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A Study on the Performance of Students' Mathematical Problem Solving Based on Cognitive Process of Revised Bloom Taxonomy

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The main objective of this research is to studying students' mathematical performance based upon cognitive dimension of Revised Bloom Taxonomy (RBT). A sample of 95 K11 school girls and 63 K11 school boys were tested on mathematics exam include 120 questions from K11 calculus book based on Revised Bloom Taxonomy. Data of this research was analyzed by MANOVA repeated measure and graphs error bars from SPSS (Statistical Package for the Social Sciences) software. Results obtained, indicate that there was difference between students' mathematical performance in each category of knowledge dimension according to cognitive process of Revised Bloom Taxonomy and students' mathematical performance would be decreased from remembering through creating in each category of knowledge dimensions. Overall, these findings could help to provide some practical implications for adapting problem solving skills and effective teaching/learning.

Keywords: Revised Bloom Taxonomy, problem solving, mathematics performance, cognitive process, knowledge dimension
MESC Classification: C30
MSC2010 Classification: 97C30.

INTRODUCTION

Mathematics is a universal subject, so much a part of life that anyone who is a partici-

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pating member of society must know mathematics. Students' mathematical achievement, however, is ultimately determined and limited by the opportunities they have had to learn. (Moenikiaa & Zahed-Babelanb, 2010). "All students must learn to think mathematically, and they must think mathematically to learn" (Kilpatrick, Swafford & Findell, 2001). So mathematics educators for improving students mathematical performance, struggle with the design and implementation of standards based mathematics curriculums, authentic mathematics assessments, and accountability programs. Since publication of Bloom's Taxonomy of Educational Objectives in 1956, numerous changes have occurred in our culture that influence how we think about and practice education. It was provided for classification of educational objectives, in particular to help teachers, administrators and researchers to discuss curricular and evaluation problems with greater precision (Bloom, 1994).

Bloom's taxonomy was first described as a hierarchical model for the cognitive domain in 1956 (Bloom *et al*, 1956). It consists of six skill levels of learning which increase in complexity starting with knowledge, comprehension, application, analysis, synthesis and evaluation. In other words, Bloom identified six levels within the cognitive domain, from simple recall or recognition of facts, as the lowest level, through increasing more complex and abstract mental levels, to the highest order which is classified as evaluation.

Kathwohl (2002) believed that Bloom explained the cognitive taxonomy as a more than a measurement tool. It could serve for example as a basis for determining for particular course or curriculum. Moreover, as a mean for determining the congruence of educational objectives, activities and assessment in a unit, course or curriculum.

A notable weakness in the original Bloom's taxonomy was the assumption that cognitive processes are ordered on a single dimension of simple to complex behavior (Furst, 1994). Moreover, the structure of the original taxonomy was a cumulative hierarchy, because the classes of objectives were arranged in order of increasing hierarchy. It was cumulative because each class of behaviors was presumed to include all the behaviors of the less complex classes (Krietzer & Madaus, 1994). This means that the mastery of each simpler category was prerequisite to mastery of the next more complex one (Krathwohl, 2002).

Recognizing some limitations of Bloom's taxonomy, new approaches and theories to learning such as Information Processing Theory (IPT), Constructivism and Metacognition make students to be responsible for their own thinking and learning. All these theories and approaches see learning as a proactive, requiring self-initiated motivational and behavioral process as well as metacognitive ones (Zimmerman, 1998). Smith, Coupland, & Stephen (1996) have suggested modifications to it in order to make it compatible with the purpose of assessing students' understanding in mathematics. As a result, the model was revisited in 2001 by Anderson and a team of cognitive psychologists'. A number of significant changes were made to the terminology and structure of the taxonomy (Anderson et al., 2001). New knowledge of how students learn as well as how teachers plan lessons, teach learners, and assess learning has been incorporated into a revision of Bloom's Taxonomy of Education Objectives. Anderson et al. (2001) have made some apparently minor but actually significant modifications, to come up with remembering, understanding, applying, analyzing, evaluating and creating. The names of six major categories in Bloom Taxonomy were changed from noun to verb forms in Revised Bloom Taxonomy. As the taxonomy reflects different forms of thinking and thinking is an active process, verbs were used rather than nouns. Revised Bloom Taxonomy employs the use of 24 verbs that creating collegial understanding of student behavior and learning outcome. The subcategories of the six major categories were also replaced by verbs and subcategories were recognized. The lowest level of the original, knowledge was renamed and become remembering. Comprehension and synthesis were re-titled to understanding and creating respectively, in order to better reflect the nature of the thinking defined in each category. To minimize the confusion, comparison image are appeared in Figure (Based on Schultz, 2005).

