The Capacity Region of Gaussian Fading Multiple Access Relay Channels with Orthogonal Components

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***Abstract*— In this paper, we investigate the capacity problem of Gaussian fading multiple access relay channel with orthogonal components from the senders to the relay receiver and from the senders and relay to the receiver. This model is motivated by the practical constraint that a node cannot send and receive at the same time or in the same frequency band. We derive a capacity region for the model where partial Channel State Information (CSI) is available at the transmitters (CSIT) and perfect CSI is available at the receiver (CSIR). Superposition block Markov encoding and multiple access channel encoding and decoding strategies are used to prove the results.**

Keywords- additive white Gaussian noise; Gaussian fading channels; channel state information; discrete multiple access relay channel; orthogonal components; discrete multiple access relay channel.

I. INTRODUCTION

The relay channel was first introduced by Van der Meulen [1]. In [2] the capacity of degraded and reversely degraded relay channels and the capacity of the relay channel with feedback as well as upper and lower bounds on the capacity of the general relay channel were established. In [3] the capacity of semi deterministic relay channel , in [4] and [5] the capacity of the relay channel with orthogonal components, in [6] the capacity of modulo-sum relay channel, in [7] the capacity of a class of deterministic relay channel and in [8], [9] capacity of a more general class of relay channels have been determined. The multiple access rely channel with orthogonal components (MARCO) was introduced in [10] and a capacity region was obtained for it in [11].

The rest of the paper is organized as follows: In section II, preliminaries and main results are introduced. We prove the main results in section III. Finally, the paper is concluded in Section IV.

II. PRELIMINARIES AND MAIN RESULTS

We divide X_k , $k \in \{1, \dots, N\}$ to orthogonal components (X_{Rk}, X_{Dk}) and send these components from the sender to the relay receiver $(X_{Rk})$and from the sender and relay to the receiver (X_{Dk}). For Gaussian fading multiple access relay channel with orthogonal components, we derive a capacity region.

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A discrete memoryless multiple access relay channel is said to have orthogonal components if the channel input-output distribution can be expressed as

$$P(y_D, y_R, x_{R1}, \dots, x_{RN,} x_{D1,} \dots, x_{DN,} x_R) = P(y_R|x_{R1}, \dots, x_{RN}, x_R)$$

$$\times P(y_D|x_{D1}, \dots, x_{DN}, x_R) P(x_R) \prod_{k=1}^{N} P(x_{Rk}|x_R) P(x_{Dk}|x_R) \qquad (1)$$

where x_R, x_{Rk} and x_{DK}, $k \in \{1, \dots, N\}$, are the inputs, y_D and y_R are the outputs, all with finite alphabets, and $P(y_D, y_R|x_{R1}, \dots, x_{RN,} x_{D1,} \dots, x_{DN,} x_R)$ is the channel probability function.

$\mathbf{H} \in R_+^{2\times(N+1)}$ is a random matrix representing the state process of the channel (the matrix of fading coefficients) and the elements of $\mathbf{H}$ are r.v.'s which belong to R_+ as follows,

$$\mathbf{H}_t = \begin{bmatrix} h_{D1,t} & \dots\ h_{DN,t} & h_{DR,t} \\ h_{R1,t} & \dots\ h_{RN,t} & 0 \end{bmatrix}, \quad 1 \le t \le n \qquad (2)$$

and $F_k \in \theta_k, k = 1, \dots, N, R$, is a r.v. representing the CSIT available at the k^{th} transmitter which is a deterministic function of H: $F_k = \gamma_k(H)$. In addition, the function $\varphi_k: \theta_k \to R_+$ satisfying the following constraint:

$$E\{\varphi_k(F_k)\} \le P_k, \quad k = 1, \dots, N, R \qquad (3)$$

that denotes the power allocation policy for the i^{th} transmitter and relay. $\alpha_k: \theta_k \to [0,1]$ and $\beta_k: \theta_k \to [0,1]$, $k = 1, \dots, N$, are two arbitrary (limited) deterministic functions that take their values from the interval [0,1]. At the t^{th} transmission $(t = 1, \cdots, n)$, $X_{Rk,t}, X_{Dk,t}$ $(k = 1, \dots, N)$ and $X_{R,t}$ are sent and the channel outputs are

