Energy and momentum in one dimensional open channel flow


Discusser:
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Although I found the paper interesting and comprehensive, I would like to convey my understanding of the subject and discuss more about the question raised and the recommendation made by the authors.

The authors begin with the fact that there is confusion in the literature about the use of the energy and momentum equation in one dimensional analysis of open channel flow. To reduce this confusion, by applying the fundamental laws of motion to a control volume of flow, they develop the following equations:

\[ \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \quad \text{Continuity Equation} \]

\[ S_f = S_e - \frac{g}{\rho g R} \left( \frac{\partial Q}{\partial x} \right) \quad \text{Friction slope} \]

and:

\[ S_e = S_e - \frac{1}{2gQ} \left( \frac{\partial (\rho Q^2)}{\partial x} \right) \quad \text{Energy slope} \]

In the above equations, all the symbols are the same symbols used by the authors.

If the general assumptions associated with the above fundamental equations such as one dimensionality of the flow and hydrostatic pressure distribution are ignored, the equations fully describe the flow in an open channel. As a problem in classical mechanics, it is necessary to use another relationship called constitutive relationship and apply boundary conditions to solve the equations and to obtain the parameters of interest such as depth and discharge. The constitutive relationship can be the Manning equation or any other resistance equation. In the process of solving the above equations, the following two questions, which are interdependent, can be raised:

1. Which equation, the energy or the momentum equation, should be used to analyze the problem? The authors clearly show that the results of the analysis are different for gradually varied flow in compound channels.

2. Does the constitutive relationship define the energy slope or the friction slope?

In order to answer these questions, I would like to refer to two local phenomena in the open channel flow and relate them to the one dimensional gradually varied flow. In general, it can be said that when the friction slope or boundary shear stresses are noticeable and unknown but energy losses are negligible or known independently, the energy equation is a good start to analyze the problem. An example of such analysis is the analysis of the flow over a smooth step placed in the bottom of a channel. In this system, after analyzing the flow using the energy equation and obtaining the depth of flow over the step, the momentum equation can be used to find the unknown drag force (resulting from the distributed shear stresses) of the flow on the step. A different problem is the hydraulic jump. In the analysis of this phenomenon, the energy loss is noticeable and unknown while it is commonly assumed that the boundary shear stresses are negligible. Therefore, the momentum equation is a good start. After analyzing the flow using the momentum equation, the energy equation can be used to find an equation that defines the loss of energy in the jump. It is worth considering that in both examples, momentum and energy equations are wisely used to analyze the problems. The above discussion can be extended to one dimensional analysis of open channel flow. In gradually varied flow problems, both boundary shear stresses and energy losses are noticeable and unknown. Therefore, the use of momentum or energy equation depends on which of the two slopes, friction slope or energy slope, is really defined by the resistance equations.

A review of the traditional methods for estimating the resistance coefficient, n, in the Manning equation (Chow, 1955; French, 1985) reveals to me that this is energy slope, which is evaluated by the Manning equation. Even in an inverse procedure for determining Manning’s n, a method classified as photographic method by French (1985), this is the gradually varied form of the energy equation which is used to estimate n.

In general, if the apparent mathematical complexity of the energy equation compared to that of the momentum equation is ignored, it is expected that the results of applying the energy equation together with the use of the Manning equation be closer to the measured values in situations where the results may be significantly different such as gradually varied flow in compound channels. This statement assumes that the constitutive relationship (second order loss equation) is governed by the thermodynamic behavior of the system. The mathematical complexity does not play any important role in the analysis of the gradually varied flow under steady state conditions. After analyzing the flow using...
the energy equation, the momentum equation can be used to evaluate the friction slope. In a next step, the friction slope equation \( S_f = \frac{1}{2} \frac{\nu}{f g R} \) can be used to estimate the distribution of average shear stresses, if this information is required.

As a final point, considering the possibility of occurrence of more than one critical depth in compound channels (Chaudhry, 1993) and that this phenomenon may affect the behavior of the water surface profiles in compound channels, some information about the values of the parameters such as possible critical depths, normal depth and downstream control depth in the experimental studies conducted by the authors would have been informative.

References


Discussions:

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The paper highlights a sensitive part of one-dimensional open channel flow modelling. The authors clarify the meaning of energy and momentum equation for such modelling. Nevertheless, as they point it out, it does not yet completely solve the problem to know whether friction laws like the Manning’s one fit better the energy equation or the momentum one for non-uniform flow modelling. The discussers suggest that a further investigation of the physical meaning of the energy slope \( S_e \) and momentum slope \( S_m \) could help in the selection. It will not answer the question definitely but could give some insights, as illustrated by two examples : local losses in a channel contraction or expansion; momentum transfer correction in compound channels.

