Self-scheduling approach for large consumers in competitive electricity markets based on a probabilistic fuzzy system

M. Zarif1 M.H. Javidi1 M.S. Ghazizadeh2

1Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
2Power and Water University of Technology (PWUT), Iran

E-mail: MA_ZA722@stu-mail.um.ac.ir; Zarif.mahdi@ieee.org

Abstract: In competitive energy markets, large energy consumers are confronted with risks associated with energy prices. As the volatility in price of electricity is higher than that of other energy sources, this problem may become more pronounced for large electricity consumers. Hence, large industries should always seek sound paradigms for maximisation of their profit considering the risk caused by uncertainty within the planning horizon. In this study, a new fuzzy-based decision-making system for procurement of electricity from different sources is proposed which helps large industries to reach a compromise between the profit and risk. The simple but efficient technique of fuzzy $\alpha$-cuts is applied for modelling the mentioned electricity pool price uncertainty in our problem. By using this modelling, a decision is made to maximise the profit of a consumer for different $\alpha$-cuts corresponding to different levels of uncertainty. Therefore a range of decision-making with different strategies of profit-uncertainty is established and the consumer, depending on being a risk-taker or being risk-averse, can choose the appropriate strategy. To evaluate the proposed approach, a cement factory in north-eastern part of Iran (Khorasan Province) has been selected as the industrial consumer through a 12-week scheduling plan.

Notation

Abbreviations used throughout the paper are stated below:

### Real variable

- $D_i^m$ electricity purchased from market at time $i$ (MWh)
- $D_i^{SG,d}$ part of the load of the industrial consumer supplied by energy of block $h$ of consumer owned generators at time $i$ (MW)
- $D_i^{SG,s}$ self-generated electricity sold at time $i$ (MW)
- $D_i^SG$ self-generated power at time $i$
- $E_i$ electricity purchased from contract $l$ in week $t$ (MWh)
- $A_L^{(a)}$ left-end point of interval
- $A_R^{(a)}$ right-end point of interval
- $E_i^{IP}$ percentage of demand supplied through bilateral contracts
- $D_i^{SG,d}$ percentage of demand supplied through self-generation
- $a_1, \ldots, a_{10}$ membership function parameters

### Stochastic variable

- $\lambda_i$ day-ahead pool price prediction at time $i$ ($$/MWh)
- $R_i$ real-time pool price prediction at time $i$ ($$/MWh)

### Set

- $L$ set of bilateral contracts

### Binary variable

- $r_1(t), r_2(t)$ random variables with normal distribution in $[0, 1]$

### Index

- $t$ week index
- $l$ bilateral contract index
- $i$ period index
- $B$ index of type of bilateral contracts
- $u$ index of self-generation blocks
1 Introduction

Owing to competition in markets, large-scale industrial units always try to reduce their production costs to take advantage of the economic opportunities and to increase their profit. The cost of electricity, especially in cases where it serves as the main input source of energy for industries, dominantly influences the production cost of such industries. The uncertainty and the volatility associated with electricity prices make the process of optimising electricity consumption from different energy sources difficult. Therefore accurate modelling of electricity pool price uncertainty becomes very important for large industries.

Several researches have been focused on the problem of electricity procurement faced by a large consumer. Daryanian proposed an approach to derive the consumer response to spot prices in electricity market [1]. Kirschen has investigated the problem of medium-term profit maximisation for retailers [2]. The optimal demand for a consumer in a deregulated power market has been presented in a work by Yan and Yan [3]. Conejo and Carrion [4] proposed a framework for electricity procurement by large consumers taking into account the risk associated with cost volatility. Additionally, the optimal amount of self-produced energy to sell to the pool by the large consumer is determined. Carrion et al. [5] have developed a decision-making technique based on stochastic programming for electricity procurement by a large consumer who owned a self-generating facility. While stochastic programming approaches have been proposed in several studies for procurement allocation of electricity consumers, but scenario-based methods require some assumptions on uncertainties (e.g. means and variances) to generate scenarios. Electricity procurement for large consumers based on information gap decision theory has been presented in a study by Zare et al. [6]. They developed a decision-aid tool for large consumers, considering three different supply sources including pool market, bilateral contract and self-generation. However, in their simulations, they did not consider the participation of consumers in electricity market. Zare et al. [7] also provide a technique to derive the bidding strategy in the day-ahead market of a large consumer that procures its electricity demand in multi-market energy. In this paper an efficient fuzzy $\alpha$-cut decision-making methodology is proposed for large consumers for electricity procurement from three available sources, that is, day-ahead market, bilateral contract and self-generation. However, in their simulations, they did not consider the participation of consumers in electricity market. Zare et al. [7] also provide a technique to derive the bidding strategy in the day-ahead market of a large consumer that procures its electricity demand in multi-market energy. In this paper an efficient fuzzy $\alpha$-cut decision-making methodology is proposed for large consumers for electricity procurement from three available sources, that is, day-ahead market, bilateral contract and self-generation. Considering the uncertainty associated with the market prices. This approach requires no assumption about the probabilistic characteristics of the uncertain parameters and no scenario is needed.

