Agent-based Centralized Fuzzy Kalman Filtering for Uncertain Stochastic Estimation

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***Absratct**: In this paper, we investigate the problem of agent-based centralized fuzzy Kalman filters observing an uncertain physical process with parametric uncertainties. An agent-based sensor network is a distributed system which consists of sensors with limited computational capabilities. In our agent-based sensor network we consider sensor agents and a moderator agent. Any of these sensor agents have limited computational capabilities and also may be affected by different noises. Agents derive the information in the form of fuzzy states from their fuzzy Kalman filters, the estimated fuzzy states would be transmitted to the moderator agent for aggregation and result sharing by any sensor agent. The moderator agent fuses the fuzzy estimations to generate the global state estimations which is highly reliable.*

Keywords: agent-based sensor network, centralized fuzzy Kalman filtering, fuzzy Kalman filter, possibility, result sharing.

1. Introduction

An agent-based sensor network is a distributed system consisting of a number of sensor nodes distributed over a spatial region. The sensor networks can be categorized as homogenous multi agent systems. While the agents structures are the same in this network, the agents must receive different inputs to have different outputs, otherwise all the agents will have the same performance and consequently the system behaves as a single agent. Therefore in these systems the agents (sensors) must distribute in the considered area to benefit from the multi agent structure in accomplishing their commission which is mostly estimating and tracking. In an agent-based sensor network, agents can improve the precision of the estimated variable through collaboration and can increase confidence and decrease uncertainty in the final result by result sharing amongst themselves[11].

The standard Kalman filter (SKF), which is an optimal state estimator, is traditionally implemented using probabilistic techniques. For covering the deficiencies of SKF, possibilistic fuzzy Kalman filter is introduced in [6,7]. In this fuzzy Kalman filter which was introduced by Matia et al., the states are fuzzy and they are characterized by trapezoidal possibility distributions instead of probability distributions. After the latest research [7], in [10] we introduced a fuzzy Kalman filter that can be implied for fuzzy state estimations where not only the states are fuzzy but also the the systems parameters are fuzzy too, also we modeled the uncertainties through triangular possibility distributions. Here we aim to benefit from the multi agent systems structures to model our intelligent sensor network and benefit from this structure for uncertainty reduction in the global result of the network.

The justification of using fuzzy variables instead of random ones is that, in many applications, information about the behavior of variables is possibilistic, rather than probabilistic[7]. Through this replacement we can overcome the deficiencies of SKF in modeling asymmetric uncertainties, error accumulation by propagating the probability distribution through nonlinear systems and extracting the process model from the human knowledge [6,7].

Several researchers have implemented different versions of what we could call a *fuzzy Kalman filter*. In [9] probabilistic Kalman filters used fuzzy rules to adapt process noise and observation noise. Longo et al. [5] apply fuzzy rules combined with a probabilistic Kalman filter, where fuzzy logic is only used for sensor fusion outside the state estimation process. Oussalah et al. [8] build a kind of possibilistic Kalman filter by extending the idea of interval Kalman filter to different α - cuts, in which the filter parameters are intervals. In [4] Layne et al. employed fuzzy relations to represent both the process and observation models.

Here we apply an agent-based structure consisting of centralized possibilistic fuzzy Kalman filters for uncertain stochastic estimation. In this paper we demonstrate the uncertain states and also parameters by triangular fuzzy numbers which are a common type of possibility distributions. The organization of the paper is as follows: Section 2 introduces the possibilistic fuzzy Kalman filter for processes with parametric uncertainties and fuzzy states. Section 3 describes the performance of an agent-based structure consisting centralized possibilistic fuzzy Kalman filters. Section 4 illustrates the procedure mentioned in section 3 for the case of radar 2D tracking. Finally the conclusion is drawn in section 5.