The discussers agree with the authors when they point out that the energy slope \( S_e \) could cover the dissipation into heat of more mechanical properties than only the friction on the solid boundary. It means that, while the momentum slope \( S_m \) corresponds to the real bed shear stress (as shown by equation 7), the energy slope \( S_e \) corresponds to the dissipation of turbulence and secondary currents into heat (23) by viscosity through the transformation into smaller scales of turbulence (Nezu and Nakagawa, 1993). It is clear that, in open channels, the main source of this turbulence and of these secondary currents is the bed shear stress : the corresponding part of energy slope \( S_e \) is thus equivalent to the momentum slope \( S_m \). Nevertheless, other turbulence sources can easily be identified like e.g. secondary currents due to centrifugal forces in bends; secondary currents due to channel contraction or expansion; momentum transfer in compound channel at the shear layer between main channel and floodplains: this leads to a difference between \( S_m \) and \( S_e \).

A classical illustration of this difference can be found for non-prismatic channels which were not considered by the authors: since the momentum (6) and energy (25) equations are presented only for prismatic channels, the distinction between the two equations does not appear for the evaluation of local losses due to expansion or contraction of flow. This distinction can be clearly demonstrated in the case of flow expansion in closed channel. In the momentum equation, no local loss is expressed but a new pressure term appears on the non-prismatic boundary. On the other hand, an explicit head loss term is necessary in the energy equation: this head loss corresponds to a recirculation of the flow behind the channel expansion. Indeed, the momentum equation is usually used to exhibit the value of the head loss in the energy one, leading to the classical Borda formula. For open channels, this shows that, if section inertia variations are taken into account in the momentum equation, local loss are only to be considered in the energy equation.

A second illustration is given by the momentum transfer in compound channels. For such channels, it is well known that classical formulae do not fit the discharge observations (Aackers, 1993). This is due to the momentum transfer at the interface between main channel and floodplains. Several flow corrections have been proposed that are theoretically based. Each of them fits a particular approach. Using the momentum equation, the effect of the interaction will be a modification of the bed shear stress distribution. This can be modelled by correcting the roughness coefficients (see e.g. Myers and Brennan, 1990). Using the energy equation, additional dissipation due to the secondary currents through the interface has to be added to the general dissipation of unmodified bed shear stress. This can be achieved either by a direct modelling of the shear layer and the corresponding shear stress at the interface (Wormleaton and Merrett, 1990) or by a modelling of the corresponding momentum flux exchange through the interface (Bousmar and Zech, 1999).

In their simulations of flow in SERC-FCF channel (Figures 6 and 7), the authors used the measured roughness coefficient in each subsection (Williams, 1998): this corresponds to a roughness correction method. While such a method is appropriate for momentum slope \( S_m \) evaluation (Beta curves), it is inadequate for energy slope \( S_e \) evaluation (Alpha curves) as it has been shown that a difference exists between those two slopes. Therefore, the discussers calculated new water profiles using the energy equation and their own correction of energy slope \( S_e \) for momentum transfer (Bousmar and Zech, 1999). In this case, a roughness coefficient close to the one measured at bankfull flow in main channel (Knight, 1992) has been used for the whole section.

Results are plotted on Figures A1 and A2 where profiles computed with the corrected energy slope are identified by squares. As expected from the authors demonstration, when the Coriolis coefficient \( \alpha \) is taken equal to 1, the profile is similar to the one obtain with the Boussinesq coefficient \( \beta = 1 \), proving the validity of the discussers’ calculations. For simulations with a variable \( \alpha \), the latter computed by using vertical divisions of the section, a notable difference appears between the profiles calculated with friction correction and those calculated with energy slope correction, mainly for the larger discharge. Unfortunately, as the experimental noise is larger than the observed differences, it is impossible to come to a definitive conclusion.
References


Discussors:

Delbert Franz

This paper reviews a topic of continuing interest, that is, the differences and similarities between the principles of conservation of momentum and of mechanical energy in open-channel flow. The authors have improved on the presentation of Yen and Wenzel(12) on this topic, by expanding the various derivative terms containing an implicit $\delta \beta / \delta x$ in (12), that is, $\delta \alpha / \delta x$ and $\delta \beta / \delta x$.