The fuzzy $\alpha$-cut technique is based on the fuzzy sets theory to model the uncertainty and imprecision in the parameters. In this paper, the uncertainty associated with electricity pool price is modelled through a fuzzy $\alpha$-cut approach. By using this modelling, the decisions for electricity procurement are made for different $\alpha$-cuts corresponding to different levels of uncertainty. Therefore the plan ends up in solutions with different strategies of various ranges in profits and different planning uncertainties. In our investigation, it is assumed that the consumer is able to sell the electricity produced by its owned generating units in real-time market.

It is also assumed that sufficient data about electricity demand, bilateral contracts and generation units are available. However, the electricity price is considered to be unknown and should be predicted by the decision maker.

The remainder of this paper is organised as follows: The procedure of uncertainty modelling is described in Section 2. Section 3 is devoted to introduce the objective function of the procurement problem. The fuzzy decision-making (FDM) system is elaborated in Section 4 and simulation results are presented in Section 5. Finally, Section 6 draws some conclusions from the presented results.
2 Uncertainty modelling through Fuzzy $\alpha$-cuts

The simple but efficient technique of fuzzy $\alpha$-cuts is applied here for modelling the mentioned uncertainties in our problem. In this approach, the uncertain parameter is considered a fuzzy number with its associated membership function and the uncertainty level is modelled by parameter $\alpha$. The $\alpha$-cut of the membership function is defined as a fuzzy set that involves all elements with membership degrees equal to or greater than $\alpha$. The wider support of the membership function (the lower value of $\alpha$) means the higher uncertainty. The higher values of $\alpha$ correspond to the higher confidence in the parameter represented by the fuzzy number [8].

Hence, values of $\alpha$ between zero and one correspond to different levels of uncertainty or risk. Therefore a range of possible decisions with different values of objective function (here profit) and different risk levels is generated for the decision-maker. As the value of uncertainty decreases, the range of profit becomes smaller. However a higher level of confidence and reliability is obtained. Based on the fuzzy sets theory, uncertainties in the parameters can be considered by fuzzy numbers

$$\tilde{A} \rightarrow [A_L^{(\alpha)}, A_R^{(\alpha)}] \quad 0 \leq \alpha \leq 1$$

In the fuzzy $\alpha$-cut approach, the membership function of the uncertain parameter is cut horizontally at different $\alpha$-levels between 0 and 1. Then to find the maximum and minimum values of the output, the model is run for different $\alpha$-levels. The obtained information is used to construct fuzziness of the output [9, 10].

The upper and lower bounds of the fuzzy interval are defined as follows

$$u_\alpha = [u_\alpha^-, u_\alpha^+] \quad (2)$$

$$u_\alpha^- = a + \alpha(b - a) \quad (3)$$

$$u_\alpha^+ = d - \alpha(d - c) \quad (4)$$

For modelling uncertain pool prices for the day-ahead market the parameters $a$, $b$, $c$ and $d$ in (3) and (4) are selected as follows

$$a_i = \lambda_i \times (1 - \text{err}) \quad (5)$$

$$d_i = \lambda_i \times (1 + \text{err}) \quad (6)$$

$$b_i = c_i = \lambda_i \quad (7)$$

where, $\lambda_i$ stands for market clearing price at time $i$ and err is the corresponding relative error in forecasting electricity price. Similarly, the real-time market price can be modelled using (3) and (4).