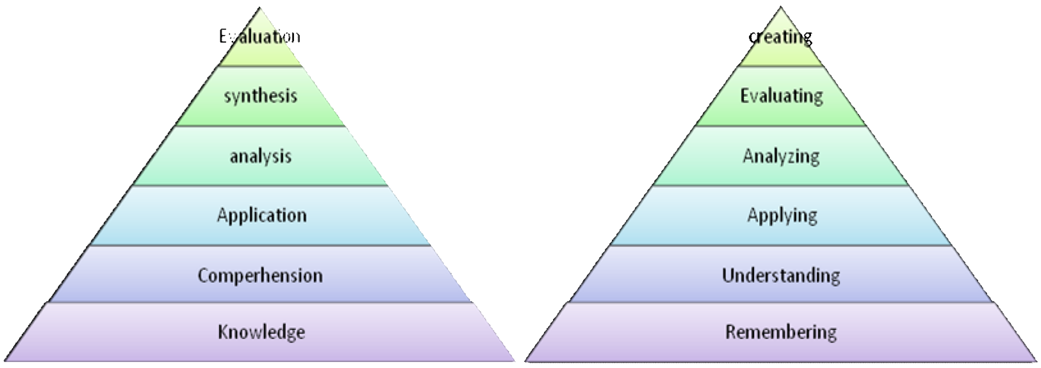


Figure 1. Original Term of Bloom Taxonomy and New Term OF RBT

The most considerable change in the RBT is the movement from one to two dimensions, which is the consequence of adding products. The Revised Bloom Taxonomy divides the noun and verb components of the original knowledge into two separate dimensions: the knowledge dimension (noun aspect) and the cognitive process dimension (verb aspect) (Krathwohl, 2002).As represented in Table 1, the intersection of the knowledge and cognitive process categories form 24 separate cells. The knowledge dimension on the side is comprised of four levels that are defined as factual, conceptual, procedural and metacognitive. The cognitive process dimension across the top of the grid consists of six levels that are defined as *Remembering, Understanding, Applying, Analyzing, Eva-*

luating and *Creating*. Each level of both dimensions of the table is subdivided.

Table1. The Two Dimensional Taxonomy

	The Cognitive Process Dimension					
Knowledge Dimension	1. Remembering	2. Understanding	3. Applying	4. Analyzing	5. Evaluating	6. Creating
A. Factual Knowledge						
B. Conceptual Knowledge						
C. Procedural Knowledge						
D. Metacognitive Knowledge						

The main aim of the present study is to identify students' difficulties associated with mathematical problem solving, in particular calculus area. The focus of this research was to provide a profile of students' mathematical performance in the different cognitive cells of the Revised Bloom Taxonomy. Thus the main question addressed here is: Is there any difference between students' performance of different cells in RBT?

In an attempt to answer this question the following objectives were sought. The first objective of the study was to discover whether there was any difference between students' mathematical performance in each knowledge categories (*i.e.*, factual, conceptual, procedural, and metacognitive) according to cognitive processes (*i.e.*, remembering, understanding, applying, analyzing, evaluating, and creating) of Revised Bloom Taxonomy. The second objective was to find whether students' mathematical performance would be decreased from remembering through creating in each category of the knowledge dimension.

METHOD

Participants

95 K11 school girls and 63 K11 school boys were selected from high schools of Mashhad (Khorasan Province) using random multistage stratified sampling design.

Procedures

The research instruments were mathematical tasks for different items in the RBT. Our test had 120 mathematics questions from K11 calculus book based on RBT that researchers were designed it and K11 mathematics teachers accepted it's validity. Each 5 questions were examined one of the cells in Revised Bloom Taxonomy. We have 24 cells so 120 questions are needed to cover all of them. Reliability coefficient (cronbache's α) for 24 cells was estimated to be 0.75. Researchers mentioned that each question may be examined more than one cells but in this research we hypothesis (without loss of generality) each question, examined just one cell. The participants answered this test in 3 parts that each part contains 40 questions:

Part one examined remembering and understanding cells (including: remembering factual knowledge, conceptual knowledge, procedural knowledge, metacognitive knowledge, understanding factual knowledge, conceptual knowledge, procedural knowledge, and metacognitive knowledge).

Part two examined applying and analyzing cells (including: applying factual knowledge, conceptual knowledge, procedural knowledge, metacognitive knowledge, analyzing factual knowledge, conceptual knowledge, procedural knowledge and metacognitive knowledge).

Part three examined evaluating and creating cells (including: evaluating factual knowledge, conceptual knowledge, procedural knowledge, metacognitive knowledge, creating factual knowledge, conceptual knowledge, procedural knowledge and metacognitive knowledge).

It seems to be more beneficial to define the new term of this taxonomy before introducing any mathematical questions based on RBT:

Remembering: Retrieving, recognizing and recalling relevant knowledge from long term memory.

Understanding: Constructing meaning from oral, written and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing and explaining.

Applying: Carrying out or using a procedure through executing or implementing.

Analyzing: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing and attributing.

Evaluating: Making judgments based on criteria and standards through checking and critiquing.

Creating: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning or producing (Anderson et al., 2001)

Here are some typical mathematical questions of each Revised Taxonomy cells. More questions would be found in the Appendix. First we explain a sample of remembering metacognitive knowledge questions:

Sample Question 1. Which method is better for solving this equation?

$$x^2 - \frac{2}{3}x - \frac{35}{9} = 0$$

1) Delta method 2) Perfect square method
3) Drawing the graph 4) Factorization method

Students for answering this question should remember the methods that they can solve quadratic equations according to the structure of the knowledge dimension of the Revised Taxonomy, strategic knowledge is metacognitive knowledge therefore according to the equations and it's coefficient, students should choose perfect square method so this question is a remembering metacognitive knowledge question. Now we describe a sample of understanding factual knowledge questions:

Sample Question 2. Which one is the symmetry axis of even functions and which one is the symmetry center of odd functions?

