Hadronization and QCD in $e^+ e^-$ Annihilation at $\sqrt{s} = 52 - 57$ GeV

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Abstract – We investigate hadronization of charged particles in $e^+ e^-$ annihilation at $\sqrt{s} = 52 - 57$ GeV in the LHC. We show significant deviations from the LUND matrix element model. Possible explanation for these features is presented in this paper. Copyright © 2011 Praise Worthy Prize.

Keywords: Hadronization, QCD Models

I. Introduction

It is generally believed that Quantum Chromodynamics (QCD) is the correct theory for the description of the strong interaction of quarks and gluons. This theory has been successfully tested at high energies but proofs that QCD predicts some basic properties observed in nature, such as confinement of quarks in hadrons, are still missing. The lack of these proofs is due both to the mathematical complexity of the theory and to the nonapplicability of the perturbation theory at low energies. Due to the high statistics of the real data at different laboratories and the considerable theoretical progress in the field of perturbative QCD, the measurements and tests of QCD have entered the high precision regime. The strong coupling constant is not too "weak" at these high energies, which increases the reliability of perturbative calculations and at the same time, non-perturbative corrections too many observables, related to the hadronization of quarks and gluons into observable hadrons, become small.

We discuss a number of measurements concerning hadronization and QCD [1]-[7]. Our measurements are compared with the QCD + fragmentation models that use QCD matrix elements at the parton level, and either string or cluster fragmentation for hadronization.

II. Experimental Procedure

The AMY detector (Fig. 1) consists of a tracking detector and shower counter inside a 3-T solenoid magnetic coil which is surrounded by a steel flux return yoke followed by a muon detection system. The charged particle tracking detector consists of a 4 layer cylindrical array of drift tubes (inner tracking chamber, or ITK) and a 40-layer cylindrical drift chamber (central drift chamber or CDC) with 25 axial layers of wires and 15 stereo layers. Charged particles are detected efficiently over the polar angle region $\cos \theta$ with a momentum resolution $\Delta p/p = 0.75% \times \left[ p_{T} (\text{GeV} / c) \right]$. Radially, outside of the CDC is a 15-shot-length cylindrical electromagnetic calorimeter (barrel shower counter or SHC) which serves as a photon detector. The detector fully covers the angular region $\cos \theta=0.73$. Selection of multi-hadron final states from $e^+ e^-$ annihilation was based on the charged particle momenta measured in the CDC and on the neutral-particle energy measured in SHC. Further details may be found in Ref. [7].

III. Hadronization and Monte Carlo Models

A schematic of hadron production in $e^+ e^-$ annihilation is shown in Fig. 2. One may divide the process into several phases:

1. A hard electroweak process in which the primary quark and antiquark may be produced off mass-shell: $e^+ e^- \rightarrow q\bar{q}$.
2. Perturbative QCD evolution of the primary $q\bar{q}$ via parton Bremsstrahlung:

Fig. 1 AMY detector

Manuscript received and revised May 2011, accepted June 2011

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3. Hadronization of partonic system:

\( (q, \bar{q}, g) \rightarrow q, \bar{q}, g \)

4. Decays of primary resonances into stable particles:

\( B, C, \phi, \Delta, \rho \rightarrow \pi^\pm, k^\pm, p, \bar{p}, \ldots (e^\pm, \mu^\pm, \tau^\pm, \nu) \)

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**Fig. 2**: Schematic of hadron production in \( e^+ e^- \) annihilation

Phases 1 and 2 are generally agreed to be calculable using perturbative techniques applied to the electroweak theory and QCD, respectively. Phases 3 and 4 are more problematic in that they are intrinsically non-perturbative processes that cannot in general be calculated from first principles. Non-perturbative phase describes the transition from partons to hadrons. During the non-perturbative “confinement” phase, the QCD coupling constant becomes so large that perturbation theory breaks down and one must resort to phenomenological models, which they belong to either of two classes:

The first class is based on the exact matrix element in first - or second – order perturbative QCD and a phenomenological fragmentation model. (the model by the Lund group)