2. Possibilistic Fuzzy Kalman Filter

Generally in the processes that we would like to estimate their states, uncertainty can be categorized to parametric uncertainties and inherent variables uncertainties. In some systems we may face both of these uncertainties simultaneously. Uncertainty can be possibilistic or probabilistic or in the other words fuzzy or stochastic. In the case of fuzzy uncertainty the information about the model parameters and the behavior of the variables are possibilistic rather than probabilistic. The variables with possibilistic uncertainties can be specified by possibility distributions. One of the simplest form of possibility distribution is triangular distribution which can be defined by three numbers as below $(E(x) \sim \pi : possibility)$:

$$E(x) \sim \pi(a,b,c),\ \pi(x) = \begin{cases} 0 & x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{b-c} & b \le x \le c \\ 0 & c \le x \end{cases} \qquad (1)$$

[a,c] interval determines the base of the triangular and b is the vertex of the triangular which possibility is one. The possibility distribution may not be, in general, symmetric

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which is the usual case of sensor uncertainty. Furthermore the area of the distribution defined by

$$\alpha_x = \int_x \pi(x)dx \qquad (2)$$

Is not in general equal to one $(\alpha_x \neq 1)$ while in the probability distribution the corresponding area must be equal to one. In comparison with most of probability distributions, possibility distributions are not generally symmetric and their area is not necessarily equal to one [6,7].

In possibilistic distributions we use the concepts of center of gravity and uncertainty instead of mean and variance. As stated in [6] the definitions of center of gravity and uncertainty are respectively defined as below

$$\tilde{x} = \frac{\int_x x\pi(x)dx}{\alpha_x},\ P = \frac{\int_x (x-\tilde{x})^2 \pi(x)dx}{\alpha_x} \qquad (3)$$

Based on the concepts of possibility distributions, we introduce the fuzzy Kalman filter, for the system where the states behavior is fuzzy rather than stochastic and the systems parameters are uncertain, demonstrated by fuzzy numbers. In our FKF the process model and the observation models are demonstrated as below

$$\bar{x}(k+1) = \bar{\phi}\bar{x}(k) + \bar{v}(k+1) \qquad (4)$$

$$\bar{z}(k+1) = H(k+1)\bar{x}(k+1) + \bar{w}(k+1) \qquad (5)$$

Where k and k+1 are two consecutive instants of time, x(k) is the state, u(k+1) is the system input and z(k+1) is the observation. In the process model, $\bar{\phi}$ is a fuzzy matrix and its elements are triangular fuzzy numbers.

$$\bar{\phi} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix},\ E(A_i) \sim \pi(a_{i1}, a_{i2}, a_{i3}) \qquad (6)$$

In our computations six fuzzy variables are involved: the state estimation based on the previous observation $\bar{x}(k \mid k)$, the process noise $\bar{v}(k+1)$, the state estimation without having integrated the next observation $\bar{x}(k+1 \mid k)$, the measurement noise $\bar{w}(k+1)$, the observation estimation $\bar{z}(k+1)$ and the state estimation after having integrated the last observation $\bar{x}(k+1 \mid k+1)$, each of them has an associated possibility distribution. The center of gravity and uncertainty matrix of them are as below:

$$U[\bar{x}(k \mid k)] = P(k \mid k) \qquad (7)$$

$$\tilde{v}(k+1) = 0, U[\bar{v}(k+1)] = Q(k+1) \qquad (8)$$

$$U[\bar{x}(k+1 \mid k)] = P(k+1 \mid k) \qquad (9)$$

$$\tilde{w}(k+1) = 0, U[\bar{w}(k+1)] = R(k+1) \qquad (10)$$

$$U[\bar{x}(k+1 \mid k+1)] = P(k+1 \mid k+1) \qquad (11)$$

The fuzzy Kalman filter steps are exactly the same as in the probabilistic case. The only difference is in the uncertainty propagation where we have to propagate a fuzzy variable through the uncertain system. In this order any state estimation is accompanied by at least 3 runs of the KF for the 3 points (a,b,c) (Figure (1)) of triangular fuzzy variables and parameters, these are the boundary points of 2 α_levels of $\alpha = 0,1$ where b is the α_cut of $\alpha = 1$ and $[a,c]$ is the α_cut of $\alpha = 0$. The estimation procedure in any of these 3 runs is demonstrated as below $(i = 1,2,3)$

