Proceeding of ASME 2012 Fluids Engineering Division Summer Meeting

July 8-11, 2012

Puerto Rico, U.S.A.

Paper Number FEDSM2012-14023

Mixed convection in cylindrical annulus with rotating outer cylinder and constant magnetic field with effect in radial direction

Asghar B. Rahimi **

** Professor, Faculty of Engineering, Ferdowsi University of Mashhad, P. O. Box No. 91775-1111, Mashhad, Iran (corresponding Author: rahimiab@yahoo.com)

ABSTRACT:

In the present study, mixed convection of a fluid in the fully developed region in horizontal concentric cylindrical annulus with different uniform wall temperatures is numerically investigated in both of steady and unsteady states in the presence of radial MHD force as well as consideration of heat generation due to viscous dissipation. Also, cylinders length is assumed to be infinite**.** Moreover, radiation heat transfer from the hot surface is assumed to be negligible. Buoyancy effects are also considered along with Boussinesq approximation. The forced flow is induced by the cold rotating outer cylinder in slowly constant angular velocity with its axis at the center of annulus. Investigations are made for various combinations of non-dimensional group numbers, Reynolds number (Re), Rayleigh number (Ra), Hartmann number (Ha), Eckert number (Eck), and annulus gap width ratio (σ_0). Finite volume scheme consisting of tri-diagonal matrix algorithm (TDMA) is used to solve governing equations which are continuity, two-dimensional momentum and energy by SIMPLE algorithm. The numerical results reveal that the flow and heat transfer are suppressed more effectively by imposing an external magnetic field. Furthermore, it is found that the external magnetic field causes the fluid velocity and temperature to be suppressed more effectively. Moreover, it will be shown that viscous dissipation terms have significant effects in situations with high values of Eckert and Prandtl numbers and low values of Reynolds numbers.

Key words: Combined convection, steady, transient, cylindrical annulus, viscous dissipation, constant magnetic field.

1- INTRODUCTION

The study of magnetohydrodynamic (MHD) flow of electrically conducting fluids such as liquid metals has gained an increasing attention due to its wide industrial applications either in engineering and geophysics to control unwanted convective flows in cases such as design of MHD power generators, modification of the solidification processes of metals, optimization of crystal growth processes, plasma industries and cooling of nuclear reactors. Although the analysis of flow and heat transfer in cylindrical annulus has been investigated in several literatures, but there is not any significant research conducted using magnetic field. Recently, the laminar convection in either vertical or horizontal concentric annulus has received attention by various investigators because of its wide applications such as in heat exchangers, cooling systems in electrical devices, solar collectors, cooling of turbine rotors and cooling of high speed gas bearings. In one of the primary studies, Rokerya and Iqbal [1] investigated the effect of viscous dissipation in the thermal energy balance for viscous flow in the vertical concentric annuli. Up to date, most works for mixed-convection problems have been performed for the flows in vertical rotating systems [2-4]. Relatively few studies, however, have been made for the flows in horizontal annuli. Fusegi [5] and Lee [6-7] are from those investigators that performed their simulations about the annuli with a heated rotating inner cylinder. Busse and Finocchi [8] and Petry et al. [9] carried numerical investigations about a problem of the onset of convection in a cylindrical annulus with conical end boundaries on the presence of magnetic fields. Exact solutions for fully developed natural convection in open-ended vertical concentric annuli under a radial magnetic field have been presented by Singh et al. [10]. The flow patterns in rotating cylinders are categorized into three basic types according to the number of eddies by J.S.Yoo [11]. Some attentions have also been devoted to laminar magneto-hydrodynamics convection heat transfer about a vertical flat plate [38] and in a rectangular cavity [39]. Discussion about the stability of the circular Couette flow under imposing of a homogeneous magnetic field has been analyzed by Leschhorn et al. [40]. Moreover, Barletta et al. [41] in a recent paper have studied the combined forced and free flow of an electrically conducting fluid in a vertical annular porous medium surrounding a straight cylindrical electric cable with radially varying magnetic field.