$$Y_{R,t} = \sum_{k=1}^{N} h_{Rk} X_{Rk,t} + Z_{1t} \qquad (4)$$

$$Y_{D,t} = \sum_{k=1}^{N} h_{Dk} X_{Dk,t} + h_R X_{R,t} + Z_{2t} \qquad (5)$$

where N is the number of senders, $\mathbf{Z_i} = (Z_{i1}, \dots, Z_{it}, \dots, Z_{in})$, $i = 1,2$, is a sequence of independent identically distributed (i.i.d) normal random variables with zero mean and variance N_i , $i = 1,2$,.

Now, we express main results as two following theorems.

Theorem 1. The capacity region of N-source fading multiple access relay channels with orthogonal components denoted by $\mathcal{C}_{\text{MARCO}}^{\text{F}}$ is given by

$$\mathcal{C}_{\text{MARCO}}^{\text{F}} = \cup \left\{ \begin{array}{c} (R_1, \ldots, R_N) \in R_+^N \\ \forall A \subseteq \{1, \ldots, N\}: \\ \sum_{k \in A} R_k \leq \min\langle I_{Rk} + I_{Dk}, I_k \rangle \end{array} \right\} \quad (6)$$

$$I_{Rk} = I(\{V_{Rk}\}_{k \in A}; Y_R | H, \{V_{Rk}\}_{k \notin A}, X_R)$$

$$I_{Dk} = I(\{V_{Dk}\}_{k \in A}; Y_D | H, \{V_{Dk}\}_{k \notin A}, X_R)$$

$$I_k = I(\{V_{Dk}\}_{k \in A}, X_R; Y_D | H, \{V_{Dk}\}_{k \notin A})$$

where the joint p.d.f of r.v.'s $(H, F_1, \ldots, F_N, F_R, V_{R1}, \ldots, V_{RN}, V_{D1}, \ldots, V_{DN}, X_R)$ is given by:

$$P(h, f_1, \ldots, f_N, f_R, V_{R1}, \ldots, V_{RN}, V_{D1}, \ldots, V_{DN}, X_R)$$

$$= P(h, f_1, \ldots, f_N, f_R) P_{X_R}(x_R) \prod_{k=1}^{N} P_{V_{Rk}|X_R}(v_{Rk}|x_R)\, P_{V_{Dk}|X_R}(v_{Dk}|x_R) \quad (7)$$

Furthermore, $\{g_k(.): (\mathcal{V}_{Rk}, \mathcal{V}_{Dk}) \times \theta_k \to R\}$ is a set of deterministic function such that $(X_{Dk,t}, X_{Rk,t}) = g_k\left((V_{Rk,t}, V_{Dk,t}) \times F_k\right), k = 1, \ldots, N,\ 1 \leq t \leq n$ and a set of relay function $\{g_{R,t}\}_{t=1}^{n}$ such that $X_{R,t} = g_{R,t}(Y_R^{t-1}, F_R), 1 \leq t \leq n$, satisfies the power constraint policy of the transmitters and relay: $E\{X_{Rk}^2 + X_{Dk}^2\} \leq P_k,\ k = 1, \ldots, N$ and $E\{X_R^2\} \leq P_R$. And a decoding function, $q: \mathcal{Y}_D^n \longrightarrow \mathcal{W}_1 \times \ldots \times \mathcal{W}_N = (\mathcal{W}_{R1}, \mathcal{W}_{D1}) \times \ldots \times (\mathcal{W}_{RN}, \mathcal{W}_{DN})$.

