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ORIGINAL ARTICLE

Effect of high relativistic ions on ion acoustic solitons in electron-ion-positron plasmas with nonthermal electrons and thermal positrons

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Abstract Propagation of ion acoustic solitary waves are studied in e-p-i plasmas containing high relativistic ions, Maxwell–Boltzmann distributed positrons and nonthermal electrons. Reductive perturbation method is used and the Korteweg-de Vries (KdV) equation is derived. The effects of high relativistic ions and nonthermal electrons on soliton characters are studied.

Keywords High relativistic ions · Ion acoustic solitons · Electron-ion-positron plasmas · Nonthermal electrons · Thermal positrons

1 Introduction

Solitary waves have been investigated both theoretically and experimentally in most of the branches of science. Localized solutions of nonlinear equations are also extensively studied as important objects in plasma physics (Shukla 2003; Misra and Bhowmik 2007; Verheest 1996; Tian and Gao 2007). The ion acoustic soliton (IAS) is one of the most aspects of nonlinear phenomena in modern plasma physics researches. They arise due to a fine tuning between nonlinearity and dispersion effects in the media. Investigation of nonlinear structures is carried out usually by adopting some forms of perturbation method. In small amplitude approximation of the equations, one ends up deriving some forms of nonlinear differential equations for one spatial dimension situations like Korteweg-de Vries (KdV), modified Korteweg-de Vries (m-KdV), nonlinear Schrodinger equation and etc. Such these equations have well known solutions with extended structures, like solitary wave or solitonic solutions. A great number of authors have been studied ion-acoustic solitons using the reductive perturbation technique in different situations of plasma physics (Bharuthram and Shukla 1986; Yadav and Sharma 1991). In contrast to the usual plasmas that consisting of electrons and positive ions, it has been observed that the nonlinear waves in plasmas which containing additional components such as positrons behave differently (Shukla et al. 1986). The behaviour of the electron-positron-ion plasmas helps us to find better knowledge about the early universe which assumes to be a kind of plasma (Rees 1983; Misner et al. 1973), describing the active galactic nuclei (Miller and Witta 1987), pulsar magnetospheres (Michel 1982) and also the solar atmosphere (Goldreich and Julian 1969). The positrons can be used to probe particle transport in tokomaks and since they have sufficient lifetime. In this case, two-component (e-i) plasmas become a three-component (e-i-p) medium (Shukla and Stenflo 1993; Tsintsadze et al. 1994). During the last decade, e-p-i plasmas have attracted the attention of several authors (Popel et al. 1995; Lakhina and Verheest 1997; Pakzad 2009). We know that when the ion velocity approaches the velocity of light, relativistic effects may significantly modify the behaviour of the solitary waves. Relativistic plasmas occur in a variety of situations, such as, space–plasmas (Grabbe 1989), laser–plasma interaction (Arons 1979), plasma sheet boundary layer of earth's magnetosphere (Vette 1970). This situation is also used in order to describing the Van Allen radiation belts (Ikezi 1973). Das and Paul (1985) have investigated the weakly relativistic effects on ion-acoustic wave propagation in one dimension using the KdV equation for cold plasmas but without electron inertia effects. Nejoh (1987) has inves-