The objective of the present study is to investigate numerically the problem of mixed convection of a fluid in the fully developed region between two horizontally concentric cylinders with infinite lengths in the presence of variable magnetic field with radial MHD force direction and considering the effects of viscous heat dissipation in the fluid in both of steady and unsteady states. The forced flow is induced by the cold rotating outer cylinder in slowly constant angular velocity with its axis at the center of annulus. Buoyancy effect is also considered through Boussinesq approximation. Investigations are made for various combinations of non-dimensional group numbers such as the Reynolds number, Rayleigh number, Prandtl number, Hartmann number, Eckert number and the annulus gap width ratio.

2- PROBLEM FORMULATION

The schematic diagram of the annular duct under consideration and coordinate system are shown in Fig. 1. At first, two infinitely long horizontal concentric circular cylinders filled with incompressible fluid are fixed and held at the same temperature of T_0. Suddenly the outer cylinder starts to rotate in the counter-clockwise direction with constant angular velocity ω with it's axis at the center of the annulus. Also, the temperature of the inner cylinder is increased to higher value of T_i, concurrently. The cylindrical annulus is filled with viscous, Newtonian and incompressible fluid. The fluid is permeated by a uniform external magnetic field B_o along axial direction. We assume that the magnetic field induced by the motion of fluid and the heating produced by electromagnetic fields are taken to be small enough. Further, the viscous dissipation terms in the energy equation are considered completely due to viscosity of the fluid. It is assumed that all other thermo-physical properties of the fluid are constant except for the density change that is taken in Boussinesq approximation. Density variation can be described by the following equation.

$$\rho = \rho_0 (1 - \beta (T - T_0)) \qquad (1)$$

Where β is the thermal expansion coefficient, such that

$$\beta = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial T}\right)_P \tag{2}$$

Dimensionless forms of variables are put into consideration by the following definitions

$$r = \frac{r^*}{D}, \quad V_r = \frac{V_r^*}{R_o\Omega}, \quad V_\phi = \frac{V_\phi^*}{R_o\Omega}, \quad \theta = \frac{T - T_o}{T_i - T_o}, \quad P = \frac{P^*}{\rho\left(R_o^2\Omega^2\right)}, \quad t = \frac{t^*}{D/\left(R_o\Omega\right)} \tag{3}$$

Taking into account the above assumptions and dimensionless variables, the governing equations describing a set of conservation of laws can be written as follows

$$\vec{\nabla}.\vec{V} = 0 \tag{4}$$

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\cdot\vec{\nabla}\right)\vec{V} = -\vec{\nabla}P + \frac{1}{\mathrm{Re}}\nabla^2\vec{V} + \frac{Ra}{\Pr \mathrm{Re}^2}\theta\left[(\cos\phi)\vec{e}_r - (\sin\phi)\vec{e}_\phi\right] - V_r\frac{Ha^2}{\mathrm{Re}}\vec{e}_r \tag{5}$$

$$\frac{\partial \theta}{\partial t} + \left(\vec{V}\cdot\vec{\nabla}\right)\theta = \frac{1}{\mathrm{Re}\Pr}\nabla^2\theta + \frac{Eck}{\mathrm{Re}}\{2\left[\left(\frac{\partial V_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial V_\phi}{\partial \phi} + \frac{V_r}{r}\right)^2\right] + \left(\frac{\partial V_\phi}{\partial r} - \frac{V_\phi}{r} + \frac{1}{r}\frac{\partial V_r}{\partial \phi}\right)^2\} \tag{6}$$