1) x-axis, origin of coordinate 2) y-axis, origin of coordinate
3) Line: $y = x$, point: $(-1, -1)$ 4) x-axis, point: $(-1, -1)$

Knowledge of terminology ,according to the structure of the knowledge dimension of the Revised Taxonomy is a factual knowledge and students for answering this question should know the definition of even and odd functions and also should know the definition of symmetry center and axis so interpreting and inferring these definition lead students to choose, choice2 also according to the structure of the cognitive process of the RBT interpreting and inferring are part of understanding so this question is a understanding factual knowledge question. For the third sample we choose applying conceptual knowledge questions:

Sample Question 3. Consider that profit or loss of a factory is a function of

$$t:\ f(t) = 2t^2 - 4t - 6.$$

When the factory doesn't have any profit or loss?

1) $-1,\ 1$ 2) $3,\ 1$ 3) $-1,\ 3$ 4) $-3\ \ -1$

In RBT conceptual knowledge defined as the interrelationships among the basic elements within a larger structure that enable them to function together and also it has Knowledge of theories, models, and structures so students for answering this questions should apply theories about where functions are equal to zero therefore this is a applying conceptual knowledge questions. Now we explain sample of analyzing conceptual knowledge questions:

Sample Question 4. In the equation $|x-\alpha|+|x-b|=k>0$, how many of these statements are true?

A) If $k>|b-a|$ then the equation has two roots.
B) If $k=|b-a|$ then the equation has an infinite root
C) If $k<|b-a|$ then the equation has no root.

1) 0 2) 1 3) 2 4) 3

For answering this question, students should differentiating and organizing this equation $|x-\alpha|+|x-b|=k>0$ and should know its graph to answer it. For drawing its graph, students should know the concept of absolute value and its graph also organizing and differentiating are part of analyzing (according to cognitive process dimension) so this is an analyzing conceptual knowledge question. A sample of evaluating procedural knowledge questions is shown below:

Sample Question 5. Which one is equal to $y=x+2$?

1) $y=\dfrac{x^2-4}{x-2}$ 2) $y=2+\sqrt{x^2}$

3) $y=\dfrac{x^3+2x^2+x+2}{x^2+1}$ 4) $y=\sqrt{x^2+4x+4}$

According to the knowledge dimension of the Revised Bloom Taxonomy, procedural knowledge defined as How to do something; methods of inquiry, and criteria for using skills, algorithms, techniques, and methods. Students for answering this question should have the ability of evaluating (consist of checking and critiquing), domain of these 5 functions and their equations to determine equal functions so this is a procedural knowledge question. For the last sample we describe a creating metacognitive knowledge question:

Sample Question 6. In which condition $f: R \to Q$ is continuous?

1) When the range of f are natural numbers. 2) When f is an injective function.
3) When f is a surjective function. 4) When f is a constant function.

Researchers should note that creating questions that used in this study, chosen from objectives that doesn't exist directly in the K11 mathematics book so students need to think and use their mathematics knowledge to create new objectives and theories. And also we know that conditional knowledge is a part of metacognitive knowledge (according to knowledge dimension of RBT) so this question is a creating metacognitive knowledge question. In this study researchers are comparing students' mathematical performance in each cognitive process, in knowledge dimension of the Revised Bloom Taxonomy.

RESULTS

Comparing factual knowledge in cognitive process dimension

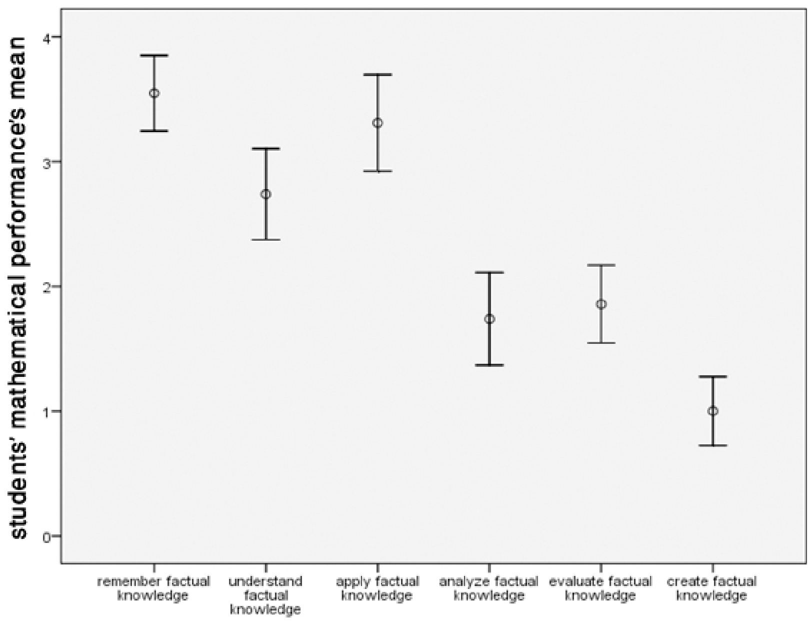
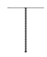