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tigated the same situation in the warm plasmas. Kalita et al. (1996) have studied the existence of solitons with considering the complete fluid equation for electrons. El-Labany and Shaaban (1995) have considered nonlinear ion-acoustic waves in weakly relativistic plasmas consisting of warm ion-fluid with non-isothermal electrons through the modified equations. Pakzad and Javidan (2011) have investigated nonlinear ion-acoustic shock waves in weakly relativistic plasmas containing nonthermal electrons. Nejoh and Sanuki (1994) also have studied the large amplitude Langmuir and ion-acoustic waves in relativistic two fluid plasmas with deriving the pseudo potential. Understanding the behaviour of multi species plasmas containing cold or warm ions with Boltzmann distribution has been extensively considered for the last few years. However, it has been found that the electron and ion distributions play the crucial role in characterizing the physics of the nonlinear wave structures. They further offer a considerable increase in richness and variety of the wave motion, which can be found in plasmas. Moreover, they have significant influence on the conditions required for the formation of the waves. Modulation of nonlinear waves in slowly responding collisional plasmas containing nonthermal species has been investigated in Misra and Roy Chowdhury (2002, 2006). Based on observations of solitary wave structures with density depletion made by Freja and Viking satellites (Dovner et al. 1994), Cairns et al. (1995a, 1995b) have considered a plasma model consisting nonthermally distributed electrons and ions and emphasizing the role of this distribution on the characterization of wave structures (Mamun 2000; Bandyopadhyay and Das 2001; Carins et al. 1996). It may be noted that plasmas with different temperatures and masses frequently occur in space environment. Particularly, two temperature electrons are very common in laboratory and space plasmas. It is found that nonthermal distributions are common features of the auroral zone (Lundin et al. 1987; Hall et al. 1991). In this paper, the ion acoustic solitary waves in plasmas consisting of nonthermal electrons, positrons with Boltzmann distribution and high relativistic ions have been studied.

2 Basic equations and derivation of the KdV equation

Let us consider one-dimensional, collisionless, unmagnetized relativistic plasmas with nonthermal electrons and thermal positrons. The nonlinear dynamics of the low frequency ion-acoustic solitons in the three component plasmas are governed by the following set of equations

$$\begin{aligned}
&\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0\\
&\frac{\partial (\gamma u)}{\partial t}+u\frac{\partial (\gamma u)}{\partial x}+\frac{\partial \phi}{\partial x}=0\\
&\frac{\partial^2 \phi}{\partial x^2}=n_e-n-n_p
\end{aligned}\tag{1}$$

where n and u are the ion number density and ion fluid velocity respectively. ϕ and γ are electrostatic potential and relativistic factor respectively. For high relativistic plasmas parameter γ is approximated by its expansion up to term $\frac{u^4}{c^4}$.

$$\gamma=\left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}}\cong 1+\frac{u^2}{2c^2}+\frac{3u^4}{8c^4}\tag{2}$$

In order to considering the effects of nonthermal electrons, the electron number density n_e is modeled as (Bandyopadhyay and Das 2001)

$$n_e=(1-\beta\phi+\beta\phi^2)e^{\phi}\tag{3}$$

where $\beta=\frac{4\alpha}{1+3\alpha}$ and α is a parameter that determines the population of nonthermal (fast) electrons (Kalita et al. 1996). The positrons are assumed to be in thermal equilibrium, with the density of

$$n_p=pe^{-\sigma\phi}\tag{4}$$

In the equation set (1), the densities of the plasma species are normalized by the unperturbed electron density n_{e0}, the ion velocity is normalized by the ion acoustic speed $c_i=\sqrt{T_e/m}$, space variables are normalized by the electron Debye length $\lambda_D=\sqrt{T_e/4\pi n_0 e^2}$, time variable is normalized by the electron plasma period $T=\sqrt{m_e/4\pi n_{e0}e^2}$ and electrostatic potential is normalized by $\frac{T_e}{e}$. Note that $p=\frac{n_{p0}}{n_{e0}}$ represents the relative positron concentration in e-p-i plasma and $\sigma=\frac{T_e}{T_p}$ is the ratio of electron temperature to positron temperature.

The reductive perturbation method can be used for investigating the behavior of nonlinear ion acoustic waves. The stretched coordinates are defined as follows

$$\xi=\varepsilon^{\frac{1}{2}}(x-\lambda t),\quad \tau=\varepsilon^{\frac{3}{2}}t\tag{5}$$

where ε is a small parameter which characterizes the strength of the nonlinearity and λ is the phase velocity of propagated wave. Dependent variables are expanded as follows

$$\begin{aligned}
&n=(1-p)+\varepsilon n_1+\varepsilon^2 n_2+\cdots\\
&u=u_0+\varepsilon u_1+\varepsilon^2 u_2+\cdots\\
&\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\cdots
\end{aligned}\tag{6}$$