Note that the MHD body force appeared on the right hand side of r-momentum equation is evaluated due to imposed axial variable magnetic field according to Lorentz force [24]. In Equations [4-6] $\vec{V}$ is the dimensionless velocity vector and P is the dimensionless acting hydrodynamic pressure. Let us define the dimensionless group numbers

$$\mathrm{Re} = \frac{R_o\Omega D}{\nu}, \; Ra = \frac{g\beta(T_i - T_o)D^3}{\nu\alpha}, \; \Pr = \frac{\nu}{\alpha}, \; Ha = B_0 D\sqrt{\frac{\sigma}{\rho\nu}\frac{R_i^2}{D^2}}, \; Eck = \frac{R_o^2\Omega^2}{c_p\left(T_i - T_o\right)}$$

$$\sigma_o = 2R_i/(R_o - R_i) \quad \text{and} \quad D = R_0 - R_i \tag{7}$$

The dimensionless form of the initial conditions for the present problem is defined as

$$V_r = V_\phi = \theta = 0, \quad at\ t = 0 \tag{8}$$

Moreover, the dimensionless form of the boundary conditions are given by

$$V_r = 0, V_\phi = 0, \; \theta = 1 \text{ at } r = r_i \tag{9}$$

$$V_r = 0 \quad , V_\phi = 1, \; \theta = 0 \quad \text{at } r = r_0 \tag{10}$$

If pure conduction heat transfer rate in the dimensionless form and in the absence of any fluid motion is defined as

$$Nu_{cond} = 1/\ln(r_o/r_i) \tag{11}$$

Then the local Nusselt number along the inner and outer cylinders can be expressed as the actual heat transfer divided by this quantity as

$$Nu_i(\phi) = -(r\frac{\partial \theta}{\partial r}) / Nu_{cond} \qquad \text{at } r = r_i \tag{12}$$

$$Nu_0(\phi) = -(r\frac{\partial \theta}{\partial r}) / Nu_{cond} \qquad \text{at } r = r_0 \tag{13}$$

Also, the average Nusselt numbers at both of inner and outer cylinders are calculated, respectively, by

$$Nu_i = \frac{1}{2\pi}\int_0^{2\pi} Nu_i(\phi) d\phi \tag{14}$$

$$Nu_0 = \frac{1}{2\pi}\int_0^{2\pi} Nu_0(\phi) d\phi \tag{15}$$

The important note is predicting the same result by both expressions in Eq. (12) and (13) for steady state solution and in the absence of heat generation. The net circulation of fluid in the direction of cylinder rotation is defined by:

$$\Gamma = |\Psi_2 - \Psi_1| \tag{16}$$

3- NUMERICAL PROCEDURE

A finite volume procedure is used to numerically solve the governing equations describing the sets of laws by the numerical code accomplished completely by the authors. The important problem on the overall solution procedure is the pressure gradients in both of momentum equations in r and ϕ directions. The remedy to solve this problem is by using the staggered grids for the velocity components through the iterative SIMPLE algorithm in a specified domain with prescribed boundary and initial conditions. In the iterative SIMPLE algorithm, the central differencing method with the accuracy of second order was introduced to obtain the discretized equations for the diffusion terms on the right hand side of equations (5) and (6). In order to calculate the value of each of the velocity components and temperature of the convective terms at control volume faces, the power-law differencing scheme is used with the accuracy of second-order at low aspect ratio of the convective mass flux per unit area to diffusion conductance at cell faces and with the accuracy of first-order at high aspect ratio. Also, a fully implicit transient scheme is employed for calculations in unsteady state. To assess grid independence of the numerical scheme, the distributions of Nusselt number on the inner cylinder were initially tested with different $(r * \phi)$ mesh sizes of 18*5, 32*9, 56*17, 100*31 and 180*55 in Fig. 2a. In this set of mesh sizes, as it can be seen, the coefficient of 1.8 was used in each test to increase the number of mesh grids in both directions of r and ϕ. It was found that the variations of Nu number distributions on the inner cylinder were not significant between $(r * \phi)$ mesh sizes of (100*31) and (180*55). Hence, a 100*31 grid in r-ϕ directions was applied for the computational domain in cylindrical annulus. A fine non-uniform grid spacing is used in r-direction to capture the rapid changes such as the grid lines being closer packed near the inner and outer walls. On the other hand, a uniform mesh was implemented in ϕ-direction. Fig.2b illustrates a sample of computational meshes used in this investigation. The value of stream function ψ on the stagnant boundary of inner cylinder is considered as zero. The other values of ψ are calculated by using distributions of velocity components computed from numerical solution by way of integration method. After imposing the sudden changes, an iterative SIMPLE algorithm is then repeated such that the desired convergence of governing equations is attained for each of time steps till the steady state conditions are obtained. The convergence criterion for the numerical solution in transient state is obtained by comparing the residue of two repeated sequences of each of the momentum and energy equations in a specific time domain characterized by the definition of time step as follows