Expanding these terms involves some assumptions on the variation of the distribution of the point velocities, v, even if the channel is prismatic. (Figure 2 implies that the channel is prismatic because no forces acting on the channel sides are shown.) The expansions leading to Eqs. 29 and 30 assume that, $\delta \beta / \delta x = (\delta \beta / \delta y) \delta y / \delta x$ and similarly for $\alpha$. Taking $\delta \beta / \delta x$, using the defining integral for $\beta$, shows that

$$\int_A \frac{\partial \beta}{\partial x} dA = \frac{\delta}{\partial A} \int_A \beta dA$$

must be true at each cross section for the assumption on $\beta$ to be valid. A similar requirement holds for $\alpha$ as well. If the lateral inflow is zero, then the point velocities must not depend explicitly on $x$ but only implicitly through variation of the depth, $y$. The requirement with lateral inflow present is more complex. In this case it is sufficient to have velocity distributions be similar at each section with the maximum velocity varying with distance in direct proportion to variation of the mean velocity. If Eq. A-0 is not valid, then extra terms must appear in Eqs. 29 and 30 as well as in the equations below. Turbulent velocity profiles in boundary layers are not truly similar (Daily and Harleman, 1966, p. 244) so that Eq. A-0 is probably not satisfied exactly in any flow other than uniform flow. Because the distribution of the point velocities is only known approximately and to be consistent with the authors’ assumption, we use the same assumption here.

Assuming that Eq. A-0 is satisfied, Eq. 30 is a marked improvement over the result given for the difference between $S_f$ and $S_f$ in Yen and Wenzel(12). However, using a Froude number in Eq. 30 that does not reflect the effects of a non-uniform velocity distribution masks its meaning.

To see this, restrict Eq. 30 to steady flow, and also ignore lateral inflow. Equation 30 can then be written as

$$S_f = S_A + \frac{\partial \gamma}{\partial \lambda} \frac{Q^2}{gA} \left( \frac{2}{A} \frac{\partial \alpha}{\partial y} \right)$$

Bilalock and Sturm(1981, 1983) and Franz(1982) show that

$$\dot{F}_\alpha = \frac{Q^2}{gA} \left( aB - \frac{A \partial \alpha}{2 \partial y} \right)$$

and

$$\dot{F}_\beta = \frac{Q^2}{gA} \left( bB - \frac{A \partial \beta}{2 \partial y} \right)$$

Fig. A1. Water profile in SERC-FCF channel at discharge of 0.5513 m$^3$/s.

Fig. A2. Water profile in SERC-FCF channel at discharge of 0.2821 m$^3$/s.
are Froude numbers, where the “hat” is included in the symbol to denote that the pressure distribution is hydrostatic. Equation A-1 then becomes

$$S_e = S_f + \frac{\partial y}{\partial x} \left( F_a - F_b^f \right) \quad (A-4)$$

Jaeger (1956, pp. 93-119) shows that critical flows defined by the energy and momentum principles are identical if the effects of non-uniform velocity and streamline curvature are included. Blalock and Sturm (1981, 1983) showed that Eqs. A-2 and A-3 predict the same critical depth values from measurements in a laboratory compound channel with essentially uniform flow. Thus, under the assumptions inherent in the equation derivation, $F_a = F_b^f$, and therefore Eq. A-4 shows that $S_f = S_e$ in a prismatic channel without lateral inflow.

This result is unexpected. However the equations must show that result. Their derivation assumed a hydrostatic pressure distribution and the equations cannot detect any effects of deviations from that assumption. The Froude numbers computed using $\alpha$ and $\beta$ based on measurements in a real flow will not produce values using Eqs. A-2 and A-3 that are equal. The real flow will have a non-hydrostatic pressure and therefore the approximate Froude numbers will differ slightly. This difference will not give the correct difference between $S_f$ and $S_e$, however.

To find a difference between the two slopes, we must have a non-hydrostatic pressure distribution in the equations used to define the difference. Yen and Wenzel (1972) introduced coefficients that reflect the effect of a non-hydrostatic pressure distribution on the energy and momentum equations. If their Eq. 37 is expanded and simplified in the same way as Eq. 38, we find, taking $\cos \theta = 1$ in their equations, that

$$S_e = S_f + \frac{\partial y}{\partial x} \left[ \frac{gA}{k} \left( \alpha B - \frac{A \partial \alpha}{2 \partial y} \right) \right]$$

$$- \frac{\partial y}{\partial x} \left[ \frac{gA}{k} \left( \beta B - \frac{A \partial \beta}{2 \partial y} \right) \right] - \left( m \left[ 1 + \frac{\partial B}{A} \right] + \frac{\partial m}{\partial y} \right) (F_e - 1) \quad (A-5)$$

where $k$ is the coefficient in the energy equation defined by $f_s(p + \rho g)vdA = kpgQv$; and $m$ is the coefficient in the momentum equation defined by $f_s pdA = mpqAy$.