By substituting (6) into (1) using (5) and collecting the terms in the different powers of ε, one can derive the following equations in the lowest order of ε

$$\begin{aligned} & n_1 = \frac{(1-p)u_1}{\lambda - u_0}, \quad u_1 = \frac{(\lambda - u_0)(1-\beta+p\sigma)\phi_1}{(1-p)}, \\ & n_1 = (1-\beta+p\sigma)\phi_1 \\ & (\lambda - u_0)^2 = \frac{1-p}{(\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4})(1-\beta+p\sigma)} \end{aligned} \tag{7}$$

For the higher orders of ε, we have

$$\begin{aligned} & -(\lambda-u_0)\frac{\partial n_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial (n_1 u_1)}{\partial \xi} + (1-p)\frac{\partial u_2}{\partial \xi} = 0 \\ & \frac{\partial n_2}{\partial \tau} + u_2\frac{\partial n_1}{\partial \xi} + u_1\frac{\partial n_2}{\partial \xi} + n_1\frac{\partial u_2}{\partial \xi} + n_2\frac{\partial u_1}{\partial \xi} = 0 \\ & \left[\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4} - (\lambda-u_0)\left(\frac{3u_0}{c^2} + \frac{15u_0^3}{2c^4}\right)\right]u_1\frac{\partial u_1}{\partial \xi} \\ & \quad - (\lambda-u_0)\left(\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4}\right)\frac{\partial u_2}{\partial \xi} \\ & \quad + \left(\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4}\right)\frac{\partial u_1}{\partial \tau} + \frac{\partial \phi_2}{\partial \xi} = 0 \\ & \frac{\partial^2 \phi_2}{\partial \xi^2} + (p\sigma^2 - 1)\phi_1\phi_2 - \frac{1}{2}\left(\beta + \frac{1}{3} + \frac{1}{3}p\sigma^3\right)\phi_1^3 = 0 \end{aligned} \tag{8}$$

where $\gamma_1 = 1 + \frac{3u_0^2}{2c^2} + \frac{15u_0^4}{8c^4}$ and $\gamma_2 = \frac{3u_0}{c^2} + \frac{15u_0^3}{2c^4}$.

Finally the KdV equation is derived from (7) and (8) as

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{9}$$

where

$$\begin{aligned} & A = \frac{1}{2}\left[\frac{-(\lambda-u_0)^3(1-p\sigma^2)\gamma_1}{(1-p)} + \frac{3}{(\lambda-u_0)\gamma_1} - \frac{\gamma_2}{\gamma_1^2}\right], \\ & B = \frac{(\lambda-u_0)^3\gamma}{2(1-p)} \end{aligned} \tag{10}$$

The stationary solution of (9) is given by

$$\phi_1 = \phi_0 \sec h^2 \frac{(\xi - U\tau)}{w} \tag{11}$$

in which U is constant velocity of solitary wave. The ion acoustic wave amplitude (ϕ_0) and its width (w) are given as

$$\phi_0 = \frac{3U}{A}, \quad w = 2\sqrt{\frac{B}{U}} \tag{12}$$

3 Discussion and results

Equation (8) shows that the coefficients A and B are functions of positron density (p), relative temperature (σ), relativistic factor ($\eta = \frac{u_0}{c}$) as well as nonthermal parameter (β). The positron density affected on A and B through the density ratio p. Some researchers have studied one-dimensional (Das and Paul 1985; Nejoh 1987) and two-dimensional (Nejoh and Sanuki 1994) cases of ion acoustic solitary waves in weakly relativistic plasmas containing ions and electrons and the results of their studies are comparable with the presented results of this paper. The above results are also congruent with Gill et al. (2007) for weakly relativistic plasma with the Boltzmann distribution of electron ($\beta = 0$). The obtained results correctly reduce to Nejoh (1987) for electron-ion ($p = 0$) weakly relativistic plasmas with Boltzmann distributed electrons. The above results also are in agreement with the results of Kaur et al. (2009) for electron-ion ($p = 0$) weakly relativistic plasmas with nonthermal electrons. We use the numerical computation for doing quantitative analysis on the results. Relativistic effect influences on the equations through the parameter $\eta = \frac{u_0}{c}$. The relative values of the plasma parameters are used to characterizing the existence of solitons of different types.