The second class of models uses perturbative QCD calculation in a leading logarithmic approximation (LLA) to calculate a shower of quarks and gluons. (the Webber model)

Since it is also necessary in phase 4 to simulate the interaction of particles with detectors, which can only be done in a deterministic fashion, Monte Carlo event generators have been developed for the complete simulation of hadronic event production in \( e^+ e^- \) annihilation and are now essential components of data analysis. In this paper we shall discuss some of the very popular Monte Carlo event generators; they are JETSET, HERWIG and BIGWIG. In the JETSET the perturbative phase can be generated either according to parton showers (PS), [1] or according to the exact second matrix element (ME)[2]. The latter allows for at most four partons in the final state. The transformation of these partons into observable hadrons is based on the relativistic massless string model. HERWIG and BIGWIG incorporate Marchesini-Webber parton shower and cluster fragmentation [3]. The philosophy here is to outline the main features of the generators.

In the context of their use as tools in understanding and correcting the data, no attempt will be made to justify these models on phenomenological grounds, and the outline will necessarily be brief.

Both JETSET and HERWIG implement electro-weak matrix elements for the production of a primary \( q\bar{q} \), as well as a perturbative QCD “parton shower” evolution of the system into a set of low-virtual-mass quarks and gluons. More formally, the latter is based on a probabilistic parton branching process that is derived from a leading + partial next-to-leading logarithmic resummation of the QCD matrix elements [4]. JETSET and HERWIG implement the parton branching process slightly differently, a discussion of which is beyond the scope of this paper, but both generators have a parameter \( A \) that characterises the scale of strong interactions, as well as a parameter \( Q_0 \) that characterises the minimum virtual-mass scale of the parton evolution.

JETSET implements the “string model” of jet fragmentation, illustrated in Fig. 3.

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**Fig. 3**: Schematic of hadronization in JETSET

The string-fragmentation scheme considers the color field between the partons, i.e., quarks and gluons, to be the fragmenting entity rather than the partons themselves. The string can be viewed as a color flux tube formed by gluon self-interaction as two colored partons move apart. Energetic gluon emission is regarded as energy-momentum carrying “kinks” on the string. When the energy stored in the string is sufficient, a \( q\bar{q} \) pair may be created from the vacuum. Thus the string breaks up repeatedly into color singlet systems as long as the invariant mass of the string piece exceeds the on-shell mass of a hadron. The \( q\bar{q} \) pairs are created according to the probability of a tunneling process \( \exp(-\pi m_{q\bar{q}}^2 / k) \) which depends on the transverse mass squared \( m_{q\bar{q}}^2 = p_{T,q\bar{q}}^2 + m_q^2 \) and the string tension \( k = 1\text{GeV} \). The transverse momentum \( p_{T,q\bar{q}} \) is locally compensated...
between quark and antiquark. Due to the dependence on the patron mass $m_q$ and/or hadron mass, $m_h$, the production of strange and, in particular, heavy-quark hadrons is suppressed. The light-cone momentum fraction $z = (E + p_i)_{L} / (E + p_i)_{Q}$, where $p_i$ is the momentum of the formed hadron $h$ along the direction of the quark $q$, is given by the string-fragmentation function:

$$f(z) = \frac{1}{z} (1-z)^a \exp\left(-\frac{b m_{Q,h}}{z}\right)$$  \hspace{1cm} (1)$$

where $a$ and $b$ are free parameters. These parameters need to be adjusted to bring the fragmentation into accordance with measured data, e.g., $a=0.11$ and $b=0.52$ GeV$^{-1}$ as determined in Ref.5.

The Webber model employs leading logarithmic parton-shower evolution and includes soft-gluon interference effects by angular ordering of successive gluon branchings. A highly virtual $q\bar{q}$ pair is generated and allowed to radiate gluons, which subsequently branch into more gluons or $q\bar{q}$ pairs according to the leading logarithmic QCD probabilities. The branching stops when the parton virtuality becomes less than cut off parameter $Q_c$.