Figure 2. Comparing factual knowledge in cognitive process dimension

Using MANOVA repeated measures we obtain a p-value less than 0.001 regarding Hotelling's statistic so the hypothesis of equality of mean's students' mathematical performance in these 6 cells (remembering factual knowledge, understanding factual knowledge, applying factual knowledge, analyzing factual knowledge, evaluating factual knowledge, creating factual knowledge) were rejected. Graph error bar has shown the difference between students' mathematical performance in factual knowledge according to cognitive process of the Revised Bloom Taxonomy:

The circle in graph 1 shows the mean of the response variable and each of



shows upper and lower boundaries for a 95 percent confidence interval which means, the mean of variable with 95 percent probability is in the range that the graph denoted and also we should say that two or more mean's groups haven't significant difference if there is a horizontal line that intersects corresponding vertical lines.

So from this graph, It can be seen that: students are more successful in answering remembering factual knowledge questions than understanding, analyzing, evaluating and creating factual knowledge questions. There isn't any significant difference between answering remembering factual knowledge questions and applying factual knowledge questions nevertheless students in this study are a little better in answering remembering factual knowledge questions than applying factual knowledge questions. There are not any significant differences between answering analyzing factual knowledge questions and evaluating factual knowledge questions. Students are less successful in answering creating factual knowledge questions than each 5 parts.

Comparing conceptual knowledge in cognitive process dimension

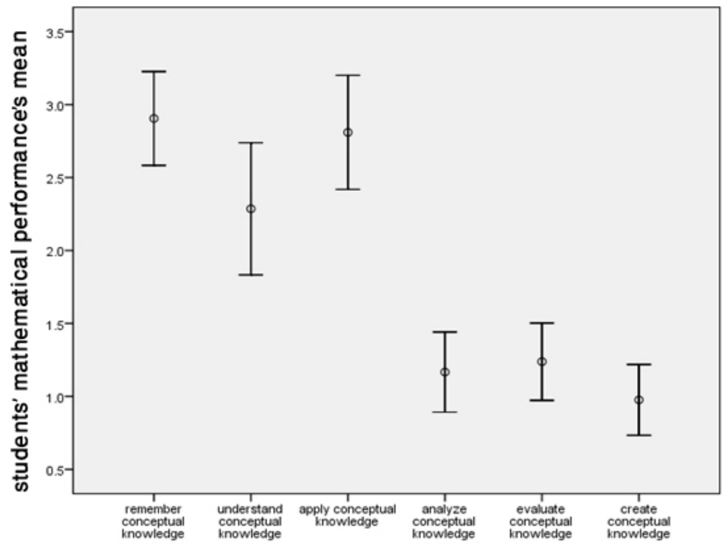


Figure 3. Comparing conceptual knowledge in cognitive process dimension

Using MANOVA repeated measures we obtain a p-value less than 0.001 regarding Hotelling's statistic so the hypothesis of equality of mean's students' mathematical performance in these 6 cells (remembering conceptual knowledge, understanding conceptual knowledge, applying conceptual knowledge, analyzing conceptual knowledge, evaluating conceptual knowledge, creating conceptual knowledge) were rejected. Graph error bar has shown the difference between students' mathematical performance in conceptual knowledge according to cognitive process of the Revised Bloom Taxonomy:

So from this graph, It can be seen that: Students are more successful in answering remembering conceptual knowledge questions than analyzing, evaluating and creating conceptual knowledge questions. There isn't any significant difference between answering remembering conceptual knowledge questions, understanding conceptual knowledge questions and applying conceptual knowledge questions nevertheless Students in this study are better in answering remembering and applying conceptual knowledge questions than understanding conceptual knowledge questions. There isn't any significant difference between answering analyzing conceptual knowledge questions, evaluating conceptual knowledge questions and creating conceptual knowledge questions nevertheless Students in this study are a little better in answering evaluating conceptual knowledge questions than creating conceptual knowledge questions.