It is interesting to compare presented results with those of Gill et al. (2007). Similar coefficients "A" and "B" which have been appeared in Lundin et al. (1987) (with some simplifications) are as follows:

$$\begin{aligned} & A' = \frac{1}{2}\left[\frac{-(\lambda-u_0)^3(1-p\sigma^2)\gamma_1'}{(1-p)} + \frac{3}{(\lambda-u_0)\gamma_1'} - \frac{\gamma_2'}{\gamma_1'^2}\right], \\ & B' = \frac{(\lambda-u_0)^3\gamma'}{2(1-p)} \end{aligned} \tag{13}$$

where $\gamma_1' = 1 + \frac{3u_0^2}{2c^2}$ and $\gamma_2' = \frac{3u_0}{c^2}$. Therefore differences between our results and the results of Gill et al. (2007) with $\beta = 0$ become

$$\begin{aligned} & \Delta A = A - A' = -\frac{2(1+p^2\sigma^2) + p(1+\sigma^2+6\sigma)}{\sqrt{(1-p)(1+p\sigma)^3\gamma_1'^3}}\left(\frac{3u_0^4}{4c^4}\right) \\ & \Delta B = B - B' = -\sqrt{\frac{1-p}{(1+p\sigma)^3}}\left(\frac{3u_0^4}{8c^4\gamma_1'}\right) \end{aligned} \tag{14}$$

Equations (14) clearly show that both parameters A and B find smaller values in comparison with similar parameters of Gill et al. (2007); however differences are very small. Therefore the soliton height and its width become larger when soliton velocity approaches the velocity of light.

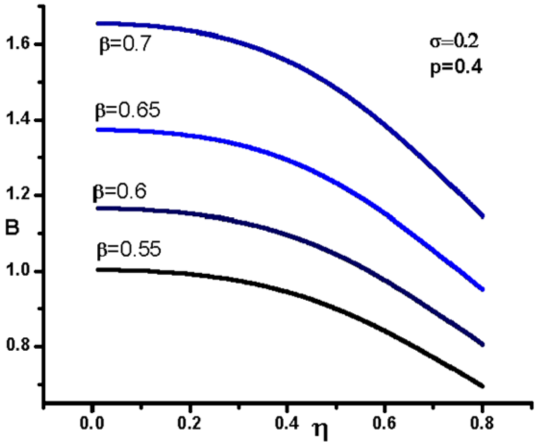

Fig. 1 The parameter "B" as a function of relativistic parameter η with different values of β

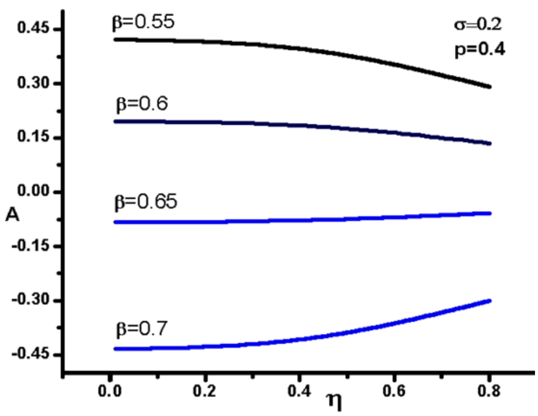

Fig. 2 The parameter "A" as a function of relativistic parameter η with different values of β

Now we can investigate the behavior of soliton amplitude and its width propagated in this kind of plasma medium.

Equation (12) shows that Soliton width (amplitude) has a proportional growth with B ($1/A$). Figure 1 presents B as a function of $\eta = \frac{u_0}{c}$ for different values of β. This figure shows that B decreases when η increases. This means that Soliton width decreases when ion velocity approaches to light velocity. B also increases with an increasing β. Thus soliton width increases when the plasma contains fast electrons.

Figure 2 shows "A" as a function of η for different values of β. This figure demonstrates that A find positive and negative values. Therefore both positive and negative solitons can be created in this plasma medium. This figure also shows that A has small changes respect to η. On the other hand, both positive and negative values of A decreases when η increases. Thus soliton amplitude decreases with an increasing η.