1) Prediction

$$x_i(k+1 \mid k) = \phi_i(k)x_i(k \mid k) + v_i(k+1) \qquad (12)$$

$$P_i(k+1 \mid k) = \phi_i P_i(k \mid k)\phi_i^T + Q(k+1) \qquad (13)$$

Where $\bar{x}(k \mid k)$ and $\bar{v}(k+1)$ are independent.

2) Observation

$$z_i(k+1) = H(k+1)x_i(k+1 \mid k) + w_i(k+1) \qquad (14)$$

$$S_i(k+1) = H(k+1)P_i(k+1 \mid k)H^T(k+1) + R(k+1) \qquad (15)$$

3) Correction

$$x_i(k+1 \mid k+1) = [I - W_i(k+1)H(k+1)]x_i(k+1 \mid k) + W_i(k+1)z_i(k+1) \qquad (16)$$

$$P_i(k+1 \mid k+1) = [I - W_i(k+1)H(k+1)]P_i(k+1 \mid k) \qquad (17)$$

The gain $W_i(k+1)$ which minimizes the uncertainty matrix $P_i(k+1 \mid k+1)$ is obtained by

$$W_i(k+1) = P_i(k+1)H^T(k+1)S_i^{-1}(k+1) \qquad (18)$$

Therefore in any time instance (k+1) we can obtain the fuzzy state $\bar{x}(k+1 \mid k+1)$, fuzzy Kalman gain $\bar{W}(k+1)$ and fuzzy uncertainty matrix:

$$x(k+1 \mid k+1) = \pi(x_1(k+1 \mid k+1), x_2(k+1 \mid k+1), x_3(k+1 \mid k+1)) \qquad (19)$$

$$\bar{W}(k+1 \mid k+1) = \pi(W_1(k+1 \mid k+1), W_2(k+1 \mid k+1), W_3(k+1 \mid k+1)) \qquad (20)$$

$$\bar{P}(k+1 \mid k+1) = \pi(P_1(k+1 \mid k+1), P_2(k+1 \mid k+1), P_3(k+1 \mid k+1)) \qquad (21)$$

3. Centralized Fuzzy Kalman Filter in an Agent-based Network

In an agent-based sensor network each sensor agent makes noisy observations of the states of the uncertain stochastic system (e.g. physical phenomena or moving object) and report his estimations to the moderator agent. the moderator agent aggregate the local estimations to result a global confidential estimation. In the proposed agent-based network, each sensor agent can estimate the location of the object with acceptable precision in a specified rejoin, named as sensing region. The sensor agents with higher sensing abilities have a sensing region with a larger radios. If the object locates in the sensing region of each sensor agent, then the sensor agent will estimate the object location through his own fuzzy Kalman filter. The sensor agents that the object is located in the overlapped part of their sensing regions, transfer their results to the moderator agent for result sharing and aggregation. Therefore the location of the object determines that which sensors must collaborate with each other through moderator for state estimatiom. Now assume that we have n agents that have estimated the fuzzy states of a system based on the procedure explained on the previous section. Any of these agents have transferred their fuzzy state estimations for aggregation to the moderator agent. Since the states are fuzzy, they must be combined through fuzzy arithmetic. The employed method for aggregating fuzzy states is introduced in [12]. For obtaining the aggregated fuzzy state, moderator should multiply the possibilities of all fuzzy states at any point

of the variable x and then normalize the obtained products between one and zero. Finally we can achieve the aggregated fuzzy number plotting the points of x (on x axis) and their corresponding normalized possibilities (on y axis). As an instance the aggregated fuzzy numbers (possibility distributions) of $E(x_1)=\pi(0.2,0.4,0.6)$ and $E(x_2)=\pi(0.3,0.5,0.7)$ depicted in Fig. (1.a), is shown in Fig. (1.b).

