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A NUMERICAL SCHEME OF HIGHER ORDER FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. We consider the numerical approximation of stochastic partial differential equation driven by additive space-time white noise. we introduce a new numerical scheme for the time discretization of the finite-dimensional SPDE, which we call the exponential Euler scheme, then present a theorem, which state the strong convergence of the above numerical scheme.

1. Introduction and Preliminaries

In this paper, we consider the numerical approximation of a SPDE with additive noise i.e. of the form

$$dU_t = [AU_t + f(U_t)]dt + dw_t, \quad U_0 = u_0 \qquad (1.1)$$

This scheme convergence with an overall rate faster than classic methods.

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2. Exponential Euler scheme

We will now apply a Galerkin projection to the spde (1.1), i.e. of the form

$$dU_t^N = [A_N U_t^N + f_N(U_t^N)]dt + dW_t^N \quad (2.1)$$

for $N \in \mathbb{N}$, where N-dimensional subspace X_N of $L_2(D)$ spanned by $\{e_1, ..., e_N\}$ and the projection $P_N := H \to H$ by $P_N v = \sum_{n=1}^{N} \langle e_n, v \rangle$,
$f_N := P_N f, \quad A_N := P_N A \quad W_t^N = P_N W_t \quad u_o^N := P_N u_0$
the SPDE (2.1) can be interpreted in the mild sense, see [4], i.e. as satisfy the integral equation

$$U_t^N = e^{A_N t u_0^N} + \int_0^t exp(A_N(t-s)) f_N(U_s^N) ds + \int_0^t exp(A_N(t-s)) dW_s^N. \quad (2.2)$$

Now, we introduce a numerical method to approximation U_t^N in time on the interval $[0, t]$, which we will call the exponential Euler scheme: let $V_0^{N,M} := u_0^N$ and define

$$V_{k+1}^{N,M} = e_N^A h V_k^{N,M} + A_N^{-1}(e^{A_n h} - I) f_N(V_k^{N,M}) + \int_{t_k}^{t_{k+1}} exp(A_N(t_{k+1} - s)) dW_s^N \quad (2.3)$$

with time-step $h = T/M$for some $M \in \mathbb{N}$ and discretization time $t_k = kh$ for $k = 0, 1, 2, ..., M$

Theorem 2.1. *suppose:*
(1) there exit sequences of real eigenvalues $0 < \lambda_1 \leq \lambda_2 \leq ...$ *and eigenfunction* $\{e_n\}_{n \geq 1}$ *of* A *such that the linear operator* $A : D(A) \subset H \to H$ *is given by*

$$Av = \sum_{n=1}^{\infty} -\lambda_n \langle e_n, v \rangle e_n$$

for all $v \in D(A)$ *with* $D(A) = \{v \in H | \sum_{n=1}^{\infty} |\lambda_n|^2 |\langle e_n, v \rangle|^2 < \infty\}$.
(2) There exit sequences $q_n \geq 0, n \geq 1,$ *of positive real numbers, a real number* $\gamma \in (0, 1)$ *such that*

$$\sum_{n=1}^{\infty} (\lambda_n)^{2\gamma - 1} q_n < \infty.$$

(3) The nonlinearity $f : H \rightarrow H$*is tow times continuously, Fréchet differentiable.*
(4) The initial value u_0 *is a* $D((-A)^{\gamma})$ *valued random variable, which satisfies* $\mathbb{E}|(-A)^{r}u_0|^4 < \infty$ *where* $\gamma > 0$ *is given in assumption 2.2 then there is a constant* $C_T > 0$ *such that*

$$sup(\mathbb{E}(|U_{t_k} - V_k|^2)^{1/2}) \leq C_T((\lambda)_N^{-\gamma} + \frac{log(M)}{M})$$

hold for all $N, M \geq 2$ *where* U_t *is the solution of SDE(1.1),* $V_k^{N,M}$ *is the numerical solution given by (2.3),* $t_k = T(K/M)$ *for* $k = o, 1, ..., M$ *and* $\gamma > 0$ *is the constant given in the above assumption*

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