Figure 3 presents the amplitude of soliton as a function of η for different values of p with $\beta = 0$. This figure shows

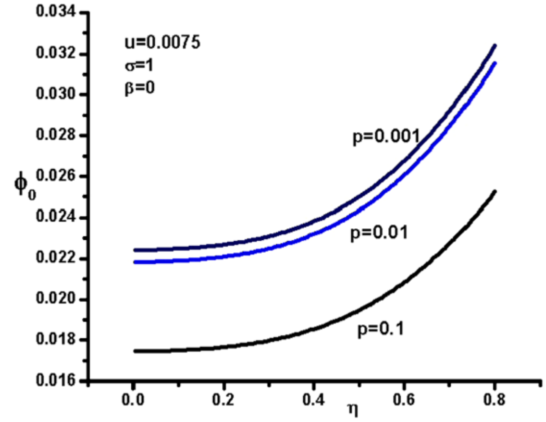

Fig. 3 The soliton height ϕ_0 as a function of relativistic parameter η with different values of p and $\beta = 0$. This figure is comparable with Fig. 1 of Gill et al. (2007)

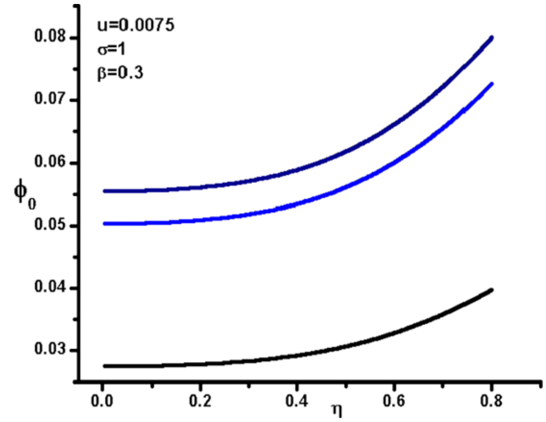

Fig. 4 The soliton height ϕ_0 as a function of relativistic parameter η with different values of p and $\beta = 0.3$

that our results in the interval $0 < \eta < 0.2$ (for weakly relativistic plasmas) confirm the results of Gill et al. (2007). Equations (14) describe differences between we have extracted the numerical results of Gill et al. (2007) by digitizing figures in this paper. It is observed that the change rate of ϕ_0 in the high relativistic limit is more than that of in weakly relativistic situation. In comparison with Fig. 1 of Gill et al. (2007), we haven't plotted the profile of ϕ_0 for $p = 0.0001$; because it is very close to curve $p = 0.001$.

Figure 4 shows the same functions of Fig. 3 but with $\beta = 0.3$. Comparing the soliton amplitude (ϕ_0) in Figs. 3 and 4 shows that the amplitude of soliton in the existence of nonthermal electrons is increased. This result also is in agreement with Fig. 2.

4 Conclusion and remarks

Propagation of solitary waves in collisionless, unmagnetized high relativistic plasmas with nonthermal electrons and thermal positrons has been studied. Maximum amplitude of the solitary wave and its width have been derived as functions of plasma parameters. It is shown that both positive and negative solitons can be created in this plasma medium. The soliton amplitude increases when relativistic parameter η increases. Therefore the amplitude of soliton in the existence of nonthermal electrons is increased. But soliton width decreases with an increasing η.

The results have been compared with the results of weakly relativistic plasmas. We can find a good agreement between the results of these situations.

As Fig. 2 clearly shows, it is possible that parameter "A" becomes zero in some special set of plasma characters. In this condition, soliton maximum amplitude goes to infinity. It is clear that our formulation of the problem fails to describe the situation. This special state can be investigated in further studies.

Plasmas containing of positrons, high relativistic ions and also relativistic electrons is an interesting medium which has not studied yet. Investigating the propagation of solitary waves in this medium can help us to find better knowledge about the effects of relativistic particles in plasmas. The results of suggested study can be compared with the results of Kalita and Das (2007).

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