At the end of cascade, when all partons have been put on the mass shell, the gluons are split into $q\bar{q}$ pairs, and each parton then joins with a neighbor of the correct color index to form a color-singlet cluster. The cluster for which the mass exceeds a certain value, $m_c$, are split into two, and all clusters are allowed to decay via a phase-space model into resonances or stable particles [5]. A schematic of the hadronization process as implemented in HERWIG is shown in Fig. 3.

HERWIG is tended to a general-purpose event generator able to simulate lepton-hadron and hadron-hadron scattering as well as $q\bar{q}$ collisions. There are some differences in such details as cluster decay between the two programs [6]. These are three important arbitrary parameters in the Webber model. The cascade virtuality cut off $Q_c$, the LLA RCD scale parameter $\Lambda_{LLA}$, and the cluster mass parameter $m_c$.

![Schematic of hadronization in HERWIG](image)

IV. Definition of the Variables

The properties of the events are analyzed in terms of commonly used global event shape variables.

It is convenient to use the momentum tensor:

$$M_{ab} = \sum_i P_{ai} P_{bi}$$  \hspace{1cm} (2)$$

Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues and $n_1, n_2, n_3$ the corresponding eigenvectors of $M$ which are ordered by $0 \leq \lambda_1 \leq \lambda_2 \leq \lambda_3$.

Let:

$$Q_0 = \frac{\lambda_2}{\sum_j \rho_j^2}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = \sum_j \rho_j^2$$  \hspace{1cm} (3)$$

These normalized eigenvalues $Q_0$ satisfy $Q_1 + Q_2 + Q_3 = 1$ and $0 \leq Q_1 \leq Q_2 \leq Q_3$, and their physical meanings are as follows:
\[ Q_1 = \min \frac{\sum_j (p_j \cdot n_j)^2}{\sum_j p_j} \]  
(4)
gives the flatness of the event;

\[ Q_2 = \min \frac{\sum_j (p_j \cdot n_j)^2}{\sum_j p_j} \]  
(5)
gives the width of the event and:

\[ Q_3 = \min \frac{\sum_j (p_j \cdot n_j)^2}{\sum_j p_j} \]  
(6)
gives the length of the event.

Colinear events are characterized by \( Q_2 \approx Q_3 \) and similarly coplanar events by \( Q_1 \approx Q_3 \).

The observables used in our measurements are defined as follows:
(a) \( S = \frac{1}{2} (Q_1 + Q_2) \), which lies in the range \( 0 < S < 1 \) and approaches 0 for thin 2-jet events and 1 for spherical events.
(b) The Aplanarity \( A = \frac{1}{2} Q_3 \), which lies in the range \( 0 < A < 0.5 \) and approaches 0 for planar events.
(c) The variable \( Q_4 \) which is defined as \( Q_4 = \frac{Q_1 - Q_2}{\sqrt{3}} \),

lies in the range \( 0 \leq Q_4 < \frac{1}{\sqrt{3}} \) and approaches 0 for spherical events. A measure of event structure that uses the linear momenta is the thrust value \( T = \sum p_i / |\sum p_i| \), where \( p_i \) refers to the momentum component along the axis for which the value of \( T \) is maximal, called the thrust axis.

(d) The major value \( M = \sum p_i / |\sum p_i| \), a thrust-like parameter where \( p_i \) refers to the momentum component along the axis perpendicular to the thrust axis that gives the largest value of \( M \) called the major axis, and the similarly defined minor value \( m \) where \( p_i \) refers to the momentum component along the so-called minor axis which is perpendicular both to the thrust axis and to the major axis.

(e) The Obliqueness \( O = M - m \), the difference between the major and minor values.