$$|\mathrm{Re}\,sidual_{m+1}^{n} - \mathrm{Re}\,sidual_{m}^{n}| \prec 10^{-8}$$

In the above expression, n refers to time and m refers to number of iteration in that time domain. Moreover, the condition needed for passing the transient situation and attaining the steady state is defined by comparing the residue of each of the governing equations at two successive time domain as:

$$|\mathrm{Re}\,sidual(n+1) - \mathrm{Re}\,sidual(n)| \prec 10^{-4}$$

It must be noticed that the time step used in this procedure depends strongly on values specified for pr and Re numbers. The criterion employed for steady state conditions in the numerical technique is defined by $|\mathrm{Re}\,sidual(m+1) - \mathrm{Re}\,sidual(m)| \prec 10^{-9}$. In Fig.2c the convergence history of ϕ-momentum equation can be seen for specified values of non-dimensional group numbers such as Re=200, Pr=2, Eck=0.0007, Ra=10000 and Ha=20.

In the next section the results of solving the governing equations along with discussions are presented.

4- PRESENTATION OF RESULTS

As we know, the dimensionless Hartman number represents the magnitude of the applied magnetic field. Here, the effect of Hartman number on streamlines and isotherms at $Ra = 10000$, $Eck = 0.0$, $\mathrm{Pr} = 1.0$, and $\mathrm{Re} = 100.0$ is presented. In the absence of magnetic field the existence of buoyancy forces and forced convection brings about the solution region in the fluid flow, but application of an outer magnetic field changes the balance of these forces in the fluid flow. The effects of application of a weak magnetic field, $Ha = 20.0$, on the streamlines and temperature contours are shown in Figs. 3 and 4. These changes are more pronounced for Ha=50.0 so that in this situation the existing eddies in the right hand side of annulus fade away and intensity of the effect of buoyancy forces on temperature contours is reduced greatly. By increase of Hartman number after around 200 not much change in natural convection of fluid is seen. In this situation the rotation of outer cylinder is the sole cause of fluid flow. Also, it can be seen from these contours that for large magnetic field such as the case $Ha = 100$, the dominant heat transfer mechanism is conduction.

In Fig. 5 the distribution of Nusselt number on the outer and inner surfaces is presented for different Hartman numbers. Increase of Hartman number which causes reduction in buoyancy forces will bring about gradual decrease of Nusselt number on both surfaces of the cylinder in such a way that at Ha=200 much changes in the region between 0 to 360 degrees can not be sensed. It is interesting to point out that in this situation where the forced convection is dominant on fluid flow the Nusselt number in the above region on both cylinder surfaces is equal to one. As we know imposing magnetic field causes change in streamlines structure and the amount of stream function on them in a way that increase of Hartman number not only causes the existing vortices in fluid flow to diminish but causes the streamlines to become concentric circles. Net circulation is a quantity related to the rate of volume flow in unit depth between the two concentric cylinders.

