Astrophys Space Sci (2012) 337:193–199
DOI 10.1007/s10509-011-0800-0

ORIGINAL ARTICLE

Propagation of solitary waves in non uniform dusty plasmas

Kurosh Javidan · Parvaneh Mohammad Zadeh

Received: 14 April 2011 / Accepted: 13 July 2011 / Published online: 8 September 2011

Abstract Interaction of dust acoustic solitary waves in plasmas consisting of medium disorders is investigated. Disorders and inhomogeneities of the medium are added to the equation of motion as perturbative terms through the medium parameters. The effects of these perturbations on the behaviour of solitary waves are studied with numerical simulations and the results are compared with theoretical predictions in a uniform media.

Keywords Dust acoustic solitary waves · Plasmas · Disorders · Numerical simulation · Non uniform media

1 Introduction

Wave propagation in nonlinear dusty plasmas has received a great deal attention because of its vital role in understanding different types of collective processes in space environments such as the early universe which assumes to be a kind of plasma, describing the active galactic nuclei, pulsar magnetosphere, the solar atmosphere as well as plasmas in laboratory devices (Tsytovich et al. 2002; Barkan et al. 1996; Homann et al. 1997; Fortov et al. 2005; Rao et al. 1990). The physical phenomena met in the dusty plasmas have been reviewed by several authors. Dusty plasmas are nonlinear media. In a nonlinear medium, a stable wave pattern can be propagated if the effects of nonlinearity and dissipation cancel out each other. This means that a very fine tuning between the medium parameters is needed in order to have a stable wave pattern. These stable patterns call solitary waves or solitons.

Rao, Shukla and Yu predicted the existence of dust acoustic solitary waves (DAWs) theoretically, about twenty years ago (Shukla 2001). In such these waves, dust particle mass creates the inertia and the restoring force is provided by the pressures of the inertia less electrons and ions. Following their pioneering work a number of laboratory experiments have been initiated, where DAWs are observed. Also a large number of theoretical investigations have been done, which provided different linear and nonlinear features of DAWs in an unmagnetized weakly coupled dusty plasmas.

In recent years several solitonic solutions have been proposed in various types of dusty plasmas using different methods. These solutions have been appeared in homogeneous and well behaved medium, while in real world the medium of propagation contains disorders and impurities which add local space-dependent potentials to the problem. Behaviour of solitary waves during the interaction with local disorders and their stability after the interaction are important subjects due to its application in real situations and also because of theoretical point of view. Motivated by this situation we have studied the interaction of solitary solutions in dusty plasmas with defects and the results are presented in this paper. Therefore a brief description of the targeted medium and its solitonic solution has been introduced in Sect. 2. Method of adding the potential to the soliton equation of motion will be described in Sect. 3. Interaction of solitons with disorders and a discussion will be presented in Sects. 4 and 5. Some conclusion and remarks will be presented in the final section.

K. Javidan (✉)
Department of Physics, Ferdowsi University of Mashhad, 91775-1436 Mashhad, Iran
e-mail: javidan@um.ac.ir

P.M. Zadeh
Islamic Azad University of Mashhad, Mashhad, Iran
e-mail: parvane.mohammadzade@yahoo.com

2 Hot adiabatic magnetized dusty plasma and its dust-ion-acoustic solitary solution

We Consider the propagation of dust ion-acoustic waves in a fully ionized magneto dusty plasma contains hot adiabatic inertial ions with number density of N_i normalized by its equilibrium value n_{i0}, hot adiabatic inertia-less electrons with number density of N_e normalized by its equilibrium value n_{e0}, and negatively charged static dust in the presence of an external constant magnetic field $\vec{B}= B_0\hat{z}$ (where $\hat{z}$ is unit vector along the z direction). Total charge neutrality at equilibrium requires that (Pakzad and Javidan 2009)