V. Data Correction

To correct the observed distributions for detector acceptance, initial-state radiation effects, and the multihadron-event selection cuts, QCD Monte Carlo programs including fragmentation models are used to simulate multihadron events. In first step, \( N_{\text{true}} \) Monte Carlo events are generated without initial-state radiation. These events yield the distributions \( n_{\text{true}}(x) \), where \( x \) represents the variable of interest, of all long-lived particles produced either at primary vertices or from the decays of all short-lived states such as \( K^0 \), strange baryons, resonances, and particles containing charm and bottom quarks. For the distributions of quantities that depend on particle masses, the known masses of the particles are used.

For the second step, events are generated including initial-state radiation and traced through the AMY detector. Energy loss, multiple scattering, photon conversion, and nuclear interactions in the material of the detector, as well as decays, are taken into account. This information is then converted into the quantities measured by the detector (e.g., drift times and pulse heights).

The events are then passed through the same reconstruction algorithms and analysis programs used for our experimental data. From the \( N_{\text{true}} \) accepted events, we obtain the detected particle distributions \( n_{\text{det}}(x) \).

The corrected distributions \( d_{\text{corr}}(x) \) as a function of a variable \( x \) are then obtained from the measured distributions \( d_{\text{meas}}(x) \) by using a bin-by-bin correction function \( C(x) \):

\[ d_{\text{corr}}(x) = C(x) d_{\text{meas}}(x) \]  
(7)

where \( C(x) \) is calculated by:

\[ C(x) = \frac{n_{\text{meas}}(x)}{N_{\text{meas}}} \frac{n_{\text{det}}(x)}{N_{\text{det}}} \]  
(8)

This method was used to compute correction functions using different models and then the results were averaged. We chose to use the Lund parton-shower (PS) model and the Lund matrix-element (ME) model. The difference between the correction functions computed from average values and from the two models individually was taken as an estimate of the systematic uncertainty in the corrections. These uncertainties were typically on the order of, or smaller than, the statistical uncertainties, except for the tails of a few distributions, and were combined in quadrature with the statistical errors. The correction factors themselves were generally close to unity, lying mainly between 0.7 and 1.4.

Because of the limited number of events available at any single energy value, the data collected from the 52-57 GeV energy region were averaged to produce the distributions. The contribution of this combining procedure to the systematic error was neglected, as investigation by additional Monte Carlo analysis found it to be small, typically on the order of 1/10 of the combined systematic and statistical errors. The combined data were assumed to correspond to data taken at an average center-of-mass energy of 55.2 GeV.
VI. Comparison with the Models

The data distributions are shown and compared with the predictions of LUND parton-shower, LUND matrix-element, and Webber QCD+ fragmentation models in Figs. 5 to 8.

As mentioned previously Sphericity is defined as \( S = \frac{1}{2} (Q_1 + Q_2) \). Ideal two jet events would have \( S = 0 \), while \( S \) would equal unity for completely isotropic events.

Aplanarity \( A = \frac{1}{2} Q_1 \) and the variable \( Q_2 = \frac{1}{\sqrt{2}} (Q_1 - Q_2) \) is also used at AMY energies. LUND ME shows some large deviations and demonstrates significant difficulties in reproducing the experimental data for most of the event shape distributions. On the other hand, both the Webber model (as incorporated in BIGWIG 4.3) and the LUND PS models provide a good description of AMY data.

![Graph showing Sphericity distribution](image1)

**Fig. 5.** Observed distributions of the quadratic global event variable Sphericity compared with Monte Carlo predictions

![Graph showing Aplanarity distribution](image2)

**Fig. 6.** Observed distributions of the quadratic global event variable Aplanarity compared with Monte Carlo predictions
VII. Conclusion

Data on hadron production by $e^+e^-$ annihilation at centre of mass energies between 52 – 57 GeV are presented and compared with the three most well known Models, which are based on QCD, but using different schemes for the fragmentation of quarks and gluons. Our data are produced better by the Webber and LUND parton shower models, but shows some deviations from the LUND matrix element model.

Acknowledgements

We would like to acknowledge the KEK staff and the AMY Collaboration for giving us the opportunity of using the AMY data for this analysis.

This work was funded by the vice president for Research & Technology of Ferdowsi University of Mashhad, Code: 2/15353, (date13/09/1389).
References


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