmax}[τ(i,j)][η(i,j)]^q & \text{if } q ≤ q_0 \\
\text{otherwise} & \end{cases}
\]  
(8)

Where \( τ(i,j) \) shows the amount of pheromone related to the \( j \)th element of variable \( i \), and \( η(i,j) \) is the heuristic function defined according to the investigated problem. In this rule, \( q \) is a random number, and \( q_0 \) is a parameter set by the user ( \( 0 ≤ q, q_0 ≤ 1 \) ). If \( q > q_0 \), the next step is selected according to proportional distribution of probability function, like the roulette wheels, assigned as follows:
\[
s = \begin{cases} 
\sum\{τ(u,j)[η(u,j)]^q \text{if } j \text{allowed } & \text{if } j = \text{allowed } \\
0 & \text{otherwise} 
\end{cases}
\]  
(9)

The significant factor of \( q_0 \) defines the range of randomness against determination of state transition rule. It is clear that the higher amounts of \( q_0 \) directs the algorithm towards deterministic decisions, while the lower amounts generates more randomness.

To avoid stagnation of the algorithm and similar to evaporation of pheromone in real world, the amount of pheromone level is changed after finishing each evaluation by applying “the local updating rule”:
\[
τ(i,j) = (1 - ρ)τ(i,j) + ρΔτ(i,j)
\]  
(10)

where \( ρ \) denotes the local evaporation coefficient. The best performance is obtained when \( Δτ(i,j) = τ_0 \) [17].

The third rule known as “the global updating rule” acts as a positive feedback and accumulates more pheromone around the best solution obtained so far:
\[
τ(i,j) = (1 - α)τ(i,j) + αΔτ
\]  
(11)

where \( Δτ \) is the inverse of the objective function and \( α \) is the global evaporation coefficient [17].

This process of evaluation and updating is repeated with \( m \) ants until the termination condition, which is usually the maximum number of cycles, is satisfied. Similar to other heuristic optimization techniques, it is important to tune the algorithm to achieve sensible results. For the tackled problem in this paper, the values presented in Table 1 are found acceptable. In addition, without damaging the overall effectiveness of ACO [17], the heuristic function is neglected due to intricacies in its definition procedure.

<table>
<thead>
<tr>
<th>Table 1: ACO parameter settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m ) &amp; ( α ) &amp; ( ρ ) &amp; ( τ_0 ) &amp; ( q_0 )</td>
</tr>
<tr>
<td>5 &amp; 0.1 &amp; 0.1 &amp; 2 &amp; 0.7</td>
</tr>
</tbody>
</table>

Since the problem is of multi-objective nature trying to minimize both maximum and RMS of displacements, an overall objective function including both concepts should be employed. Here, as the results approve its fitness and the single objectives are of similar numerical range, they are simply multiplied by each other.

**Illustrative Examples and Discussion**

The methodology outlined previously is employed to calculate the structural response of a 40-storey building with TMD. Table 2 shows the structure parameters [14].

The stiffness \( K_i \) linearly decreases as \( Z_i \) increases. TMD is installed on the top of building for the better damping of vibrations.

<table>
<thead>
<tr>
<th>Table 2: Structure parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of stories</td>
</tr>
<tr>
<td>Storey height (( Z_i ))</td>
</tr>
<tr>
<td>Storey mass (( M_i ))</td>
</tr>
<tr>
<td>Storey moment of inertia (( I_i ))</td>
</tr>
<tr>
<td>Storey stiffness</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Foundation radius (( R_0 ))</td>
</tr>
<tr>
<td>Foundation mass (( M_{f0} ))</td>
</tr>
<tr>
<td>Foundation moment of inertia (( I_{f0} ))</td>
</tr>
</tbody>
</table>
In this study, three types of ground states, namely soft, medium and dense soil are examined. A structure with a fixed base is also investigated. The soil and foundation properties are presented in Table 3.