Rearranging Eq. A-5 to show Froude numbers analogous to those used earlier, but including the effects of a non-hydrostatic pressure distribution, yields

$$S_e = S_f + \frac{\partial y}{\partial x} \left[ k + \frac{\partial k}{\partial y} \right] (F_e - 1) - \left( m \left[ 1 + \frac{\partial B}{A} \right] + \frac{\partial m}{\partial y} \right) (F_e - 1) \quad (A-6)$$

Now $F_a = F_b^f = F$ and Eq. A-6 simplifies to

$$S_e = S_f + \frac{\partial y}{\partial x} \left[ k + \frac{\partial k}{\partial y} \right] - \left( m \left[ 1 + \frac{\partial B}{A} \right] + \frac{\partial m}{\partial y} \right) (F^2 - 1) \quad (A-7)$$

where $F$ is the Froude number for the flow and is defined by

$$F^2 = \frac{Q^2 \alpha B - A \frac{\partial \alpha}{2 \partial y}}{gA^3} = \frac{Q^2 \beta B - A \frac{\partial \beta}{2 \partial y}}{gA^3} \left( 1 + \frac{\partial B}{A} \right) + \frac{\partial m}{\partial y} \quad (A8)$$

Let $p = \rho g(y - \eta) + \rho g(y, \eta, w)$, where the head increment, $\eta$, representing a deviation from the hydrostatic, must be such that $\epsilon(y, y, w) = 0$, that is, the deviation at the water surface must be zero. Here $w$ is an axis in the cross section perpendicular to $\eta$. We assume that $\epsilon$ varies only implicitly with $x$ through variation of $y$ with $x$. If $\epsilon$ depends on $x$ explicitly in addition to the implicit dependence through $y$, then the equations given here will require additional terms reflecting the variation of $\epsilon$ with $x$ when $y$ is fixed.

Substituting this value of pressure into the definitions for $m$ and $k$ and evaluating the correction terms in Eq. A-8 gives

$$k + \frac{\partial k}{\partial y} = 1 + \frac{\partial}{\partial y} \left( \frac{Q}{A} \right) \int_n v dA = 1 + \frac{\partial}{\partial y} \left( \frac{Q}{A} \right) \int_n v dA \quad (A9)$$

and

$$m \left( 1 + \frac{\partial B}{A} \right) + \frac{\partial m}{\partial y} = 1 + \frac{\partial}{\partial y} \left( \frac{Q}{A} \right) \int_n \epsilon dA = 1 + \frac{\partial}{\partial y} \left( \frac{Q}{A} \right) \int_n \epsilon dA \quad (A10)$$

(Interchange of the derivative and integral is valid as shown given the assumptions on variation of $v$ and $\epsilon$) Thus the correction terms in the Froude numbers both approach unity as the pressure approaches the hydrostatic value. Substituting Eqs. A-9 and A-10 into Eq. A-8, expanding $\partial \nu / \partial y$, and collecting terms gives

$$S_e = S_f + \frac{\partial y}{\partial x} \left[ \frac{V}{V - 1} \frac{\partial \nu}{\partial y} \right] dA + \frac{1}{\frac{Q}{A} \frac{\partial y}{\partial y}} \int_n \frac{\partial \nu}{\partial y} dA \quad (F^2 - 1) \quad (A11)$$

as the final form of the relationship for the two slopes in a prismatic channel with no lateral inflow. The lateral inflow term can be added as required. It contains all the information that exists in its original form in Yen and Wenzel (1972), assuming Eq. A-0 holds. However, it shows some interesting conclusions. First, it supports the earlier conclusion that the assumption of a hydrostatic pressure distribution can only give the result that $S_f = S_e$. Second, it shows that the relationship between the two slopes depends on the Froude number and that the slopes are equal at a section if $F = 1$ at the section. Third, it shows that the difference between the two slopes is the product of three factors, two of which are usually small. If the flow is sub-critical then the third factor is at least smaller than 1.0. This suggests that in the usual range of flows described as gradually varied, the difference between $S_f$ and $S_e$ is small, when lateral inflow is absent. Yen, Wenzel, and Yoon (1972) discuss some of the effects of lateral inflow.