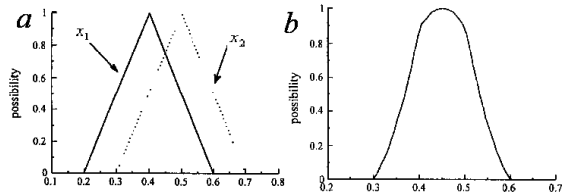

Figure 1: (a).Fuzzy numbers X1 and X2, (b) The aggregation of X1 and X2

But it should be noticed that for obtaining a reliable aggregation the moderator should influence the fuzzy state with less uncertainty more than other estimations.
Therefore the estimations must be weighted due to the defuzzified amount of their uncertainties. In this order any single agent would transfer its state estimation and its corresponding uncertainty to the moderator agent. The moderator agent defuzzifies the fuzzy uncertainties by considering the center of gravity of each uncertainty. Afterwards the fuzzy states would be weighted by the inverse of defuuzified uncertainties which should be normalized. In this way the moderator agent changes the fuzzy states based on their weights. To demonstrate that how this weighting influences the result sharing process. we consider that X_1 and X_2 depicted in Fig. (1.a) are 80% and fully reliable (weight=1) respectively, hence they would change to Figure (2.a) based on their weights and their aggregation can be shown in Fig. (2.b). It is considerable that the modal value of the aggregated fuzzy number (0.475) is nearer to the modal value of state X_2 which is more reliable than state X_1.

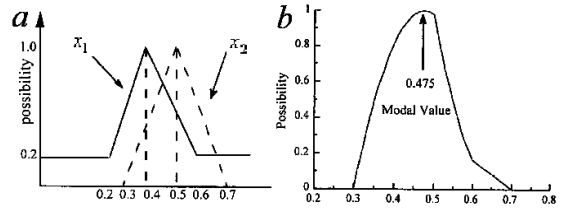

Figure. 2. (a).Weighted fuzzy numbers, (b). The aggregation of weighted fuzzy numbers

4. Case study 2D Radar Tracking

Through the example of 2D radar tracking demonstrated by the following uncertain system

$$\overline{X}(k+1)=\begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix}\overline{X}(k)+\overline{v}(k+1) \quad (22),$$

$$E(A_1)=\pi(1,1,1)=1, E(A_2)=\pi(0.5,1,1.5),$$

$$E(A_3)=\pi(1,1,1)=1,\ E(A_4)=\pi(0,0,0)=0 \quad (23)$$

$$\overline{z}(k+1)=\begin{bmatrix}1 & 0\end{bmatrix}\overline{X}(k)+\overline{w}(k+1) \qquad (24)$$

We show that, the more the sensor agents cooperate in result sharing, the higher is the precision and the lower is the uncertainty of the final result. At the first time the radar tracking is accomplished by an agent-based network consists of 2 sensor agents sensing the object and being affected by different noises and a moderator agent. In the second time we add another sensor agent to our network to investigate the effect of sensor agents increment.
Suppose that the object is located at

$$X(4)=\begin{bmatrix}\overline{x}(4)\\ \overline{y}(4)\end{bmatrix}$$ With $E(\overline{x}(4\,|\,4))\sim\pi(0.17,0.4,0.77)$

$E(\overline{y}(4\,|\,4))\sim\pi(0.28,0.6,0.88)$.