Comparing procedural knowledge in cognitive process dimension

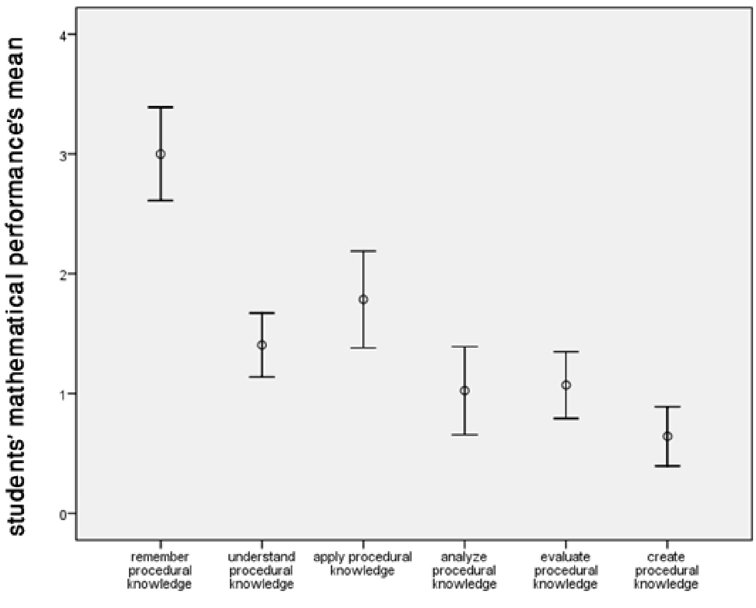


Figure 4. Comparing procedural knowledge in cognitive process dimension

Using MANOVA repeated measures we obtain a p-value less than 0.001 regarding Hotelling's statistic so the hypothesis of equality of mean's students' mathematical performance in these 6 cells (remembering procedural knowledge, understanding procedural knowledge, applying procedural knowledge, analyzing procedural knowledge, evaluating procedural knowledge, creating procedural knowledge) were rejected. Graph error bar has shown the difference between students' mathematical performance in procedural knowledge according to cognitive process of the Revised Bloom Taxonomy:

So from this graph, It can be seen that: Students are more successful in answering remembering procedural knowledge questions than each 5 parts. There isn't any significant difference between answering understanding procedural knowledge questions and applying procedural knowledge questions nevertheless Students in this study are better in answering applying procedural knowledge questions than understanding procedural knowledge questions. There isn't any significant difference between answering analyzing procedural knowledge questions, evaluating procedural knowledge questions and creating procedural knowledge questions nevertheless learners in this study are better in answering evaluating procedural knowledge questions than creating procedural knowledge questions.

Comparing metacognitive knowledge in cognitive process dimension

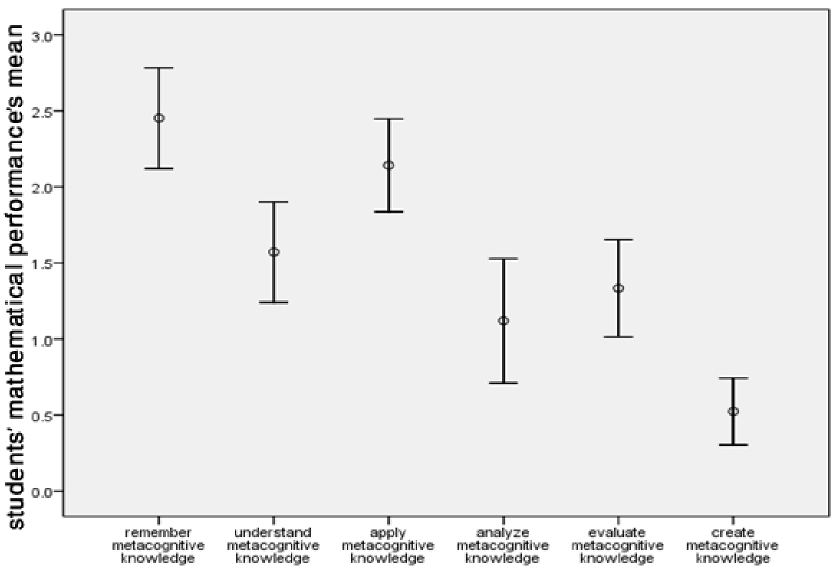


Figure 5. Comparing metacognitive knowledge in cognitive process dimension

Using MANOVA repeated measures we obtain a p-value less than 0.001 regarding

Hotelling's statistic so the hypothesis of equality of mean's students' mathematical performance in these 6 cells (remembering metacognitive knowledge, understanding metacognitive knowledge, applying metacognitive knowledge, analyzing metacognitive knowledge, evaluating metacognitive knowledge, creating metacognitive knowledge) were rejected. Graph error bar has shown the difference between students' mathematical performance in metacognitive knowledge according to cognitive process of the Revised Bloom Taxonomy:

So from this graph, It can be seen that: Students are more successful in answering re ember metacognitive knowledge questions than understanding, analyzing, evaluating and creating metacognitive knowledge questions. There isn't any significant difference between answering understanding metacognitive knowledge questions and applying metacognitive knowledge questions nevertheless Students in this study are better in answering applying metacognitive knowledge questions than understanding metacognitive knowledge questions. There isn't any significant difference between answering analyzing metacognitive knowledge questions and evaluating metacognitive knowledge questions nevertheless students in this study are a little better in answering evaluating metacognitive knowledge questions than analyzing metacognitive knowledge questions. Students are less successful in answering creating metacognitive knowledge questions than each 5 parts.