$$n_{e0} + n_{d0}Z_{d0} = n_{i0} \tag{1}$$

Z_{d0} is the unperturbed number of charges on the dust particles. The dynamics of the dust ion-acoustic waves in one dimensional form in such a dusty plasma system is governed by Anowar and Mamun (2008)

$$\frac{\partial N_i}{\partial T} + \vec{\nabla}\cdot\left(N_i\,\vec{U_i}\right) = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \vec{U_i}\cdot\vec{\nabla}\right)\vec{U_i} = -\vec{\nabla}\,\psi + \Omega_{ci}\,\vec{U_i}\times\hat{z} - \nu\,\vec{U_i} - \frac{\alpha}{N_i}\,\vec{\nabla}\,p_i, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \vec{U_i}\cdot\vec{\nabla}\right)P_i + \gamma_i P_i\,\vec{\nabla}\cdot\vec{U_i} = 0, \tag{4}$$

$$\vec{\nabla}\,\psi - \Omega_{ci}\,\vec{U_e}\times\hat{z} - \frac{1}{N_e}\,\vec{\nabla}\,P_e = 0, \tag{5}$$

$$\frac{\partial P_e}{\partial T} + \vec{U_e}\cdot\vec{\nabla}\,P_e + \gamma_e P_e\,\vec{\nabla}\cdot\vec{U_e} = 0, \tag{6}$$

$$\nabla^2\psi = \mu N_e - N_i + \frac{Z_d n_{d0}}{n_{i0}}$$

where $\vec{U_i}$ ($\vec{U_e}$) is the ion (electron) fluid velocity normalized by the ion-acoustic velocity C_i, Ω_{ci} is the ion cyclotron frequency normalized by the ion plasma frequency $\omega_{pi} = \sqrt{\frac{4\pi e^2 n_{i0}}{m_i}}$. ν is the ion-dust collision frequency normalized by the ion plasma frequency ω_{pi}. ψ is the electrostatic wave potential normalized by $\frac{k_B T_{e0}}{e}$, $\alpha = \frac{T_{i0}}{T_{e0}}$, $\mu = \frac{n_{e0}}{n_{i0}}$, $\gamma_i = \frac{C_p^i}{C_v^i}$, $\gamma_e = \frac{C_p^e}{C_v^e}$, $C_p^i(C_p^e)$ is the specific heat of the ion (electron) at constant pressure, and $C_v^i(C_v^e)$ is the specific heat of the ion (electron) at constant volume. $P_i(P_e)$ is the ion (electron) pressure normalized by $n_{i0}k_B T_{i0}(n_{e0}k_B T_{e0})$. The time variable T is normalized by the ion plasma period ω_{pi}^{-1}, and the space variable is normalized by the Debye radius $\lambda_{pi} = \sqrt{\frac{k_B T_{e0}}{4\pi e^2 n_{i0}}}$.

Anowar and Mamun have derived the following equation of motion for small values of electro static wave potential ψ using the reductive perturbation method Anowar and Mamun (2008)

$$\frac{\partial\psi}{\partial\tau} + A\psi\frac{\partial\psi}{\partial\xi} + B\frac{\partial^3\psi}{\partial\xi^3} = 0 \tag{7}$$

where

$$A = \frac{3l_z^2 V_P^2 - 2l_z^4\alpha\gamma_i + l_z^4\alpha\gamma_i^2}{2V_P(V_P^2 - l_z^2\alpha\gamma_i)} - \frac{\mu(2-\gamma_e)(V_P^2 - l_z^2\alpha\gamma_i)^2}{2l_z^2 V_P\gamma_e^2}, \tag{8}$$

$$B = \frac{(V_P^2 - l_z^2\alpha\gamma_i)^2}{2l_z^2 V_P} + \frac{(1-l_z^2)V_P^3(\Omega_{ci}^2 - \nu^2)}{2l_z^2(\Omega_{ci}^2 + \nu^2)} \tag{9}$$

where l_z is directional cosine of the wave vector in the direction of the external magnetic field and $V_P = \frac{l_z}{\sqrt{\mu}}(\gamma_e + \alpha\mu\gamma_i)$ is phase speed.