This means $P[\overline{x}(4\,|\,4)]=0.016$, $P[\overline{y}(4\,|\,4)]=0.015$ and we suppose that $\overline{x}(4\,|\,4)$ and $\overline{y}(4\,|\,4)$ are independent then $P(4\,|\,4)=\begin{bmatrix}0.016 & 0\\ 0 & 0.015\end{bmatrix}$

The object is located in the overlapped region of 2 sensor agents (Fig. (3.a)) by the following descriptions, therefore they do the procedure of estimation based on the mentioned computations in section 3. The information and the results of these computations in the last step (correction part) for each sensor is as below:
Sensor agent 1:

$$\overline{v}=\begin{bmatrix}\overline{v}_1\\ \overline{v}_2\end{bmatrix}, E(\overline{v}_1)\sim\pi(0.09,0.1,0.11),$$

$E(\overline{v}_2)\sim\pi(0.09,0.1,0.11)$, $Q=1.68\times10^{-5}$

$E(\overline{w})\sim\pi(-0.23,0,0.23)$, $R=0.0088$

It should be noticed that based on (24) the sensor output is just the $\overline{x}$ coordinate from the estimated state $\overline{X}=\begin{bmatrix}\overline{x}\\ \overline{y}\end{bmatrix}$, therefore the mentioned results are limited to $\overline{x}$ coordinate results.
Sensor agent 1 results:

$W_1(\overline{x}(5))=0.95737$, $\overline{x}_1(5\,|\,5)=0.974482$

$P_1(\overline{x}(5\,|\,5))=8.42\times10^{-4}$, $W_2(\overline{x}(5))=0.77898$

$\overline{x}_2(5\,|\,5)=1.0221013$, $P_2(\overline{x}(5\,|\,5))=6.85\times10^{-3}$

$W_3(\overline{x}(5))=0.255719$, $\overline{x}_3(5\,|\,5)=1.893136$,

$P_3(\overline{x}(5\,|\,5))=0.0111468$

Sensor agent 2:

$$\overline{v}=\begin{bmatrix}\overline{v}_1\\ \overline{v}_2\end{bmatrix}, E(\overline{v}_1)\sim\pi(0.14,0.15,0.16),$$

$E(\overline{v}_2)\sim\pi(.14,0.15,0.16)$, $Q=1.68\times10^{-5}$

$E(\overline{w})\sim\pi(-0.11,0.12,0.35)$, $R=0.0088$

Sensor agent 2 results:

$W_1(\overline{x}(5))=0.95737$, $\overline{x}_1(5\,|\,5)=0.976553$,

$P_1(\overline{x}(5\,|\,5))=8.42\times10^{-4}$, $W_2(\overline{x}(5))=0.778987$,

$\overline{x}_2(5\,|\,5)=1.033151$, $P_2(\overline{x}(5\,|\,5))=6.85\times10^{-3}$

$W_3(\overline{x}(5))=0.255719$ $\overline{x}_3(5\,|\,5)=1.928794$

$P_3(\overline{x}(5\,|\,5))=0.0111468$

These two sensor agents transfer their estimations and their uncertainties to the moderator agent.
In the second time we add a third sensor agent to the agent-based network that can cooperate with sensor1 and 2 in the process of radar tracking as depicted in Fig. (3.b).
Sensor agent 3:

$$\bar{v} = \begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \end{bmatrix}, E(\bar{v}_1) \sim \pi(0.04,0.05,0.06),$$

$$E(\bar{v}_2) \sim \pi(0.04,0.05,0.06), Q = 1.68 \times 10^{-5}$$

$$E(\bar{w}) \sim \pi(-0.35,-0.12,0.11),\ R = 0.0088$$

Sensor agent 3 results:

$$W_1(\bar{x}(5)) = 0.95737\ \bar{x}_1(5|5) = 0.9722905$$

$$P_1(\bar{x}(5|5)) = 8.42 \times 10^{-4},\ W_2(\bar{x}(5)) = 0.778987,$$

$$\bar{x}_2(5|5) = 1.011050,\ P_2(\bar{x}(5|5)) = 6.85 \times 10^{-3}$$

$$W_3(\bar{x}(5)) = 0.2557196,\ \bar{x}_3(5|5) = 1.8559,$$

$$P_3(\bar{x}(5|5)) = 0.0111468$$

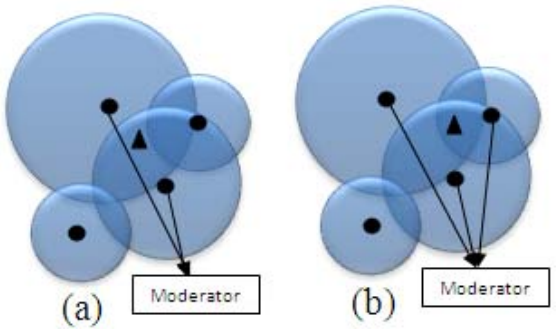

Figure. 3 An agent-based network (▲:object, ●:sensor agent)

In the next step the 3 sensor agents transfer their estimations and their uncertainties to the moderator agent.

In the situations where the agent-based network consists of 2 and 3 agent sensors the moderator agent receives all sensor agents estimations. The moderator defuzzifies and normalizes the outputs uncertainties as below:

a) Defuzzification

$$E(P_{sensor1}(x(5|5))) = E(P_{sensor2}(x(5|5))) =$$

$$E(P_{sensor3}(x(5|5))) = \pi(0.000842,0.00685,0.011146)$$

the defuzzified amount is 0.0063. This uncertainty (0.0063) is obviously lower than the corresponding uncertainty in the previous time instant which is equal to 0.016. It should be noticed that even the highest amount of output uncertainty (0.011146) which is possible, is lower than 0.016.

b) Normalization

In this section moderator agent reverses and then normalizes the defuzzified amounts of the sensors uncertainties to determine the estimations weights. As it is obvious while the defuzzified uncertainties are equal, all the weights would be equal to one. Therefore the estimated states will remain unchanged. For the first and second agent-based network which respectively consist of 2 and 3 sensor agents, the moderator agent obtains the aggregated fuzzy state 1 and 2 from the first and second network (shown in Fig. 4). We discussed in the defuzzification part that all sensors estimations are more accurate in comparison with their previous estimations, due to the amount of their defuzzy and fuzzy uncertainties. Also we can show that the aggregated state estimation of the network with 3 sensor agents is more reliable than individual agents estimations since through (3) the uncertainty in the nature of the aggregated outputs 2 is equal to 0.0405 which is lower than the uncertainty in the 3 estimated states which are respectively equal to 0.0440, 0.0476, 0.0417. furthermore the uncertainty of aggregated output 2 (0.0405) is lower than the aggregated output 1 (0.0441), this comparison shows that the increment of sensor agents in the space can decrease the amount of uncertainty.

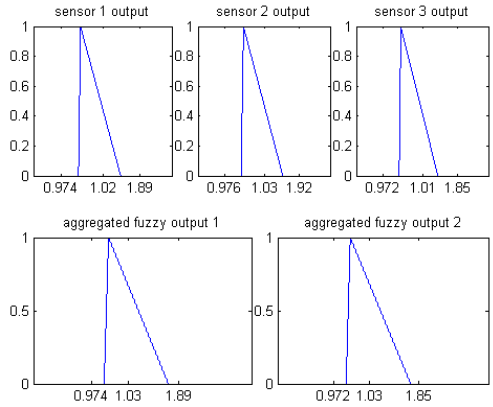

Figure 4: output estimation

5. Conclusion

In this paper we introduce an agent-based structure for centralized fuzzy Kalman filters, which can be implemented in the object tracking applications where the measured location of any object is accompanied by uncertainties. Also it was shown that the recommended fuzzy Kalman filter can cover the parameters uncertainties of the system and benefiting from the multi agent structure we can reduce the existing uncertainties in the final result. As shown in the 2D radar tracking case study it can be easily concluded that the aggregated output estimation is more reliable while the value of its uncertainty is less than the uncertainties of individual agents estimations.

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