3 Problem formulation

The problem is to propose a decision-making system for energy consumption of a typical industry to help large industries to reach a plan for energy procurement from available sources, considering the present uncertainties. The details of the interchanged energy with outside environment for a typical industrial consumer are shown in Fig. 1.

The consumer can purchase electricity from electricity markets or through bilateral contracts. To elaborate the proposed energy diagram, the consumer is assumed to be also capable of purchasing fuel from bilateral contracts to meet its demand and to produce electricity by its owned electricity generating facilities. Furthermore it is assumed that the industry may trade energy, including electricity and fuel with outside markets. The presented model in Fig. 1 covers all the options of the industrial consumer, but we mainly focus on the uncertainty of electricity pool price and for the sake of simplicity we exclude selling bilateral contracts and gas trading in our mathematical formulation. Based on the proposed structure, the profit function for the industrial consumer can be derived as follows

$$\text{Prof} = \sum_{i=1}^{I} P_i \times V_i + \sum_{i=1}^{I} R_i \times D_i^{SG} - \sum_{i=1}^{I} \lambda_i \times D_i^{SG} - \sum_{i=1}^{I} \sum_{j=1}^{T} \lambda_{ij} \times E_{ij} - \sum_{i=1}^{I} \text{cost}(D_i^{SG}) \quad (8)$$

This function is composed of five components. The first component shows the gross revenue obtained through selling the production of the industry. The second term expresses the revenue obtained through selling the generated electricity to the real-time market. The cost of electricity purchased from the day-ahead market, the cost of buying from bilateral contracts and the cost of generated electricity are expressed by third, fourth and fifth components, respectively.

As mentioned before, the problem is to determine the amount of energy procurement from day-ahead market, bilateral contracts, through its owned generating facilities and the amount of electricity to be sold in real-time market over the planning horizon by developing a decision-making system such that the consumer’s profit is maximised.

![Energy diagram for a typical industrial consumer](image-url)
As described above, the function of the decision-making system may be formulated as an optimisation problem as follows (see (9) and (10)).

The production cost of the self-generation facility, modelled by (10), is represented by a three-block piecewise linear function [9].

Contract constraints

\[
E_i^{\text{min}} \leq \sum_{j \in T_i} E_{ij} \leq E_i^{\text{max}} \quad (11)
\]

\[
E_{ij} = 0, \quad \text{if } i \notin T_i \quad (12)
\]

\[
0 \leq E_{ij} \leq E_i^{\text{max}} \quad (13)
\]

The maximum and minimum levels of purchased energy from bilateral contracts are stated by constraint (11). Constrained (12) requires that the purchase of energy from contract \( l \) can only be made during its validity period \( T_i \). Constraint (13) imposes the maximum power that can be supplied by contract \( l \) in every period.

Self-generation constraints

\[
E_0 \leq D_i^{\text{SG,d}} + D_i^{\text{SG,s}} \leq E_{\text{cap}}, \quad i = 1, \ldots, I \quad (14)
\]

Constraint (14) shows the generating limits of the self-generation unit.

Consumer’s constraints

\[
D_i^{\text{SG,d}} + D_i^{\text{m}} + \sum_{j \in L} E_{ij} = P_i^D \quad (15)
\]

Constraint (15) states that in every period, the sum of supplied demands by day-ahead market, bilateral contracts and self-generation over the planning horizon must be equal to the demand of the consumer. The structure of the proposed FDM system for solving the problem is explained in the next section.

\section{Fuzzy decision-making system}

Fuzzy systems, because of their capability in addressing uncertainties and ambiguities, have been used in a wide range of decision-making applications [11–15]. Herein, we have proposed a FDM system for the procurement problem. The proposed system is composed of two Mamdani fuzzy sub-systems referred to as FDM\(_1\) and FDM\(_2\). The fuzzy membership functions have been selected to be in triangular form with a variable base. Triangular membership functions have been used in many FDM systems [16] and can be easily employed in fuzzy \( \alpha \)-cuts uncertainty modelling. The variable base is determined such that the profit is maximised.

