

## **Multi Objective Optimization of Turning Process using Grey Relational Analysis and Simulated Annealing Algorithm**

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**Abstract.** Multi objective optimizing of machining processes is used to simultaneously achieve several goals such as increased product quality, reduced production time and improved production efficiency. This article presents an approach that combines grey relational analysis and regression modeling to convert the values of multi responses obtained from Taguchi method design of experiments into a multi objective model. The proposed approach is implemented on turning process of St 50.2 Steel. After model development, Analysis of Variance (ANOVA) is performed to determine the adequacy of the proposed model. The developed multi objective model is then optimized by simulated annealing algorithm (SA) in order to determine the best set of parameter values. This study illustrates that regression analysis can be used for high precision modeling and estimation of process variables.

### **Introduction**

One of the most important methods in production of metal parts is machining. Turning is the most widely used machining processes that may result in high precision and quality and increased productivity. However, the quality of final product and its production cost heavily depend of process parameters values. Single purpose control and process optimization can't satisfy economic demands such as reducing time and costs with maintaining quality at the same time. This is because quality improvement usually increases production costs and thus productivity decreases. The use of traditional optimization methods such as differential measures and enumeration of all possible solutions is not very efficient and accurate. Use of metaheuristic algorithms in such incidents can improve speed and accuracy of the computations [1]. In many industrial applications related to the turning process, control and optimization of surface quality (SR) and material removal rate (MRR) are the most important performance measures and considered as the main responses [1-3].

Simulated annealing (SA) algorithm is one of efficient innovative optimization algorithms in solving optimization problems. This algorithm was introduced in 1982 by "Kirkpatrick", "Gelatt" and "Vecchi" [4]. Adaptability and ease of programming over the optimization problems and tolerability of feasible non improving solutions are the most important features of this algorithm.

In this article, a multi objective optimization model for CNC turning of St 50.2 Steel, has been developed and solved using SA algorithm. The model has been converted to a weighted multiple performance characteristics Grey Analysis method. The multiple performance characteristics include material removal rate (MRR) and surface roughness (SR). Three important machining parameters; namely cutting speed, feed rate and depth of cut are considered as the input process parameters. The analysis of variance (ANOVA) is also conducted to estimate the relative effect of each process parameters.

### Design and experimental conditions

Many factors affect the material removal rate and surface roughness in turning process. The important machining parameters include feed rate (F), depth of cut (D) and cutting speed (S). Fig. 1 schematically illustrates the turning mechanism with respect to its process variables.

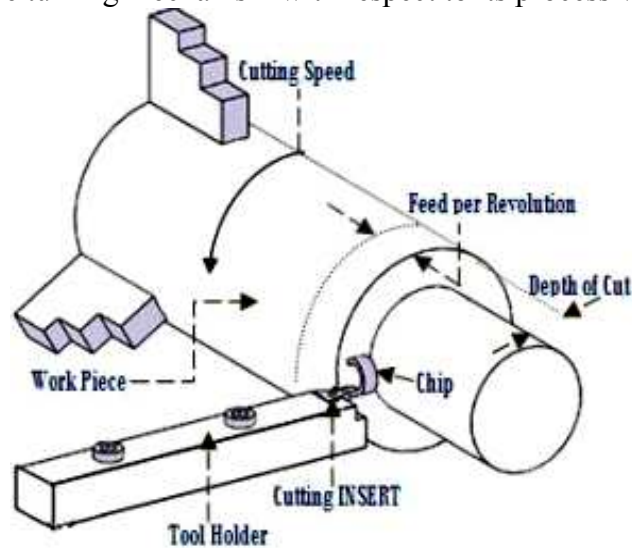


Figure 1. Machining parameters in turning process

Basically material removal rate is related to these three machining parameters. Material removal rates are determined by the following equation [1].