### Table 3: Parameters of the soil and foundation

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Swaying damping $C_s$ (Ns/m)</th>
<th>Rocking damping $C_r$ (Nsm)</th>
<th>Swaying stiffness $K_s$ (N/m)</th>
<th>Rocking stiffness $K_r$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Soil</td>
<td>2.19×10^8</td>
<td>2.26×10^10</td>
<td>1.91×10^9</td>
<td>7.53×10^11</td>
</tr>
<tr>
<td>Medium Soil</td>
<td>6.90×10^8</td>
<td>7.02×10^10</td>
<td>1.80×10^10</td>
<td>7.02×10^12</td>
</tr>
<tr>
<td>Dense Soil</td>
<td>1.32×10^9</td>
<td>1.15×10^11</td>
<td>5.75×10^10</td>
<td>1.91×10^13</td>
</tr>
</tbody>
</table>

Table 4 represents the first 3 natural and damped frequencies of the structure, considering and ignoring SSI effects. The TMD design variables set in such a way that all the first 3 frequencies of the structure are covered, and damping ratio ($\xi$) is always less than unity.

### Table 4: Natural and damped frequencies of the structure

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Damping</td>
<td>With Damping</td>
<td>With Damping</td>
<td>With Damping</td>
</tr>
<tr>
<td>Soft Soil</td>
<td>-0.02±1.08</td>
<td>-0.24±4.45</td>
<td>-0.62±7.42</td>
</tr>
<tr>
<td>Without Damping</td>
<td>1.09</td>
<td>4.44</td>
<td>7.40</td>
</tr>
<tr>
<td>Medium Soil</td>
<td>-0.02±1.54</td>
<td>-0.21±4.57</td>
<td>-0.58±7.55</td>
</tr>
<tr>
<td>Without Damping</td>
<td>1.54</td>
<td>4.58</td>
<td>7.58</td>
</tr>
<tr>
<td>Dense Soil</td>
<td>-0.02±1.60</td>
<td>-0.21±4.58</td>
<td>-0.58±7.57</td>
</tr>
<tr>
<td>Without Damping</td>
<td>1.61</td>
<td>4.59</td>
<td>7.59</td>
</tr>
<tr>
<td>Fixed Base</td>
<td>-0.03±1.64</td>
<td>-0.21±4.59</td>
<td>-0.58±7.58</td>
</tr>
<tr>
<td>Without Damping</td>
<td>1.65</td>
<td>4.60</td>
<td>7.60</td>
</tr>
</tbody>
</table>

The optimized parameters of TMD obtained by ACO are presented in Table 5 for the three soil types. Table 6 shows the RMS and maximum displacement (in meters) for the three ground states, considering and ignoring TMD.

### Table 5: The optimized TMD parameters

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Mass (kg)</th>
<th>Damping (Ns/m)</th>
<th>Spring Stiffness (N/m)</th>
<th>$\omega_n$ (rad/s)</th>
<th>$\omega_d$ (rad/s)</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Soil</td>
<td>1.825×10^6</td>
<td>0.222×10^6</td>
<td>3.178×10^6</td>
<td>1.320</td>
<td>1.318</td>
<td>0.046</td>
</tr>
<tr>
<td>Medium Soil</td>
<td>1.369×10^6</td>
<td>0.262×10^6</td>
<td>1.732×10^6</td>
<td>1.125</td>
<td>1.121</td>
<td>0.085</td>
</tr>
<tr>
<td>Dense Soil</td>
<td>1.995×10^6</td>
<td>0.487×10^6</td>
<td>2.196×10^6</td>
<td>1.049</td>
<td>1.042</td>
<td>0.116</td>
</tr>
<tr>
<td>Fixed Base</td>
<td>1.995×10^6</td>
<td>0.545×10^6</td>
<td>2.191×10^6</td>
<td>1.048</td>
<td>1.039</td>
<td>0.130</td>
</tr>
</tbody>
</table>

Table 6 shows that the maximum feasible reduction of building oscillations is about 50% for maximum and RMS of total displacements. Furthermore, it can be seen that the soil type brings important effects on the structure vibrations. Generally, the soil with higher stiffness would decrease the maximum and RMS of total displacements, and slightly affects the maximum possible reduction. Comparing fixed base model with other three models indicates that ignoring the soil characteristics would result in the underestimation of TMD’s frequency and overestimation of TMD’s damping ratio. It also brings the underestimation of the maximum and RMS of displacement.