\begin{align*}
\text{Max Profit} \quad (9) \\
\text{Subject to: } \text{cost}(D_i^{\text{SG}}) = \\
\begin{cases}
\text{MC}_1 \times D_i^{\text{SG}} & \text{if } D_i^{\text{SG}} \leq E_i \\
\text{MC}_1 \times E_i + \text{MC}_2 \times (D_i^{\text{SG}} - E_i) & \text{if } D_i^{\text{SG}} > E_i \\
\sum_{u=1}^{n_u} S_u \times E_u + \text{MC}^{n_u-1} \times (D_i^{\text{SG}} - E_{u-1}) & \text{if } D_i^{\text{SG}} \leq E_{u-1} \\
\vdots \\
\sum_{u=1}^{n_u} S_u \times E_u + \text{MC}^{n_u} \times (D_i^{\text{SG}} - E_{u-1}) & \text{if } D_i^{\text{SG}} \leq E_{n_u-1} \\
\end{cases}
\end{align*}

Fig. 2 shows the structure of the proposed FDM system. The input and output signals of the fuzzy systems are presented in Section 4.1. According to Fig. 2, the FDM\(_1\) determines the demand allocation to bilateral contracts and FDM\(_2\) chooses the amount of procurement from self-generation facility and day-ahead market.

\subsection{Design of fuzzy rules}

The typical fuzzy rule for FDM\(_1\) and FDM\(_2\), are determined from human knowledge as follows: If \( x_1 = A_1 \) and \( x_2 = A_2 \), then \( y = B \), where, \( A_1 \) and \( A_2 \) in the above equation are membership functions of the inputs and \( B \) is membership function of the output. FDM\(_1\), which determines the amount of procurement from bilateral contracts, has two fuzzy input variables, including \( (\lambda_{ij} - \lambda_i) \) and \( (\lambda_{ij} - MC_u) \) and one output, that is, contribution of contracts in demand \((E_i^D)\). To have a reasonable input range, the mentioned variables are normalised as follows

\begin{align*}
\text{input variable 1: } & \frac{\lambda_{ij} - \lambda_i}{\lambda_i} \times 100 \\
\text{input variable 2: } & \frac{\lambda_{ij} - MC_u}{MC_u} \times 100 \\
\end{align*}

The rule table for FDM\(_1\) is presented in Table 1. The second fuzzy system, that is FDM\(_2\), is designed to determine the market and self-generation shares. Table 2 summarises the rules for FDM\(_2\). This system includes two inputs, as follows

\begin{align*}
\text{input variable 1: } & \frac{\lambda_i - m_i}{m_i} \times 100 \\
\text{input variable 2: } & \frac{MC_u - m_i}{m_i} \times 100 \\
\end{align*}

\begin{table}[ht]
\centering
\begin{tabular}{|c|c|}
\hline
\text{Max Profit} & (9) \\
\hline
\text{Subject to: } & \text{cost}(D_i^{\text{SG}}) = \\
\hline
\end{tabular}
\end{table}
Furthermore each output variable of FDM has five membership functions, namely, negative large (NL), negative small (NS), zero (Z), positive small (PS) and positive large (PL).

The output of FDM is the percentage of demand supplied by self-generation, that is, $D_{SG}^{\%}$. The contribution of day-ahead market in demand can be found by

$$D_{DA}^{\%} = 100 - (D_{SG}^{\%} + E_{PS}^{\%})$$

Each input variable of FDM$_1$ and FDM$_2$ has five membership functions, namely, negative large (NL), negative small (NS), zero (Z), positive small (PS) and positive large (PL). Furthermore each output variable of FDM$_1$ and FDM$_2$ has four membership functions, including Z (zero), S (small), M (medium) and L (large).

### 4.2 Adjusting fuzzy membership functions

The proper functioning of the proposed FDM system depends on appropriate tuning of membership functions. In this paper, optimal values for the base of triangular membership functions should be determined. Particle swarm optimisation algorithms, which are population-based evolutionary search techniques, have been widely used for fuzzy membership function tuning [17–19]. Furthermore, PSO algorithms are fast and easy to code and are suitable for solving high-dimensional problems [17].