On introducing the new variable $\zeta = \xi - U_0\tau$, where U_0 is a constant velocity, solitary solution of (7) in the stationary frame is given as

$$\psi = \psi_m\,\mathrm{sech}^2\,(\zeta/\Delta) \tag{10}$$

where peak amplitude ψ_m and width Δ of soliton, respectively, are given by

$$\psi_m = \frac{3U_0}{A}, \tag{11}$$

$$\Delta = \sqrt{\frac{4B}{U_0}} \tag{12}$$

3 DAWs in an inhomogeneous media

Investigation of DAWs commonly is done in a homogeneous and ordered nonlinear medium. The dust-acoustic solitary wave is a localized nonlinear wave which arises due to a delicate balance between nonlinearity and dispersion effects. This means that a very fine tuning between these effects is needed for propagating stable solitons in the media.

In real situation the medium of propagation is disordered and inhomogeneous. It is clear that such defects disturb the parameters of the medium and therefore change the strength of nonlinear and dispersive terms. It is a very critical point which should be taken into account when we make a model for soliton propagation. Moreover, stability of a moving solitary wave and its characters after the interaction with the medium disorders and impurities is another important problem. Plasmas are not homogeneous; they are full of fluctuations, which come from their stochastic and random nature and we have to consider these randomness.

The effects of medium disorders and impurities can be added to the equation of motion as a perturbative term (Kivshar et al. 1991; Javidan 2010). These effects also can be taken into account by making some parameters of the equation of motion to be function of space or time (Piette and Zakrzewski 2007; Javidan 2006). There still exists another interesting method which is mainly suitable for working with topological solitons (Hakimi and Javidan 2009; Javidan 2008). In this method, one can add such effects to the Lagrangian of the system by introducing a suitable non-trivial metric for the back ground space-time, without missing the topological boundary conditions. The third method is inapplicable for the KdV equation, because it is not a Lorentz invariant model. In this paper, the medium disorders are modeled using space dependent local functions.

Consider the plasma medium which has been explained in the previous section. Characteristics of propagated solitary waves of the KdV equation in this medium are specified by (10), (11) and (12). On the other hand characters of solitary wave are functions of medium details specified by (8) and (9). Particles in the medium are not distributed uniformly with same densities and same temperatures in all around the media. There are some fluctuations around the average values of particles identifications. Therefore in a real case, parameters α, μ, Ω_{ci}, ν or l_z are space dependent functions. Suppose that in a specific location of the medium, the ratio of $\alpha = \frac{n_{e0}}{n_{i0}}$ becomes a little different from its background value α_0. A simple model can be introduced by adding a delta function perturbation to the parameters of the equation of motion. For small disorders we can write

$$\alpha(\xi) = \alpha_0 + \epsilon\delta(\xi - \xi_0) \quad (13)$$

Disorders in other parameters can be define in the same way. In this situation the parameters "A" and "B" of (7) become functions of space (ξ). These parameters can be written as

$$\begin{aligned} A(\xi) &= A_0 + A_1\delta(\xi - \xi_0) \\ B(\xi) &= B_0 + B_1\delta(\xi - \xi_0) \end{aligned} \quad (14)$$

where A_0 and B_0 satisfy unperturbed KdV equation (7) with solution (10). However the KdV equation is nonlinear, but for small amplitude perturbations soliton amplitude reads (Kivshar et al. 1991; Fei et al. 1992)

$$\Psi(\xi) = \psi + \psi_1 \quad (15)$$

where ψ_1 is a small localized function which induced from the medium perturbation. Inserting (14) and (15) into (7) and integrating over the space (ξ) we have

$$\frac{\partial\psi_1}{\partial\tau'} + A_0\psi\psi_1 + B_0\frac{\partial^2\psi_1}{\partial\xi^2} + \left[A_1\psi\frac{\partial\psi}{\partial\xi} + B_1\frac{\partial^3\psi}{\partial\xi^3}\right]_{\xi=\xi_0} = 0 \quad (16)$$