### Table 6: Vibration with and without TMD

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$u_{max}$</th>
<th>RMS</th>
<th>$u_{max}$</th>
<th>RMS</th>
<th>$u_{max}$</th>
<th>RMS</th>
<th>%Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Soil</td>
<td>4.23</td>
<td>2.03</td>
<td>2.08</td>
<td>1.03</td>
<td>50.85</td>
<td>49.16</td>
<td></td>
</tr>
<tr>
<td>Medium Soil</td>
<td>2.67</td>
<td>1.23</td>
<td>1.25</td>
<td>0.45</td>
<td>53.38</td>
<td>63.29</td>
<td></td>
</tr>
<tr>
<td>Dense Soil</td>
<td>2.26</td>
<td>0.86</td>
<td>1.09</td>
<td>0.38</td>
<td>52.07</td>
<td>55.63</td>
<td></td>
</tr>
<tr>
<td>Fixed Base</td>
<td>2.02</td>
<td>0.70</td>
<td>1.04</td>
<td>0.36</td>
<td>48.50</td>
<td>48.25</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2 shows Tabas earthquake acceleration spectrum, which was about 7.7 Richter and occurred in 16th September 1978 in Tabas, Iran. Figures 3 and 4 present the time response of structure (meter vs. second) with dense soil (with greater frequency) needs TMD with greater damping ratio.
and without TMD for the soft soil, respectively. Comparing these figures, it can be seen that the displacement is effectively reduced due to the TMD device. It is also evident that the displacement pattern is somehow different in the mentioned figures because of TMD effects. However, the TMD displacement pattern is nearly the same as the structure, while its amplitude is about 4 times greater than the building amplitude of vibration.

Figure 3: Time response of structure with TMD for soft soil

The time responses of structure with and without TMD for the medium soil are presented in figures 5 and 6, respectively. According to these figures, it is clear that the TMD has efficiently decreased the displacement of floors. Comparing these figures, it can be seen that the displacement patterns of the structure are nearly the same, but differ from the TMD pattern. The TMD amplitude is about 5 times greater than the building amplitude of vibration in this case.

Figure 5: Structure time response with TMD for medium soil

Figure 6: Structure time response without TMD for medium soil

The time responses of structure with and without TMD for the dense soil are shown in figures 7 and 8, and for the fixed base model are presented in figures 9 and 10, respectively. Considering these figures, the TMD has obviously decreased the displacement of stories. Comparison between these figures reveals that the displacement patterns of the structure are similar, but differ from the TMD pattern. However, The TMD and the structure displacements of dense soil and fixed base model are highly similar to each other. The maximum TMD displacement is about 5 times greater than the maximum building displacement of vibration in this case.

Figure 7: Structure time response with TMD for dense soil

Figure 8: Structure time response without TMD for dense soil
buildings. The understanding of earthquake oscillations, and helps the TMDs; considering soil effects. This study improves the mitigation of high-rise buildings. It is also implied how are advantageous devices earthquake vibration response of structures. It is also shown that the TMDs

Conclusions

In this paper, a mathematical model is developed to obtain the earthquake response of a high-rise building with TMD, considering SSI effects. The model is based on the time domain analysis. The ant colony optimization technique is utilized to obtain the optimum parameters for TMD. Mass, damping and spring stiffness quantities of TMD are assumed as the design variables, and the objective is to decrease the maximum and RMS of displacements.

The results show that the soil characteristics greatly influence on the favorite TMD parameters. It is indicated that the soil type also severely affects the time response of structures. It is also shown that the TMDs are advantageous devices earthquake vibration mitigation of high-rise buildings. It is also implied how ACO can be employed for the design of optimum TMDs; considering soil effects. This study improves the understanding of earthquake oscillations, and helps the designers to achieve the optimized TMD for high-rise buildings.

References