In PSO, particles flow in a multi-dimensional search space and the position of each particle is tuned based on the experiences gained by him and his neighbours. In this paper, we adopt a gbest PSO algorithm. In gbest algorithm the new position of the particle ($x_i(t)$) is found by adding the velocity component, as follows

$$x_i(t + 1) = x_i(t) + v_i(t + 1)$$

$$v_i(t + 1) = v_i(t) + c_1 r_1(t) [y_i(t) - x_i(t)]$$

$$+ c_2 r_2(t) [y(t) - x_i(t)]$$

As mentioned earlier, the membership functions are in triangular form (isosceles triangle) with variable base. Therefore each membership function has two parameters to be tuned. Table 3 shows the computational complexity of the problem. For each input/output membership function, the following constraints must be added to the main optimisation problem in (8).

For input membership functions:

$$a_i < a_{i+1}, \quad i = 1, 2, \ldots, 9$$

For output membership functions:

$$a_i < a_{i+1}, \quad i = 1, 2, \ldots, 7$$

### 5 Simulation results and discussion

To evaluate the proposed approach, we have applied it to determine the procurement plan of a large industrial consumer. A cement factory in north-eastern part of Iran (Khorasan Province) has been adopted as the industrial consumer. Because of large electricity consumption, cement factories are considered as important consumers for electric utilities in Khorasan-Iran. Therefore proper plan for electricity procurement from different sources, in a cement factory, can lead to valuable savings.

In the process of a typical cement plant, there are four main energy consuming departments, including: crusher, raw mill, cement kiln and cement mill departments.

The goal is to provide a 12-week scheduling plan for procurement of electricity consumption in the factory supplied through day-ahead market, self-generation and bilateral contracts. The objective function, as shown in (8), is to maximise the profit subject to constraints in (10)–(15).

#### 5.1 Input data

The input data for our simulations, including: electricity market prices and electricity consumption of the industry, are obtained from a cement plant located in Khorasan-Iran. In our simulations, each day is divided into three different time sections namely; peak, shoulder and valley hours. The

### Table 1: Rule table for FDM$_1$

<table>
<thead>
<tr>
<th>CG</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>NS</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>Z</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>S</td>
</tr>
<tr>
<td>PS</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>PL</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
</tr>
</tbody>
</table>

### Table 2: Rule table for FDM$_2$

<table>
<thead>
<tr>
<th>PM</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL</td>
<td>M</td>
<td>M</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>NS</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>S</td>
</tr>
<tr>
<td>Z</td>
<td>L</td>
<td>M</td>
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<td>S</td>
</tr>
<tr>
<td>PS</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>PL</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>M</td>
</tr>
</tbody>
</table>

$$m_i = \frac{\left(\sum_{j=0}^{T}(x_j(t)) \cdot \alpha_j \right) / 21 + MC_u}{2}$$

The output of FDM$_2$ is the percentage of demand supplied by self-generation, that is, $D_{SG}^{\%}$. The contribution of day-ahead market in demand can be found by

$$D_{DA}^{\%} = 100 - (D_{SG}^{\%} + E_{PS}^{\%})$$

### Table 3: Complexity of the optimisation problem

<table>
<thead>
<tr>
<th>Decision-making system</th>
<th>Number of inputs</th>
<th>Number of outputs</th>
<th>Number of input and output membership functions</th>
<th>Number of unknown parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDM$_1$</td>
<td>2</td>
<td>1</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>FDM$_2$</td>
<td>2</td>
<td>1</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>total</td>
<td>4</td>
<td>2</td>
<td>28</td>
<td>56</td>
</tr>
</tbody>
</table>
hours assigned to each of these three time sections are as

\[
\text{Valley} = \{1, 2, 3, 4, 5, 6, 7, 8\}
\]
\[
\text{Shoulder} = \{9, 10, 15, 16, 17, 18, 23, 24\}
\]
\[
\text{Peak} = \{11, 12, 13, 14, 19, 20, 21, 22\}
\]

Hence, for a 12-week planning horizon, there will be \(3 \times 7 \times 12 = 252\) periods. Also, in each week three bilateral contracts for the three time sections are assumed. These contracts are referred as; peak (P), shoulder (S) and valley (V) contracts, respectively. Each contract is specified by its period, minimum and maximum levels of energy consumption and the contracted price for electricity during the period. In our simulations, the contracted period is assumed to be one week.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of particles</th>
<th>Size of particles</th>
<th>Number of iterations</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>100</td>
<td>56</td>
<td>150</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4 shows energy consumption limits of each bilateral contract for electricity consumption during the planning period. Fig. 3 shows the contracted price of electricity for peak, shoulder and valley sections during planning schedule. Table 5 presents technical data for a consumer’s self-generating facility.