5- CONCLUSIONS

In the present work, finite volume method was used to solve the set of governing equations by SIMPLE algorithm. The investigations were carried out in the wide range of 0<Ha<200, 1000<Ra<30000, 0<Eck<0.5, 0.5<Pr<20 and $0.5<\sigma_0<5$ of the flow with different Re numbers to explore the effects of the above dimensionless group numbers on the overall flow patterns and heat transfer rates. The results show that the application of the magnetic field changes the balance of the buoyancy and forced convection forces and is in the direction of suppressing the patterns of the streamlines as well as the temperature contours and in turn the heat transfer. Increase of magnetic field strength, therefore, reduces the Nusselt number on both surfaces of the cylinders. As we know, the increase of the value of Eckert number causes the influence of the viscous dissipation terms to enhance which in turn suppresses the fluctuations of the local Nusselt number on the cylinder surfaces. Also, it was noticed that with increasing the Eckert number of the high-Prandtl flow, the heat generated due to viscous dissipation causes the fluid temperature in the vicinity of inner cylinder to be larger than the cylinder temperature. Furthermore, the obtained results depict that the effects of increase of the Prandtl number and decrease of the ratio of inner cylinder diameter to the outer and inner radius difference like the effect of magnetic field are in the direction of suppressing the flow and heat transfer. Moreover, the results of unsteady state were presented in different cases to complete the results studied in the present work.

6- REFERENCES

[1] Rokerya, M.S. and Iqbal, M., "Effects of viscous dissipation on combined free and forced convection through vertical concentric annuli ", Int. J. of Heat and Mass Transfer, 14, pp. 491-495, 1971.

[2] Gue, Z.Y. and Zhang, C.M., "Thermal drive in centrifugal fields - mixed convection in a vertical rotating cylinder", International Journal of Heat and Mass Transfer, 35, pp. 1635-1644, 1992.

[3]Saarawi, M.A. and Khamis, M., "Induced flow in uniformly heated vertical annuli with rotating inner walls", Numerical Heat Transfer, 12, pp. 493-508, 1987.

[4] Hessami, M.A., Davis, G.V.D., Lenonardi, E. and Reizes, J.A., "Mixed convection in vertical cylindrical annuli", International Journal of Heat and Mass Transfer, 30, pp. 151-164, 1987.

[5]Fusegi, T., Farouk, B. and Ball, K.S., "Mixed-convection flows within a horizontal concentric annulus with a heated rotating inner cylinder ", Numerical Heat Transfer, 9, pp. 591-604, 1986.

[6] Lee, T.S., "Laminar fluid convection between concentric and eccentric heated horizontal rotating cylinders for low-Prandtl-number fluids", International Journal for Numerical Methods in Fluids, 14, pp. 1037-1062, 1992.

[7] Lee, T.S., "Numerical computation of fluid convection with air enclosed between the annuli of eccentric heated horizontal rotating cylinders", Computers and Fluids, 21, pp. 355-368, 1992.

[8]Busse, F.H., Finocchi, F.,"The onset of thermal convection in a rotating cylindrical annulus in the presence of magnetic field", Physics of the Earth and Planetary Interiors, 80, pp. 13-23, 1993.

[9] Petry, M., Busse, F.H. and Finocchi, F., "Convection in a rotating cylindrical annulus in the presence of a magnetic field ", Eur. J. Mech., B/Fluids, pp. 817-833, 1997.

[10] Singh, S.K., Jha, B.K. and Singh, A.K., "Natural convection in vertical concentric annuli under a radial magnetic field ", Heat and Mass Transfer, 32, pp. 399-401, 1997.

[11] Yoo, J.S.,"Mixed convection of air between two horizontal concentric cylinders with a cooled rotating outer cylinder ", Int. J. Heat Mass Transfer, 41, pp. 293-302, 1998.

[38]Aydin, O. and Kaya, A., "MHD mixed convection of a viscous dissipating fluid about a permeable vertical flat plate ", Applied Mathematical Modeling, 33, pp. 4086-4096, 2009.

[39] Grosan, T., Revnic, C., Pop, I. and Ingham, D.B., " magnetic field and internal heat generation effects on the free convection in a rectangular cavity filled with a porous medium ", International Journal of Heat and Mass Transfer, 52, pp. 1525-1533, 2009.