Sender 1 chooses an index $w_1 = (w_{R1}, w_{D1})$ uniformly distributed over $[1:2^{nR_{R1}}] \times [1:2^{nR_{D1}}]$ and sends the corresponding codeword over the channel. Other Senders do likewise. Assuming that the distribution of messages over the product set $\mathcal{W}_1 \times \ldots \times \mathcal{W}_N$ is uniform, we define the average probability of error for the $\left((2^{nR_1}, \ldots, 2^{nR_N}), n\right)$ code as follows:

$$P_e^{(n)} = \frac{1}{2^{n(\sum_{k=1}^{N} R_k)}} \times \sum_{(w_1, \ldots, w_N) \in \mathcal{W}_1 \times \ldots \times \mathcal{W}_N} \Pr\{g(Y_D^n) \neq (w_1, \ldots, w_N) | (w_1, \ldots, w_N) \text{ has been sent}\}$$

A rate $(R_1, \ldots, R_N)$ is said to be achievable for the Gaussian fading multiple access relay channel with orthogonal components if there exists a sequence of $\left((2^{nR_1}, \ldots, 2^{nR_N}), n\right), R_k = R_{Rk} + R_{Dk}, k = 1, \ldots, N,$ codes with $P_e^{(n)} \to 0$.

Theorem 2. The capacity region of N-source Gaussian fading multiple access relay channels with orthogonal components denoted by $\mathcal{C}_{\text{MARCO}}^{\text{GF}}$ is given by

$$\mathcal{C}_{\text{MARCO}}^{\text{GF}} = \cup \left\{ \begin{array}{c} (R_1, \ldots, R_N) \in R_+^N \\ \forall A \subseteq \{1, \ldots, N\}: \\ \sum_{k \in A} R_k \leq \min(\acute{I}_{Rk} + \acute{I}_{Dk}, \acute{I}_k) \end{array} \right\} \quad (8)$$

$$\acute{I}_{Rk} = E_H\left[C\left(\frac{\sum_{k \in A} h_{Rk}^2 \varphi_k(F_k)\left(1 - \alpha_k^2(F_k)\right)\left(1 - \beta_k(F_k)\right)}{N_1}\right)\right]$$

$$\acute{I}_{Dk} = E_H\left[C\left(\frac{\sum_{k \in A} h_{Dk}^2 \varphi_k(F_k) \alpha_k^2(F_k)\left(1 - \gamma_k^2 \beta_k(F_k)\right)}{N_2}\right)\right]$$

$$\acute{I}_k = E_H\left[C\left(\frac{h_{DR}^2 \varphi_R(F_R) + \sum_{k \in A}\left(\begin{array}{c} h_{Dk}^2 \varphi_k(F_k) \alpha_k^2(F_k) \\ +2 h_{DR} h_{Dk} \sqrt{\gamma_k^2\, \beta_k(F_k) \varphi_R(F_R) \varphi_k(F_k) \alpha_k^2(F_k)} \end{array}\right)}{N_2}\right)\right]$$

III. PROOF OF THE MAIN RESULTS

A. Proof of Theorem 1:

Achievability part:

We begin with a brief outline of the proof. We consider B blocks, each of n symbols. We use superposition block Markov coding. A sequence of B messages $w_{1,i} \times \ldots \times w_{N,i} = (w_{R1,i}, w_{D1,i}) \times \ldots \times (w_{RN,i}, w_{DN,i}),\ i \in 1,2, \ldots, B$, will be sent over the channel in nB transmissions. In each n-block, $b = 1,2, \ldots, B+1$, we use the same set of codebooks:

$$\mathcal{C} = \{x_R^n(v), v_{R1}^n(v, m_1), \ldots, v_{RN}^n(v, m_N), v_{D1}^n(v, u_1), \ldots, v_{DN}^n(v, u_N)\}$$

$$v \in [1:2^{nR_1}] \times \ldots \times [1:2^{nR_N}], m_k \in [1:2^{nR_{Rk}}],\ u_k \in [1:2^{nR_{Dk}}]$$

Now we proceed with the proof of achievability using a random coding technique.