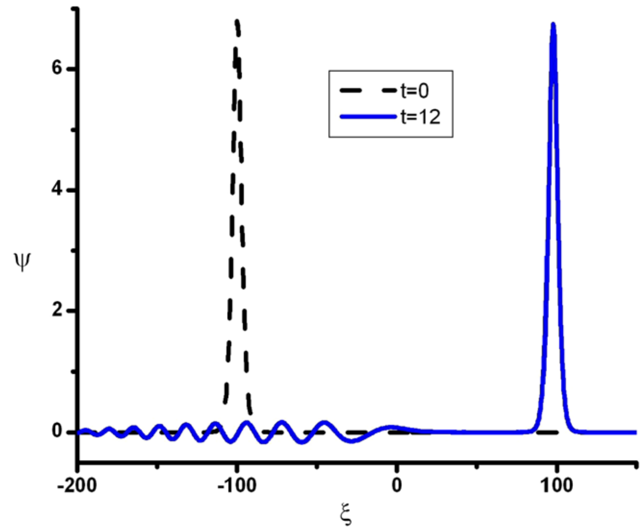

Fig. 1 Solitary wave profiles before and after the interaction with a perturbation located in $\xi = 0$. Soliton moves from initial position $\xi_0 = -100$ with initial velocity $U_0 = 0.8$. Radiated energy is propagated backward starting from the perturbation location

where ξ_0 is the location of the perturbation. In the above derivation we have used the assumption that ψ_1 is a localized delta-like function. This equation is an approximation for the evolution equation (16) describes the evolution equation of perturbation amplitude, which is a Schroedinger type equation with external nonlinear potential $-A_0\psi\psi_1 - [A_1\psi\frac{\partial\psi}{\partial\xi} + B_1\frac{\partial^3\psi}{\partial\xi^3}]_{\xi=\xi_0}$. The second term is a function of soliton characters and also location of perturbation. This term causes an energy radiation started from the perturbation location. Figure 1 presents the results of direct numerical solution of (7) for a solitary wave before and after the interaction with perturbation $A(\xi) = 0.2 + 0.05\delta(\xi)$. This figure clearly shows that some energy radiation propagates backward started from the location of perturbation. Our simulations are strongly in agreement with this prediction which will explained in the next section.

4 Numerical simulations

Several simulations have been performed using various functions as models of perturbation in different parameters of the medium. The smooth and slowly varying function $ae^{-b(\xi-c)^2}$ has been used as delta function in simulations which are reported below. Parameter "a" controls the strength of the perturbation, "b" represents its width, and "c" adjusts its position. If $a > 0$, perturbation has an additive effect respect to the background value of the parameter and for $a < 0$ we have a subtractive perturbation.

Simulations have been setup using 4th order Runge-Kutta method for time derivatives. Space derivatives were expanded using finite difference method with double precision