$$M = S \cdot F \cdot D \quad (1)$$

In this equation, S is cutting speed ( $\text{m}\cdot\text{min}^{-1}$ ), F is feed rate ( $\text{mm}\cdot\text{rev}^{-1}$ ) and D is the depth of cut (mm). Another measure of machining is surface quality, usually measured in terms of surface roughness. Surface roughness is one of the most important parameter of turned parts that affects such characteristics as fatigue resistance, cleaning ability, assembly tolerances and friction coefficients [6]. Cutting speed, feed rate and depth of cut have significant effect on average surface roughness ( $R_a$ ) [7]. Nevertheless, in most cases MRR and SR are acting opposite. Improving SR may cause the MRR to decrease and vice versa. Therefore, these two measures should be considered simultaneously. In this paper based on the experimental data obtained by Taguchi design of experiments matrix, mathematical models are developed to relate input parameters to output response characteristics.

The 16 sets of data used for modeling, are obtained using L16 orthogonal array matrix in Taguchi Design of Experiments [5]. The tests were performed under the following conditions:

- Machine tool: Tezsan-Oncu 260-330.600-C CNC Lathe
- Cutting tools: SECO inserts (SECO-DCMT11T304-F1) with the tool nose radius of 0.4mm
- Work parts: St 50.2 Steel with the length of 100mm and 25mm in diameter

Surface finish was measured using an automatic digital Hommel Surface Roughness Tester (T20). Table I, lists the ranges of machining parameters. Each parameter is considered to have 4 levels.

TABLE I. Experimental Machining Parameters

Cutting parameters	Range of change
Cutting speed (S)	100 - 250 ( $\text{m}\cdot\text{min}^{-1}$ )
Feed rate (F)	0.1 - 0.3 ( $\text{mm}\cdot\text{rev}^{-1}$ )
Depth of Cut (D)	0.5 - 2 (mm)

In Table II, the output results for all 16 test runs are given.

### Grey Relational Analysis And Modeling

The grey theory, first proposed by "Deng [7]", avoids the inherent defects of conventional, statistical methods and only requires a limited set of datas to estimate the behavior of an unknown system. During the past two decades, with hard work by scholars, the grey theory has been successfully applied to research in industry, social systems, ecological systems, economy, geography, traffic, management, education, environment, etc [7-9].

TABLE II. Experimental Conditions and Results

NO.	S (m.min)	F (mm.rev <sup>-1</sup> )	D (mm)	MRR (cm <sup>3</sup> .min <sup>-1</sup> )	SR (m)
1	100	0.15	0.5	7.5	2.08
2	150	0.15	0.5	11.25	1.48
3	200	0.15	0.5	15	0.93
4	250	0.15	0.5	18.75	1
5	100	0.15	1	15	3.28
6	150	0.3	1	45	2.38
7	200	0.3	1	60	2.5
8	250	0.3	1	75	2.5
9	100	0.3	1.5	45	0.5
10	150	0.3	1.5	67.5	0.95
11	200	0.1	1.5	30	2.15
12	250	0.1	1.5	37.5	0.55
13	150	0.1	2	30	0.38
14	200	0.1	2	40	0.73
15	250	0.1	2	50	0.78
16	100	0.3	2	60	1.25

Suppose in a system there are n series of data (number of run tests) and in each series m responses (number of dependent variables). Test results is then determined by  $y_{i,j}$  ( $i = 1,2, \dots, n$  and  $j = 1,2, \dots, m$ ). In Grey Relational analysis of this system following steps are performed [8,9]:

a) Data normalizing of each response to avoid the effect of adopting different units and reduce the variability:

$$Z_{i,j} = \frac{(y_{i,j} - \min(y_{i,j}, i = 1,2, \dots, n))}{(\max(y_{i,j}, i = 1,2, \dots, n) - \min(y_{i,j}, i = 1,2, \dots, n))} \quad (2)$$

$$Z_{i,j} = \frac{(\max(y_{i,j}, i = 1,2, \dots, n) - y_{i,j})}{(\max(y_{i,j}, i = 1,2, \dots, n) - \min(y_{i,j}, i = 1,2, \dots, n))} \quad (3)$$

When the higher value of a response is desired, the relation (2) is used for normalizing which is named "the-higher-the-better" criteria. Thus, material removal rate is normalized by this equation. When the lower value of a favorable response is desired, equation (3) is used for normalizing; termed "the-lower-the-better" criteria. By the same token, equation (3) is used to normalize observed surface roughness (SR).