Random codebook generation: First fix a choice of $P_{X_R}(x_R) \prod_{k=1}^{N} P_{V_{Rk}|X_R}(v_{Rk}|x_R)\, P_{V_{Dk}|X_R}(v_{Dk}|x_R)$

1) Generate $2^{n(\sum_{k=1}^{N} R_k)}$ independent identically distributed n-sequences x_R^n, each drawn according to $P(x_R^n) = \prod_{t=1}^{n} P(x_{R,t})$. Index them as $x_R^n(v)$, $v = (v_1, \ldots, v_N) \in [1:2^{nR_1}] \times \ldots \times [1:2^{nR_N}]$.
2) For each $x_R^n(v)$, generate $2^{nR_{Rk}}, k = 1, \ldots, N$, conditionally independent n-sequence x_{Rk}^n drawn according to $P\left(v_{Rk}^n | x_R^n(v)\right) = \prod_{t=1}^{n} P\left(v_{Rt,t} | x_{R,t}(v)\right)$. Index them as $v_{Rk}^n(v, m_k)$, $m_k \in [1:2^{nR_{Rk}}]$.
3) For each $x_R^n(v)$, generate $2^{nR_{Dk}}, k = 1, \ldots, N$, independent identically n-sequence v_{Dk}^n, each drawn according to $P(v_{Dk}^n | x_{Rk}(v)) = \prod_{t=1}^{n} P(v_{Dk,t} | x_{R,t}(v))$. Index them as $v_{Dk}^n(v, u_k)$, $u_k \in [1:2^{nR_{Dk}}]$.

Encoding: Encoding is performed in B+1 blocks, the coding strategy is shown in table I.

1) Source terminals: The messages are split into B equally sized blocks $w_{R1,b}, \ldots, w_{RN,b}, w_{D1,b}, \ldots, w_{DN,b},\ b = 1,2, \ldots, B$. In block $b = 1,2, \ldots, B+1$, the k^{th} encoder assuming observation $f_k \in \theta_k$ as CSIT, it sends $(X_{Dk}, X_{Rk}) = g_k\left(\left(V_{Rk}(w_{Rk,b}, w_{Rk,b-1}), V_{Dk}(w_{Dk,b}, w_{Dk,b-1})\right), F_k\right)$ over the channel, where $w_{Rk,0} = w_{Rk,B+1} = w_{Dk,0} = w_{Dk,B+1} = 1$.

2) Relay Terminal: After the transmission of block b is completed, the relay has seen $y^n_{R,b}$. The relay tries to find $\tilde{v}_b = (\tilde{w}_{R1,b}, \dots, \tilde{w}_{RN,b})$ such that

$$(v^n_{R1,b}(\widehat{\widehat{w}}_{R1,b-1}, \tilde{w}_{R1,b}), \dots, v^n_{RN,b}(\widehat{\widehat{w}}_{RN,b-1}, \tilde{w}_{RN,b}), x^n_{R2,b},$$

$$x^n_{R,b}\left(\tilde{v}_b = (\widehat{\widehat{w}}_{R1,b-1}, \dots, \widehat{\widehat{w}}_{RN,b-1})\right), H^n, y^n_{R,b})$$

$$\in A^n_\epsilon(V_{R1}, \dots, V_{RN}, X_R, H, Y_R) \quad (9)$$

where $\widehat{\widehat{w}}_{R1,b-1}, \dots, \widehat{\widehat{w}}_{RN,b-1}$ are the relay terminal's estimate of $w_{R1,b-1}, \dots, w_{RN,b-1}$, respectively. If one or more such $\tilde{v}_b = (\tilde{w}_{R1,b}, \dots, \tilde{w}_{RN,b})$ are found, then the relay chooses one of them, and then transmits $x^n_{R,b+1}(\widehat{\widehat{w}}_{R1,b}, \dots, \widehat{\widehat{w}}_{RN,b})$ in block b+1.