Comparing mathematics performance in cognitive performance

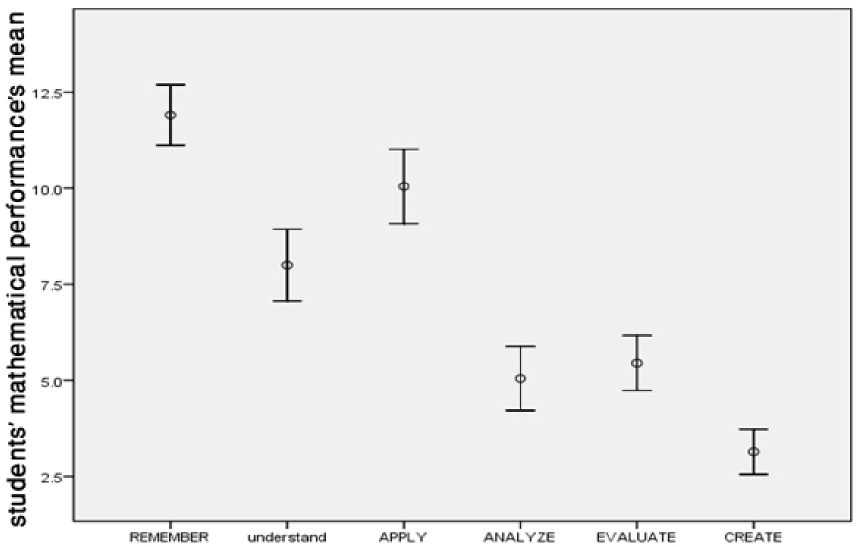


Figure 6. Comparing mathematics performance in cognitive process dimension

Using MANOVA repeated measures we obtain a p-value less than 0.001 regarding Hotelling's statistic so the hypothesis of equality of mean's students' mathematical performance in these parts were rejected. Graph error bar has shown the difference between students' mathematical performance in knowledge dimension of RBT.

From this graph, it can be seen that: Students performed better in answering remembering questions than others and they less successful in answering creating questions than others. There were significant differences between student mathematical performance in each part of cognitive processes except to students' mathematics performance in analyzing and evaluating mathematics objectives. From remembering through creating students' mathematics performance were decreased.

The first objective of the study that was to discover whether there was any difference between students' mathematical performance in each knowledge categories according to cognitive processes of Revised Bloom Taxonomy was accepted with these *p*-values:

Table2. *p*-values of each knowledge dimension

Title	P-value
Factual knowledge	Less than 0.001
Conceptual knowledge	Less than 0.001
Procedural knowledge	Less than 0.001
Metacognitive knowledge	Less than 0.001

The second objective which was to find whether students' mathematical performance would be decreased from remembering through creating in each category of the knowledge dimension was accepted because the superiority of the mathematical mean scores was in remembering, applying, understanding, evaluating, analyzing and then creating respectively. So researcher seen that mathematical performance from remembering thorough creating was decreased.

DISCUSSION

There is a strong movement in education to incorporate problem solving as a key component of the curriculum (Kirkley, 2003). The need for learners to become successful problem solvers has become a dominant theme in many national standards (AAAS, 1993; NCTE, 1996; NCTM, 1989; NCTM, 1991).

The structure of the Revised Taxonomy table matrix provides a clear, concise visual representation (Krathwhol, 2002) of the alignment between standards and educational goals, objectives, products and activities. Nowadays, mathematics teachers should make tough decisions about how to send their classroom time like pieces of huge puzzle, everything must fit properly. The RBT table clarifies the fit of each mathematics lesson plan's purpose, essential questions, goal or objectives. The Revised Taxonomy includes specific verb and product linkage with each levels of the cognitive process dimension. However, due to its 19 subcategories and two dimensional organizations, there is more clarity and less confusion about the fit of a specific verbs or product to a given level (Forehand, 2005). Thus the Bloom's Revised Taxonomy could offers mathematics teachers and even more powerful tool to help design their lessons and mathematical tasks plans.

We knew that assessment was a major issue for accreditation, and we'd paid an experienced consultant to provide us with assistance. Assessment, we were told, involves objectives. The objectives need to tie into measurable outcomes. Those outcomes need to be expressed in verb-first propositions about student behavior (Booker, 2007). Those behaviors need to reflect Revised Bloom's Taxonomy.

Since you're college teachers, none of your course objectives should be at the first level of the cognitive process and as researchers seen in this study because of mathematics teachers' methods; students' mathematical performance in higher thinking level was very weak and disappointing.

A Lot of application of using the Revised Taxonomy was explain by Krathwohl (2002) but focused of this study was in assessment. The main purpose of the present study was to highlight the pedagogically significant features of the Revised Taxonomy for example, the move from one dimension to two dimensions, the inclusion of the metacognitive knowledge category and etc. In this study, the researchers were compared students mathematical problem solving in each knowledge dimension according to cognitive processes of Revised Taxonomy.

According to results, in each category of knowledge dimension (*i.e.*, factual, conceptual, procedural, metacognitive) students performed better in remembering mathematics objective than each five parts and after that they performed better in applying mathematics objective and then understanding mathematics objectives. But generally there were not significant differences between mathematical performance in analyzing and evaluating mathematics objectives. Finally they were less successful in creating mathematics objectives (Figure 6). So researchers seen that students mathematical performance were decreased regularly. As researchers seen that in these graphs, students mathematical performance were better in applying mathematics questions than understanding mathematics questions and it happened because students can solve many mathematics problems without understanding the concepts. They just use the algorithms that suitable for the questions .Researchers seen that many students can solve questions about limit and derivative without knowing the concept of them. So teachers should note this problem and they help their students to improve their understanding of mathematics objectives. Also from this graphs researchers seen that, students had serious weakness in analyzing, evaluating and creating mathematics objectives. The researchers believed that students have not necessary skills in these categories. Therefore, textbooks and mathematics teachers should pay attention to these students difficulty. They should teach much more about knowledge of cognitive in general as well as awareness and knowledge of students own cognition in the mathematical activities, in particular problem solving. Students should learn to cope with misconceptions and correct themselves. Students should be encouraged to learn methods of questioning inquiry, criteria for using mathematics problem solving skills algorithms, techniques and methods. In addition they should help to be able to interrelate among the basic mathematical elements function together.