b) Calculating the Grey Coefficient (GRC) for the normalized values through the following equation:

$$\gamma(Z_o, Z_{i,j}) = \frac{\Delta_{\min} + \zeta\Delta_{\max}}{\Delta_{oj}(k) + \zeta\Delta_{\max}} \quad (4)$$

Where

$Z_o(k)$  is the reference sequence ( $Z_o(k)=1, k=1, 2, \dots, m$ );

$\Delta_{oj}$  is the absolute value of the difference between  $Z_o(k)$  and  $Z_{i,j}(k)$ ;  $\Delta_{oj} = |Z_o(k) - Z_{i,j}(k)|$  [9].

$\Delta_{\min}$  and  $\Delta_{\max}$  respectively are the smallest and the largest value of difference between  $Z_o(k)$  and  $Z_{i,j}(k)$ :  $\Delta_{\min} = \min |Z_o(k) - Z_{i,j}|$ ,  $\Delta_{\max} = \max |Z_o(k) - Z_{i,j}|$

$\zeta$  is the distinguishing coefficient and  $0 \leq \zeta \leq 1$

c) Computing GRG for any response:

$$\text{Grade}(Z_o, Z_{i,j}) = \sum_{k=1}^n \beta_k \gamma(Z_o, Z_{i,j}) \quad (5)$$

where  $\sum_{k=1}^n \beta_k \gamma(Z_o, Z_{i,j}) = 1$  and  $\beta_k$  is weighting factor of each response [3].

The results of experiments using above mentioned method are used for model development. The weighting of parameters depends on the relative importance of each response. When weighting coefficients of each response are equal, the value of  $\zeta$  is set to 0.5 [9]. The results for GRA are shown in Table III. In this table, the last column is the weighted Gray Relational Grade for the two process outputs.

TABLE III. Results of Grey Relational Analysis

NO.	GRC of MRR	GRC of SR	Grade
1	0.3333	0.4603	0.3968
2	0.3461	0.5686	0.4574
3	0.36	0.725	0.5425
4	0.375	0.7004	0.5377
5	0.36	0.3333	0.3466
6	0.5294	0.4202	0.4748
7	0.6923	0.4061	0.5492
8	1	0.4061	0.7031
9	0.5294	0.9235	0.5545
10	0.8181	0.7178	0.6761
11	0.4285	0.4503	0.6844
12	0.4736	0.895	0.4153
13	0.4285	1	0.7143
14	0.4909	0.8055	0.6482
15	0.5744	0.7837	0.6792
16	0.6923	0.625	0.8125

**Regression Modeling.** Regression models can be used to predict the behavior of input variables (independent variables) and grey relational grades associated with each test response results [10]. Equation (6) shows the general form of quadratic regression model.

$$\begin{aligned} \text{Grade}(S, F, D) = & a_0 + a_1 S + a_2 F + a_3 D + a_4 S^2 \\ & + a_5 F^2 + a_6 D^2 + a_7 S \times F + a_8 S \times D + a_9 F \times D \end{aligned} \quad (6)$$

In the above formula  $a_0, a_1, \dots, a_9$  are the regression coefficients to be estimated. In this study, based on the GRA data given in Table III, the regression model is developed using MINITAB software. The independent variables in model (7) are cutting speed, feed rate and depth of cut.

$$\begin{aligned} \text{Grade}(S, F, D) = & 1.47 - 0.00009 S - 11 F - 0.607 D \\ & - 0.000009 S^2 + 16.9 F^2 + 0.216 D^2 + 0.0124 S \times F \\ & - 0.000569 S \times D + 2.06 F \times D \end{aligned} \quad (7)$$

**Model validation and ANOVA results.** Analysis of variance (ANOVA) is a mathematical way to determine precision and adequacy of regression modeling. It shows how well the proposed model fits the experimental data and, therefore, represents the actual process under study [11]. It is also a powerful tool in analyzing the variable effects on the process output responses.