3) Sink Terminal: After block b, the receiver has seen $y^n_{D,b-1}$ and $y^n_{D,b}$ and try to find $\tilde{v}_{b-1} = (\tilde{w}_{R1,b-1}, \dots, \tilde{w}_{RN,b-1})$, $\tilde{w}_{D1,b-1}, \dots, \tilde{w}_{DN,b-1}$ such that

$$(v^n_{R1,b-1}(\widehat{\widehat{w}}_{R1,b-2}, \tilde{w}_{R1,b-1}), \dots, v^n_{RN,b-1}(\widehat{\widehat{w}}_{RN,b-2}, \tilde{w}_{RN,b-1}),$$

$$v^n_{D1,b-1}(\widehat{\widehat{w}}_{D1,b-2}, \tilde{w}_{D1,b-1}), \dots, v^n_{DN,b-1}(\widehat{\widehat{w}}_{DN,b-2}, \tilde{w}_{DN,b-1}),$$

$$x^n_{R,b-1}\left(\tilde{v}_{b-1} = (\widehat{\widehat{w}}_{R1,b-2}, \dots, \widehat{\widehat{w}}_{RN,b-2})\right), H^n, y^n_{R,b-1})$$

$$A^n_\epsilon(V_{R1}, \dots, V_{RN}, V_{D1}, \dots, V_{DN}, X_R, H, Y_R) \quad (10)$$

and

$$(x^n_{R,b}\left(\tilde{v}_b = (\tilde{w}_{R1,b-1}, \dots, \tilde{w}_{RN,b-1})\right), y^n_{D,b}) \in A^n_\epsilon(X_R, Y_D) \quad (11)$$

Decoding and error Analysis: It can be shown that the relay can decode reliably if

$$\sum_{k\in A} R_{Rk} \le I(\{V_{Rk}\}_{k\in A}; Y_R | H, \{V_{Rk}\}_{k\notin A}, X_R) \quad (12)$$

and the receiver can decode with arbitrarily small probability of error if

$$\sum_{k\in A} R_{Dk} \le I(\{V_{Dk}\}_{k\in A}; Y_D | H, \{V_{Dk}\}_{k\notin A}, X_R) \quad (13)$$

and receiver can decode in backward manner with arbitrarily small probability of error if

$$\sum_{k\in A} R_k \le I(\{V_{Dk}\}_{k\in A}, X_R; Y_D | H, \{V_{Dk}\}_{k\notin A}) \quad (14)$$

Converse part:

To prove the converse part, we first derive an outer bound on C^F_{MARCO} in the following lemma.

TABLE I. ENCODING STRATEGY

Block 1	Block 2	⋯	Block B + 1
$x^n_{R,1}(1, \dots, 1)$	$x^n_{R,2}(w_{R1,1}, \dots, w_{RN,1})$	⋯	$x^n_{R,B+1}(w_{R1,B}, \dots, w_{RN,}$
$v^n_{R1,1}(1, w_{R1,1})$	$v^n_{R1,2}(w_{R1,1}, w_{R1,2})$	⋯	$v^n_{R1,B+1}(w_{R1,B}, 1)$
. . .	. . .	⋯ ⋯ ⋯	. . .
$v^n_{RN,1}(1, w_{RN,1})$	$v^n_{RN,2}(w_{RN,1}, w_{RN,2})$	⋯	$v^n_{RN,B+1}(w_{RN,B}, 1)$
$v^n_{D1,1}(1, w_{R1,1})$	$v^n_{D1,2}(w_{D1,1}, w_{R1,2})$	⋯	$v^n_{D1,B+1}(w_{D1,B}, 1)$
. . .	. . .	⋯ ⋯ ⋯	. . .
$v^n_{DN,1}(1, w_{DN,1})$	$v^n_{DN,2}(w_{DN,1}, w_{DN,2})$	⋯	$v^n_{DN,B+1}(w_{DN,B}, 1)$

Lemma: C^F_{MARCO} is outer bounded by:

$$\mathcal{C}^F_{MARCO} \subseteq \bigcup \left\{ \begin{array}{c} (R_1, \dots, R_N) \in R^N_+ \\ \forall A \subseteq \{1, \dots, N\}: \\ \sum_{k\in A} R_k \le \min\langle I_{Rxk} + I_{Dxk}, I_{kx}\rangle \end{array} \right\} \quad (15)$$