[40] Leschhorn, A., Lucke, M., Hoffmann, C. and Altmeyer, S., "Stability of the circular couette flow of a ferrofluid in an axial magnetic field ", Physical Review, 79, 2009.

[41]Barletta, A., Lazzari, S., Magyari, E. and Pop, I.," Mixed convection with heating effects in a vertical porous annulus with a radially varying magnetic field ", International Journal of Heat and Mass Transfer, 51, pp. 5777-5784, 2008.

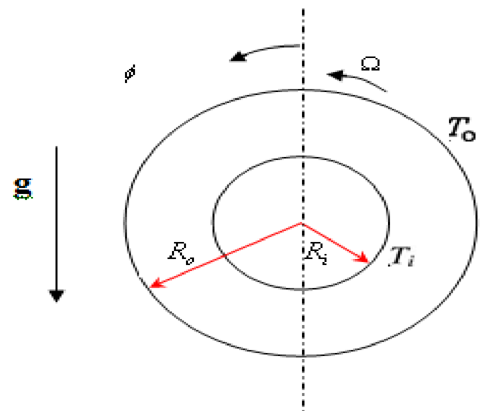


Fig.1. Configuration of the problem

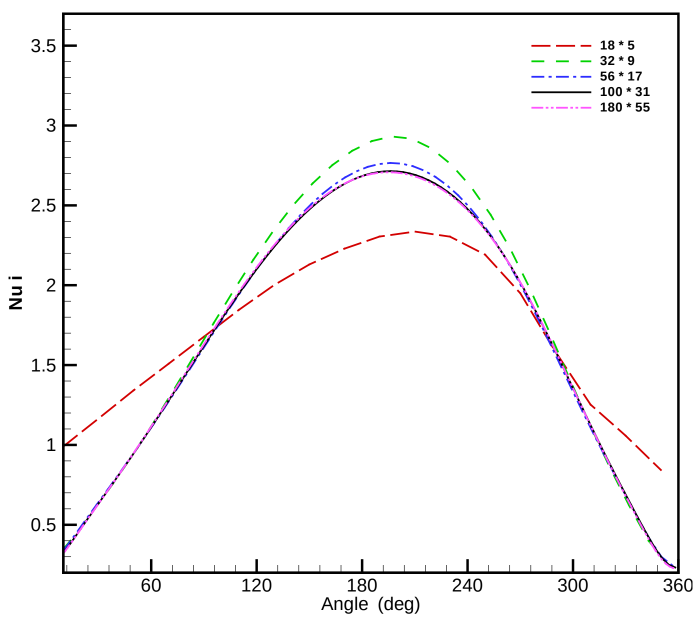


Fig. 2a. Nusselt number distributions on the inner cylinder for various mesh sizes at