According to the present study, it could be suggested that the mathematical tasks are arranged based on BRT. This method may help students to do better in different area of mathematical problem solving.

In addition this could reduce the noise and overloading of students working memory. (Alamolhodaei, 2009). When teachers replace their mathematics questions to the questions that consist of RBT 24 parts, they may find more insight of the level of students understanding. This knowledge could help them to be familiar with mathematics education problems and students difficulties.

Researchers may suggest mathematics education researcher to assess students' mathematics performance and problem solving according to Revised Bloom Taxonomy in more advance mathematical areas. This research completely comparing 24 parts of Revised Bloom Taxonomy so mathematics education researchers can be familiar to mathematics questions according to Revised Bloom Taxonomy also we can compare each factor that influence mathematics learning and teaching in 24 parts and compare them together nevertheless the important role in such researches are designing mathematics questions. Now researchers of this study are studying the influence of working memory, field dependence/independence, mathematics attention, and multiple intelligence in learning mathematics according to Revised Bloom Taxonomy.

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APPENDIX

1. A sample of remembering factual knowledge questions:

Which one is the domain of the function $\frac{f}{g}$?

1) $D_{\frac{f}{g}} = D_f \cap D_g$
2) $D_{\frac{f}{g}} = D_f \cdot D_g$?
3) $D_{\frac{f}{g}} = D_f \cup D_g - \{x \mid g(x) = 0\}$
4) $D_{\frac{f}{g}} = D_f \cap D_g - \{x \mid g(x) = 0\}$

2. A sample of remembering conceptual knowledge questions:

If an equation has one real double root then:

1) Graph of a function is tangent to x-axis
2) Graph of a function is tangent to y-axis
3) Graph of a function doesn't intersect with x-axis
4) Graph of a function doesn't intersect with y-axis

3. A sample of remembering procedural knowledge questions:

$-5 < x - 11 < 3$? Which one is equivalent to

1) $|x - 10| < 4$ 2) $|x + 2| < 4$ 3) $|x - 9| < 5$ 4) $|x + 5| < 2$

4. A sample of remembering metacognitive knowledge questions:

Which method is better for solving this equation?

$$x^2 - \frac{2}{3}x - \frac{35}{9} = 0$$

1) Delta method 2) Perfect square method
3) Drawing the graph 4) Factorization method

5. A sample of understanding factual knowledge questions:

Which one is the symmetry axis of even functions and which one is the symmetry center of odd functions?

1) X-axis, origin of coordinate 2) Y-axis, origin of coordinate
3) Line: $y = x$, point: $(-1, -1)$ 4) X-axis, point: $(-1, -1)$

6. A sample of understanding conceptual knowledge questions:
What is the ratio between the graphs of $f(kx)$ and $f(x)$, if $0 < k < 1$?

1) The graph of $f(kx)$ is more expanded than the graph of $f(x)$ in the horizontal axis
2) The graph of $f(kx)$ is more compact than the graph of $f(x)$ in the horizontal axis
3) They have no difference.
4) None of them is correct.

7. A sample of understanding procedural knowledge questions:
Which one is the range of the function

$$f(x) = \frac{x^3 - 4x}{x^2 - 2x}?$$

1) R 2) $R - \{0, 2\}$ 3) $R - \{0, 4\}$ 4) $R - \{2, 4\}$

8. A sample of understanding metacognitive knowledge questions:
Why every strictly increasing function is injective?

1) Because each strictly increasing function is continuous and each continuous function is injective.
2) The above statement can be proven according to these objectives $x_1 < x_2 \to x_1 \neq x_2$, $f(x_1) < f(x_2) \to f(x_1) \neq f(x_2)$ and the definition of strictly increasing function.
3) Because every strictly increasing function is derivable and every derivable function is injective.
4) The choice 1 and 2 are correct.

9. A sample of applying factual knowledge questions:
Which one is an application of injective function?

1) Recognizing that a function is continuous or not
2) Recognizing that a function is derivable or not
3) Recognizing that a function is invertible or not
4) None of them is correct.

10. A sample of applying conceptual knowledge questions:
Consider that profit or loss of a factory is a function of $t : f(t) = 2t^2 - 4t - 6$.
When the factory doesn't have any profit or loss?

1) –1, 1 2) 3, 1 3) –1, 3 4) –3,–1

11. A sample of applying procedural knowledge questions:
Consider $y = f(x)$ in the interval $[0, 1]$ with the range $[-1, 1]$.
Find range of $y = 2f(1-x)+1$.

1) [–2, 2] 2) [–1, 3] 3) [–1, 2] 4) [0, 3]

12. A sample of applying metacognitive knowledge questions:
Why $f(x)=1$ and $g(x) = tg\, x \cot x$ aren't equal?

1) Because they have different expressions
2) Because they have different domains
3) Because they have different ranges
4) Choice 1 and 2 are correct.