Based on Table 6, the number of unknown parameters that should be determined by optimisation problem equals 56; hence the dimension of particles is also set to be equal to 56. Furthermore, constants \(c_1\) and \(c_2\) are set equal to 2 according to [18].

The simulations are carried out using fuzzy \(\alpha\)-cuts concept. For this purpose, the day-ahead pool prices are replaced by upper and lower bounds prices found from (9)–(14). Hence, there are 22 price series. Fig. 4 shows forecasted prices of day-ahead and real-time markets. Consumer’s electricity demand is shown in Fig. 5.

### 5.2 Simulation results

Simulations are carried out to find the procurement plan of energy over the planning horizon (12 weeks). Before executing simulations, we need to specify a reasonable level of forecasting error. To do this, we first performed a series of simulations using upper level of day-ahead and real-time prices considering different levels of forecasting error.
Analysing the obtained results, it became clear that the profit did not significantly change for forecasting errors higher than 40%. This is due to the fact that for forecasting error values higher than 40%, the upper level of day-ahead pool price is much higher than contract prices and self-generation marginal cost. So the consumer prefers to completely provide its demand from contracts and self-generation rather than market. Hence, in this paper, the relative error is assumed to have a constant value of 40%. Note that the lower level of price cannot be used for this purpose, because as the day-ahead market price decreases, the profit monotonically increases until the day-ahead market price reaches zero.

As stated earlier, the \( \alpha \)-cut approach is integrated into our proposed decision-making system in order to model the uncertainties involved in our decision-making problem. This is done by replacing the term \( \lambda_i \) in (8) by upper and lower bounds of the price in (5)–(7). Then, the model is run for eleven levels of \( \alpha \), starting from 0.05 with a step value of 0.05 (ending to 0.95) and the maximum and minimum values of the profit are obtained. The maximum and minimum values of profit for different \( \alpha \)-levels are tabularised in Table 7. It is concluded from this table that considering higher levels of uncertainty (lower values of \( \alpha \)), the profit associated with the lower bound of prices increases while the profit related to the upper bound of prices is decreased.

Based on Table 7, while \( \alpha \) increases (the uncertainty decreases) the maximum and minimum profits approach to a common value which corresponds to the most certain circumstance (\( \alpha = 1 \)).

The probability density functions (PDF) of the profit, corresponding to different levels of \( \alpha \) are shown in Fig. 6. When \( \alpha \)-level decreases, the system uncertainties rise. Based on this analysis, smaller values of \( \alpha \) correspond to more uncertainties and consequently higher profits (and of course larger variance of profit). The most reliable case pertains to \( \alpha = 0.95 \) (with the smallest variance of profit). While the profit for this case (\( \alpha = 0.95 \)), as compared with all other cases, is smaller, however, in this case the probability of profit to become equal to its average value is maximum.

Investigating the effect of \( \alpha \)-cuts on procurement from pool market, through bilateral contracts and self-generation reveals interesting results. The percentage of energy supplied through each source of energy for different \( \alpha \)-levels and upper bound of pool prices are plotted in Fig. 7. If a consumer makes its decision based on the upper bound of price, it means that the consumer is risk-averse, because the upper bound of price is the worst case (the higher price) that can happen. In this case, for lower values of \( \alpha \) (more uncertainty) the consumer wishes to procure a

\[ \begin{array}{lcccccccccccc}
\text{\( \alpha \)-level} & 0.05 & 0.14 & 0.23 & 0.32 & 0.41 & 0.50 & 0.59 & 0.68 & 0.77 & 0.86 & 0.95 \\
\text{max. profit, m$ (lower bound of price)} & 0.352 & 0.338 & 0.329 & 0.328 & 0.321 & 0.312 & 0.305 & 0.296 & 0.288 & 0.277 & 0.268 \\
\text{min. Profit, m$ (upper bound of price)} & 0.168 & 0.178 & 0.187 & 0.193 & 0.205 & 0.215 & 0.223 & 0.231 & 0.239 & 0.247 & 0.257 \\
\end{array} \]

![Fig. 5](consumer-electricity-demand.png)  
**Fig. 5** Consumer’s electricity demand

![Fig. 6](pdf-profit.png)  
**Fig. 6** PDF of the profit, corresponding to different levels of \( \alpha \)
larger part of its demand from bilateral contracts and self-generation rather than market.