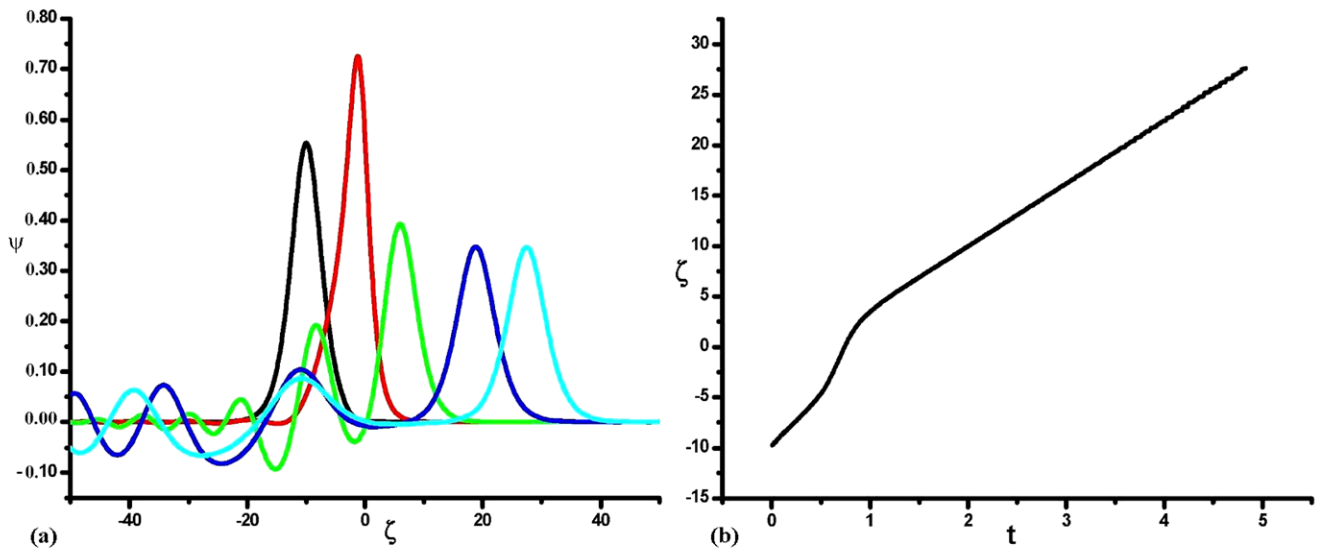

Fig. 2 Interaction of solitary wave with a perturbation on the medium parameter $\mu = \frac{n_{e0}}{n_{i0}}$ at the origin. Soliton moves from initial position $\xi_0 = -10$ with initial velocity $U_0 = 0.1$. (**a**) shows evolution of the solitary profile and (**b**) presents soliton trajectory during the interaction with the perturbation

variables. Grid spacing has been taken $h = 0.1$, 0.05 and sometimes $h = 0.01$. Time step has been chosen as $\frac{h}{\frac{A}{2}+\frac{4B}{h^2}}$ because of stability considerations. Simulations have been performed with fixed boundary conditions and solitons have been kept far from the boundaries.

Consider a plasma medium which has described in previous section, with initial parameters $\alpha = 0.2, \mu = 0.6$, $\gamma = 1, \Omega_{ci} = 0.06$ and $\nu = 0.02$. Suppose that perturbation $\mu = 0.6 + 0.2e^{-0.2\xi^2}$ exists in the origin. This perturbation can be created because of a small local doping in the density ratio $\frac{n_{e0}}{n_{i0}}$ relative to its background value. Now consider a solitary wave solution described by (10) which moves toward the origin with initial velocity $U_0 = 0.1$ from initial position $\xi_0 = -10$. The solitary wave interacts with perturbation at the origin and passes through it. Figure 2a presents the solitary wave profile during the interaction. Figure 2b demonstrates trajectory of the soliton during the interaction with the perturbation. As stated, soliton initial velocity is 0.1 before the interaction but Soliton velocity becomes about 0.06 after the interaction in somewhere far from the perturbation.

Soliton radiates an amount of energy during the interaction. one can see in the Fig. 2a that the radiated energy emerges backward relative to the direction of soliton velocity started from the initial position of the perturbation as predicted in (16). Therefore the soliton velocity after the interaction is lower than its initial velocity. Also soliton height (and thus its energy) becomes smaller than its initial value. Soliton width changes during the interaction too. Figure 3a shows the evolution of soliton height during the interaction. Peak value of the soliton is 0.56 before the interaction at its initial position far from the perturbation. Soliton finds its maximum value at the origin ($\psi_m = 0.87$) and then becomes about 0.36 after the interaction far from the origin. Figure 3b shows evolution of soliton width during the interaction. This figure demonstrates that soliton width after the interaction is greater than its value before the interaction. Numerical finding of the precise value of the soliton width is difficult because of radiation during the interaction.