13. A sample of analyzing factual knowledge questions:
Which one is false according to the equation: $ax^2 + bx + c = 0$?

1) If $\frac{c}{a} > 0$ then two roots have the same sign.

2) If $\frac{c}{a} < 0$ then two roots have different signs.

3) If $\frac{c}{a} = 1$ then two roots are inverse.

4) If $0 < \frac{-b}{a}$ then the sum of two roots are negative.

14. A sample of analyzing conceptual knowledge questions:
In the equation $|x-a| + |x-b| = k > 0$, how many of these statements are true?

A) If $k > |b-a|$ then the equation has two roots.
B) If $k = |b-a|$ then the equation has an infinite root
C) If $k < |b-a|$ then the equation has no root.

1) 0 2) 1 3) 2 4) 3

15. A sample of analyzing procedural knowledge questions:
If this equation $(x-a)(x-b)+1=0$ has two real roots then which one has two real roots?

1) $(x-a)(x-b)+2=0$ 2) $(b-x)(x-a)-2=0$
3) $(x-a)(x-b)-2=0$ 2) $(x-a)(x-b)+5=0$

16. A sample of analyzing metacognitive knowledge questions:

Which one is true?

1) $f(x)=[x]$ in real number is surjective because each line that is parallel to x axis intersects $f(x)$.
2) $f(x)=[x]$ in real number is injective because each line that is parallel to x axis just intersects with a point of $f(x)$.
3) $f(x)=[x]$ is a strictly increasing function because Its derivatives at any point is greater than zero.
4) $f(x)=[x]$ in every interval that doesn't include integer number is constant function .

17. A sample of evaluating factual knowledge questions:
The symmetry of (x, y) with respect to x-axis is...
And the symmetry of (x, y) with respect to y-axis is …

1) $(-x,\ y),(x,\ y)$ 2) $(x,-y),(-x,\ y)$
3) $(x,\ y),(-x,\ -y)$ 4) $(-x,-y),(-x,y)$

18. A sample of evaluating conceptual knowledge questions:
Which one is incorrect about $f(x)=[x]$?

1) $(n\ times)\ [x+[x+[x+\cdots]]]=n-1[x]$
2) $[nx]=[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\cdots+\left[x+\frac{n-1}{n}\right]$
3) $0 \le x-[x]<1$
4) $[x]+[-x]=\begin{cases}0 & x \in Z\\ 1 & x \in Z\end{cases}$

19. A sample of evaluating procedural knowledge questions:
Which one is equal to $y=x+2$?

1) $y=\dfrac{x^2-4}{x-2}$ 2) $y=2+\sqrt{x^2}$
3) $y=\dfrac{x^3+2x^2+x+2}{x^2+1}$ 4) $y=\sqrt{x^2+4x+4}$

20. A sample of evaluating metacognitive knowledge questions:
Why the range of $f(x)=ax^2+bx=c$ is

$$\left(-\infty, f\left(\tfrac{-b}{2a}\right)\right] \text{ or } \left[f\left(\tfrac{-b}{2a}\right),\ -\infty\right)?$$

1) Because this function has two roots.

2) Because the vertex of parabola is $\frac{-b}{2a}$ and when $a > 0$ the function is strictly increasing and when $a < 0$ the function is strictly decreasing.
3) Because the vertex of parabola is $\frac{-b}{2a}$ and when $a > 0$ the minimum point of $f(x)$ is $\frac{-b}{2a}$ and when $a < 0$ the maximum point of is $f(x)$ is $\frac{-b}{2a}$.
4) When a, c have the same sign the range of $f(x)$ is $\left(-\infty, f\left(\frac{-b}{2a}\right)\right]$ and when a, c have different sign the range of $f(x)$ is $\left[f\left(\frac{-b}{2a}\right), -\infty\right)$.

21. A sample of creating factual knowledge questions:
If p_n is circumference of *n*-gon which circumference to a circle with the radius of R and A_n is an area of *n*-gon that circumference to a circle with the radius of R and P is the circumference of the circle and A is the area of the circle and
$x_n = \frac{P_n}{P}$ and $y_n = \frac{A_n}{A}$ then for $n \geq 3$
We have:

22. A sample of creating conceptual knowledge questions:
Which one is true?

1) If $f(x)$ is polynomial with the degree of n, $f(x) = 0$ has $n-1$ real roots.
2) If $f(x)$ is polynomial with the degree of n and $(n = 2k+1)$ then $f(x) = 0$ at least has one real roots.
3) If $f(x)$ is strictly increasing function then $f(x) = 0$ has no real root.
4) All choices are correct.

23. A sample of creating procedural knowledge questions:
Let $f^{(n)}$ be the *n*-times iteration of f with itself. Now if

$$f(x) = \frac{1-x}{1+x}$$

then, find $f^{(100)}(x)$?

1) x 2) $\frac{1}{x}$ 3) $\frac{1-x}{1+x}$ 4) $\left(\frac{1-x}{1+x}\right)^{100}$

24. A sample of creating metacognitive knowledge questions:
In which condition $f : R \to Q$ is continuous?

1) When the range of f are natural numbers.
2) When f is an injective function.
3) When f is a surjective function.
4) When f is a constant function.