$$I_{Rkx} = I(\{X_{Rk}\}_{k\in A}; Y_R | H, \{X_{Rk}\}_{k\notin A}, X_R)$$

$$I_{Dkx} = I(\{X_{Dk}\}_{k\in A}; Y_D | H, \{X_{Dk}\}_{k\notin A}, X_R)$$

$$I_{kx} = I(\{X_{Dk}\}_{k\in A}, X_R; Y_D | H, \{X_{Dk}\}_{k\notin A})$$

where the joint p.d.f of r.v.'s $(H, F_1, \dots, F_N, F_R, X_{R1}, \dots, X_{RN}, X_{D1}, \dots, X_{DN}, X_R)$ is given by:

$$P(h, f_1, \dots, f_N, f_R, x_{R1}, \dots, x_{RN}, x_{D1}, \dots, x_{DN}, x_R) =$$

$$P(h, f_1, \dots, f_N, f_R) P_{X_R}(x_R) \prod_{k=1}^{N} P_{X_{Rk}|X_R}(x_{Rk}|x_R)\, P_{X_{Dk}|X_R}(x_{Dk}|x_R) \quad (16)$$

that satisfy the following power constraint:

$$E\{X_i^2\} \le P_i, i = 1, \dots, N, \; E\{X_R^2\} \le P_R. \quad (17)$$

Proof: The proof is done using Fano's inequality and the input deterministic functions. The details are omitted for the brevity.

The achievable rate region that was obtained is also an outer bound for the above outer bound i.e. the outer bound is a subset of the inner bound; consequently, it is proved that the achievable rate region is a capacity region. It is proved by substituting $(X_R, X_{R1}, \dots, X_{RN}, X_{D1}, \dots, X_{DN})$ as defined in following in the rate region described by (15); one can see that the rate region in (6) is obtained.

$$X_{Rk} = \sqrt{\varphi_k(F_k)}\left(\sqrt{\left(1 - \alpha_k^2(F_k)\right)}\, V_{Rk}\right), 0 \le \alpha_k(F_k) \le 1 \quad (18)$$

$$X_{Dk} = \sqrt{\varphi_k(F_k)}(\alpha_k(F_k)V_{Dk}), 0 \le \alpha_k(F_k) \le 1 \quad (19)$$

$$X_R = \sqrt{\varphi_R(F_R)}\left(\sum_{k=1}^{N}\sqrt{\beta_k(F_k)}\, V_{Rk}\right), \quad \sum_{k=1}^{N}\beta_k(F_k) = 1 \quad (20)$$

B. Proof of Theorem 2:

To prove of theorem let $[V_{R1}, \dots, V_{RN}, V_{D1}, \dots, V_{DN}]$ be a sequence of Gaussian distributed r.v.'s with zero mean and unit variance, independent of each other and also independent of the fading matrix **H**.
In addition, let $\varphi_k: \theta_k \to R_+, k = 1, \dots, N, R$, be a set of power allocation policy functions with satisfy the power constraints in $E\{X_{Rk,t}^2 + X_{Dk,t}^2\} \le P_i, k = 1, \dots, N, t = 1, \dots, n$, and $E\{X_R^2\} \le P_R$ and $\alpha_k: \theta_k \to [0,1]$ and $\beta_k: \theta_k \to [0,1], k = 1, \dots, N$, are two arbitrary (limited) deterministic functions that takes their value from the interval [0,1]. Define the r.v's $[X_R, X_{R1}, \dots, X_{RN}, X_{D1}, \dots, X_{DN}]$ as (18)-(20) and $\gamma_k = E(V_{Dk}V_{Rk})$.
Proof of achievability is the same as that of theorem1 with regard to the above suppositions. For converse part, by substituting $[V_{R1}, \dots, V_{RN}, V_{D1}, \dots, V_{DN}]$ and $[X_R, X_{R1}, \dots, X_{RN}, X_{D1}, \dots, X_{DN}]$ in the capacity region (6), the theorem 2 is proved. The proof is omitted for the brevity.

IV. Conclusion

We investigated the fading multiple access relay channel with orthogonal components, partial CSIT and perfect CSIR, an example of such a channel model is the cooperative uplink of some mobile stations to the base station with the help of the relay in a cellular based mobile communication system. We obtained a capacity region for this model and extended the result to the Gaussian case.

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