Re=100, Pr=0.7, Eck=0.0002, Ra=10000.0, Ha=20.0

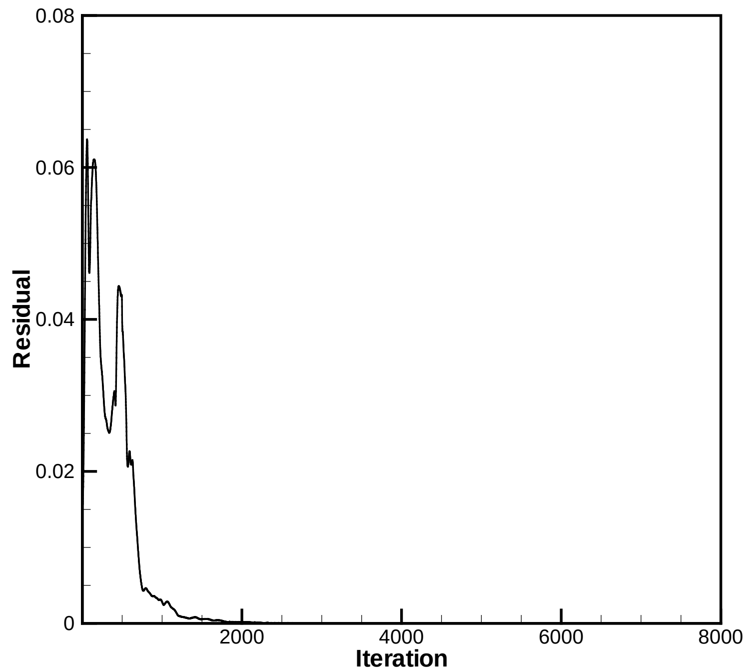
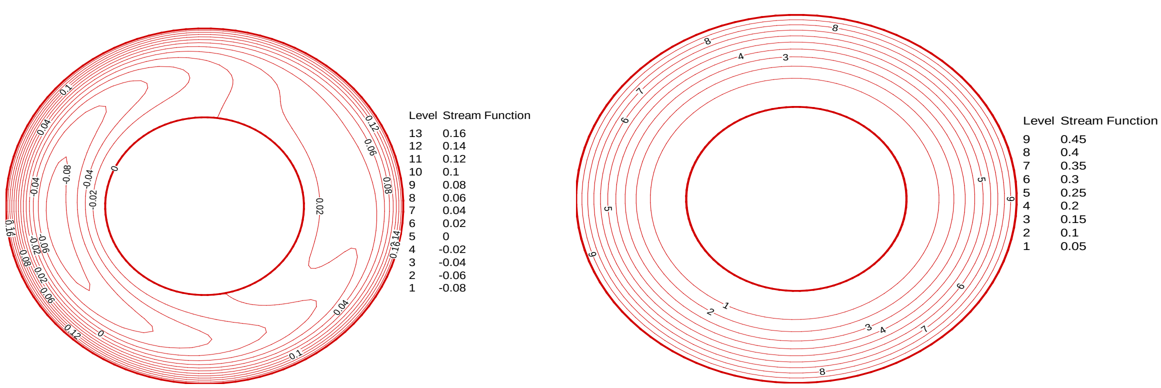


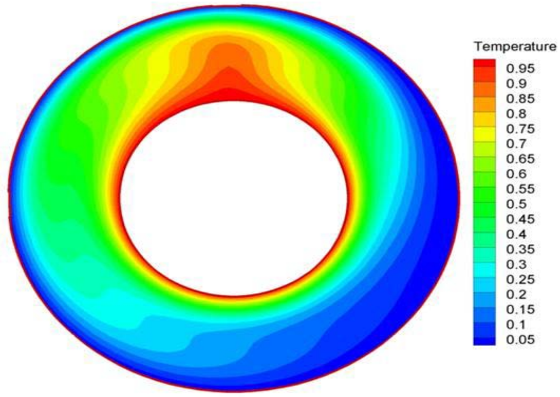
Fig.2b. Convergence history of ϕ -momentum equation for
Re=200, Pr=2, Eck=0.0007, Ra=10000 and Ha=20.