On the other hand when uncertainty of market prices decrease (higher values of $\alpha$), the proportion of contracts and self-generation decreases while the proportion of market is increased owing to the fact that the situation becomes more deterministic.

Similarly, the percentage of demand supplied by available sources for the lower bound of market prices and different levels of $\alpha$ is illustrated in Fig. 8. In this case in contrast to the former one, the risk-taking consumer tries to procure a larger part of its demand from the market for higher levels of uncertainty (lower $\alpha$-levels). But when $\alpha$ increases the proportion of contracts and self-generation are increased while that of the market is decreased. Figs. 7 and 8 contain another remarkable result. For a consumer who is risk-averse (Fig. 7), the reliability is of more importance. Therefore such a consumer is willing to hedge against variations of market prices. In the case of a risk-taking consumer (Fig. 8), the consumer wishes to fully take advantage of the uncertainties in price to reach a higher value of profit. This is also confirmed by the results presented in Table 7.

In the introduction section we mentioned shortcomings of other methods compared with the proposed method. For instance, we identified the need to a procedure to generate scenarios based on some uncertainty assumptions in scenario-based techniques such as stochastic programming.

However, to better show the advantage of the proposed method, we also solved the problem of energy procurement by the technique stated in [5]. The technique in [5] is based on the mean-risk approaches and uses conditional value at risk as the risk (CVaR) measure. The CVaR is incorporated into the objective function using weighting factor $\beta$. The larger the value of $\beta$, the more risk-averse is the decision maker. The results of simulations for different values of $\beta$ are presented in Table 8. Based on the results in table, for each value of $\beta$ a single value for the expected profit is obtained. However, in our proposed method for each $\alpha$-level, a range of profit is resulted. Hence, the decision maker can have a better analysis of the uncertain situations and the possible values for its profit.

Also in these simulations, the participation of consumers in the relevant electricity markets was not considered; however, in this study such participation has been considered. The amount of self-generated electricity sold to spot market for the lower bound of prices is shown in Fig. 9. According to this figure which corresponds to a risk-taking consumer, the electricity sold to the real-time market increases with uncertainty. This is owing to the fact that the consumer tends to benefit from lower prices that might happen as a result of price uncertainty.

6 Conclusion

In this paper, a decision-making methodology for large industrial consumers is proposed. The method considers uncertainties associated with the electricity prices of day-ahead and real-time markets. The uncertainty modelling is carried out using fuzzy $\alpha$-cuts concept. In contrast to the scenario-based techniques, for example, stochastic programming, no assumptions on the nature of uncertainties is required by the proposed approach. The fuzzy $\alpha$-cuts produce upper and lower bounds for the uncertain parameter. A risk-averse consumer can use the upper bound to hedge against possible variations of the market prices. On the other hand, a risk-taker consumer may wish to benefit from the higher values of profit that may result from the lower bound of price. Hence, rather than a single expected value (as produced by stochastic programming), a range of possible profits, associated with the $\alpha$-cut level, is resulted. According to simulation results, for higher levels of uncertainty a risk-averse consumer turns to bilateral contracts and self-generation while a risk-taking consumer enjoys from lower market prices and the resulted windfall benefits.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected profit (m$)</td>
<td>0.262</td>
<td>0.258</td>
<td>0.256</td>
<td>0.255</td>
<td>0.255</td>
</tr>
</tbody>
</table>

Table 8 Expected profit for different $\beta$

![Fig. 7 Contracts, self-generation and pool allocations against $\alpha$-levels for upper bound of market prices](image)

![Fig. 8 Contracts, self-generation and pool allocations against $\alpha$-levels for lower bound of market prices](image)

![Fig. 9 Self-generated electricity sold to real-time market for different risk aversions](image)
7 References


IET Gener. Transm. Distrib., 2012, Vol. 6, Iss. 1, pp. 50–58

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