A summary of ANOVA results for regression model have been presented in Table IV. Based on the statistical analysis results, the coefficient of determination,  $R_{adj}^2$ , for this model is equal to 89.9%. This indicates that the model has good compatibility to the actual data. The p-value of the model is also close to zero which shows the model is much better than the 0.95 confidence level. These demonstrate the appropriate compliance of the model with the actual test results.

TABLE IV. Result of ANOVA

Source	DF	Sum of Squares	Mean Squares	F Value	P Value
Regression model	9	0.216871	0.0722	4.6	0.003
Residual Error	6	0.05049	0.0042		
Total	15	0.26736	0.26736		
S = 0.0750987 R-Sq = 91.3% R-Sq(adj) = 89.9 %					

### Multi Objective Optimization

In this study, simulated annealing (SA) algorithm has been employed to determine the optimal set of machining parameters. This algorithm begins with an initial feasible solution and stepwise searches the solution domain for the optimal solution.

At each iteration a new solution in the neighborhood of the current solution is generated and evaluated. A move to new solution is then made under the following conditions:

a) If the objective functions value of the new solution is better than the current one and b) If the value or the probability function implemented in SA has a higher value than a randomly generated number between zero and one.

The probability function implemented in SA given by:

$$P = \exp \frac{\Delta F}{T_i} \quad (8)$$

In the above,  $\Delta F$  is absolute difference between the objective function of the current solution and the new solution in each step and  $T_i$  is system's temperature. In SA, as the search progresses  $T_i$  is gradually reduced according to a cooling schedule. A linear cooling schedule has been employed in this study:

$$T_{i+1} = \lambda T_i; \quad i = 0, 1, \dots, n \quad \text{and} \quad 0.9 < \lambda < 1 \quad (9)$$

For the multi objective optimization of turning operation, every feasible solution is a combination of cutting speed, feed rate and depth of cut (S, F, D) within their specified ranges. Since in our example both MRR and SR are given the same weight (50% each), the optimal solution is a combination of machining parameters that results is maximum MRR and minimum SR. Fig. 2 shows the convergence curve for the simulated annealing algorithm during the search.

The final objective function value found by the algorithm for GRG is 0.859 which corresponds to the following machining parameters:

S = 250 (m/min)

D = 1.9 (mm)

F = 0.1 (mm/rev)

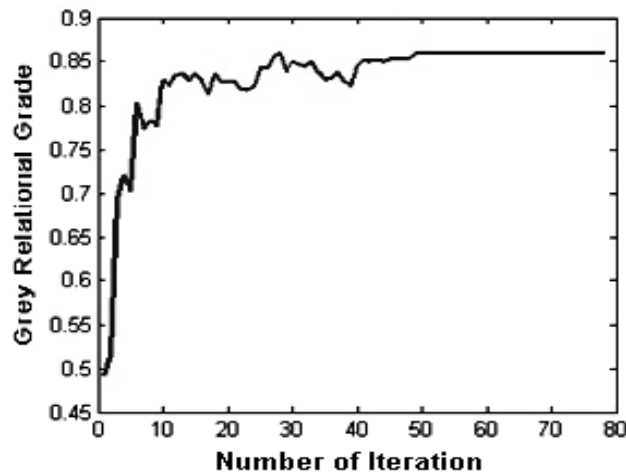


Figure 2. Simulated Annealing Algorithm performance

## Conclusion

Turning process modeling based on grey relational analysis and multiple regression models is successfully implemented and developed models is optimized with simulated annealing algorithm. Material removal rate and surface roughness are combined in a regression model using grey relational grades.

From ANOVA results of regression model, cutting speed (S), depth of cut (D) and feed rate (F) respectively are the most effective parameters in developed multi objective models. Selecting Cutting speed in 250 (m/min), Feed rate in 0.1 (mm/rev) and Depth of cut value in 1.9 mm concludes optimum machining condition in multi performance characteristics. This paper shows that multiple performance characteristics such as material removal rate and surface roughness can be improved by using this approach. Also SA due to adaptability and ease of programming is a powerful tool for optimum condition determination.

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