$Ha = 0.0$

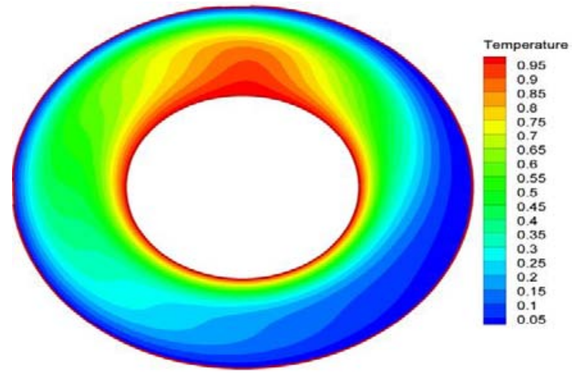


$Ha = 50.0$ $Ha = 200.0$

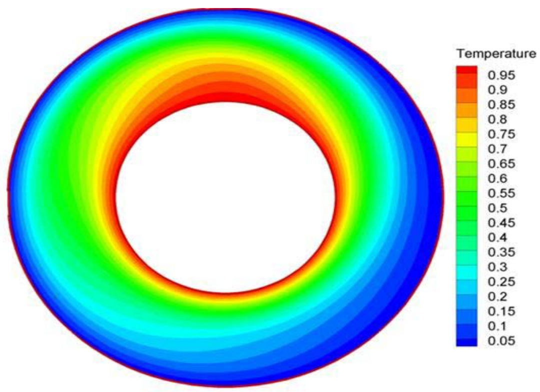
Fig.3. Effect of Hartman number on streamlines for $\mathrm{Re} = 100$, $Eck = 0.0$, $Ra = 10000$ and $\mathrm{Pr} = 1$



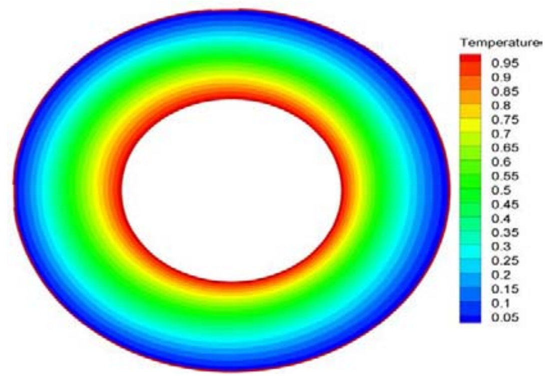
Ha=0.0



Ha=20

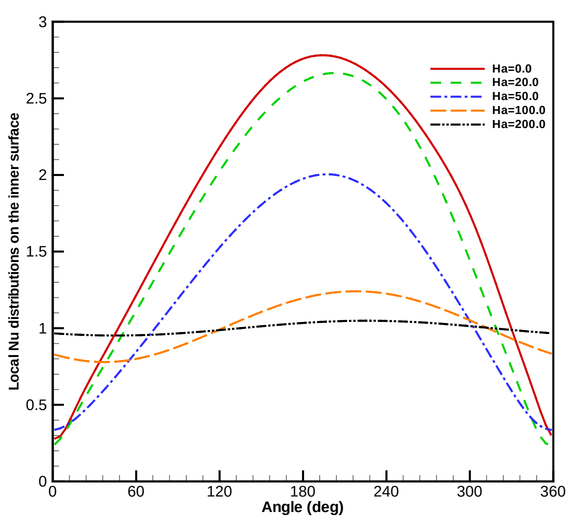


Ha=50.0

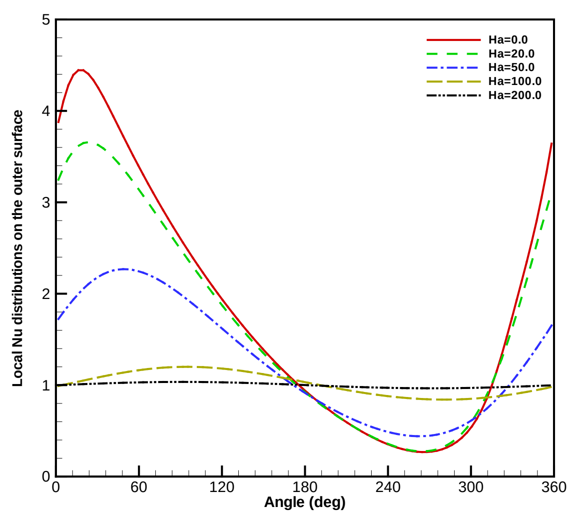


Ha=100

Fig.4. Effect of Hartman number on temperature contour for $\mathrm{Re} = 100$, $Eck = 0.0$, $Ra = 10000$ and $\mathrm{Pr} = 1$



(a)



(b)

Fig.5. Effect of Hartman number on local Nusselt number distribution on (a) inner cylinder, (b) outer cylinder, and for $\mathrm{Re} = 100$, $Ra = 10000$, $Eck = 0.0$, and $\mathrm{Pr} = 1$