All the figures strongly indicate that the soliton specifications would not get back to its initial values after the interaction with a perturbed location in the medium.

Some simulations have been performed with the same parameters but with a subtractive perturbation $\mu = 0.6 - 0.1e^{-0.2\xi^2}$. Figure 4a shows the height of the soliton as a function of time during the interaction with the perturbation. Initial value of the peak altitude is 0.55. Soliton finds its height minimum value in the center of the perturbation (0.37) and then it rises up to 0.49 after the interaction far from the perturbation. Simulations show that Soliton velocity reduces from its initial value $U_0 = 0.1$ to 0.09 after the interaction. Figure 4b presents evolution of the soliton width during the interaction. The width of the soliton changes from its initial value 0.57 to its maximum at the origin of the perturbation (0.76) and then reduces to its final value 0.60 after the interaction.

These figures also show that final values of the soliton characters is depend on its initial values and also perturbation details. Anyway final situation of the soliton is not similar to its initial state.

More interesting behaviour of a soliton during the scattering on a perturbation is seen in some very narrow win-

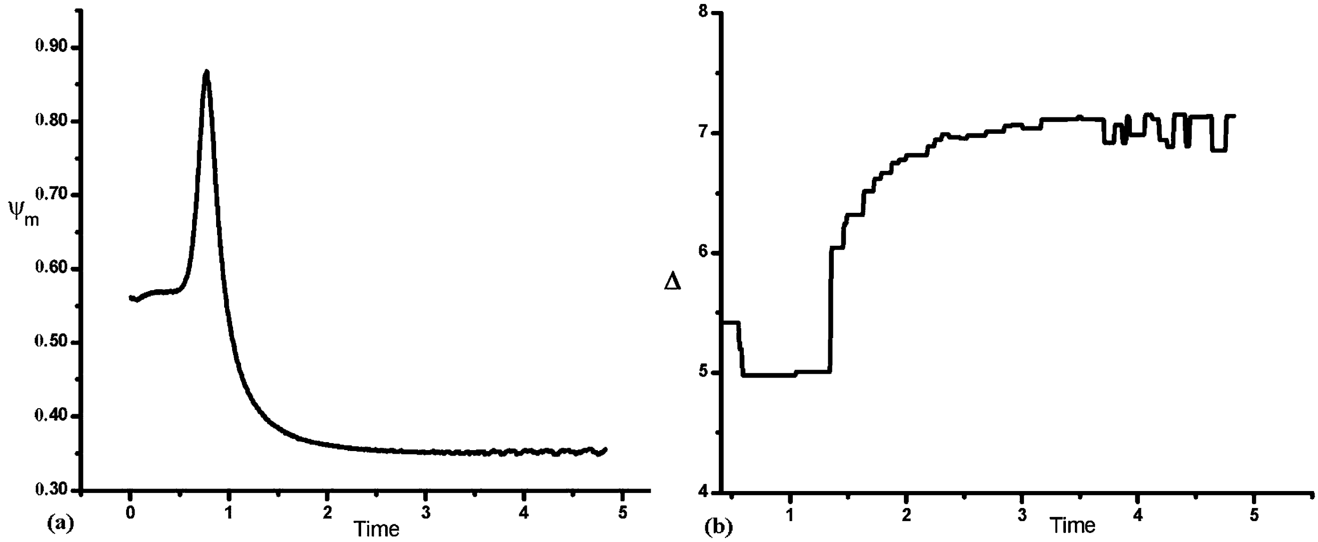

Fig. 3 (**a**) Evolution of soliton peak value during the interaction with perturbation $\mu = 0.6 + 0.2e^{-0.2\xi^2}$. (**b**) Soliton width as a function of time during the interaction

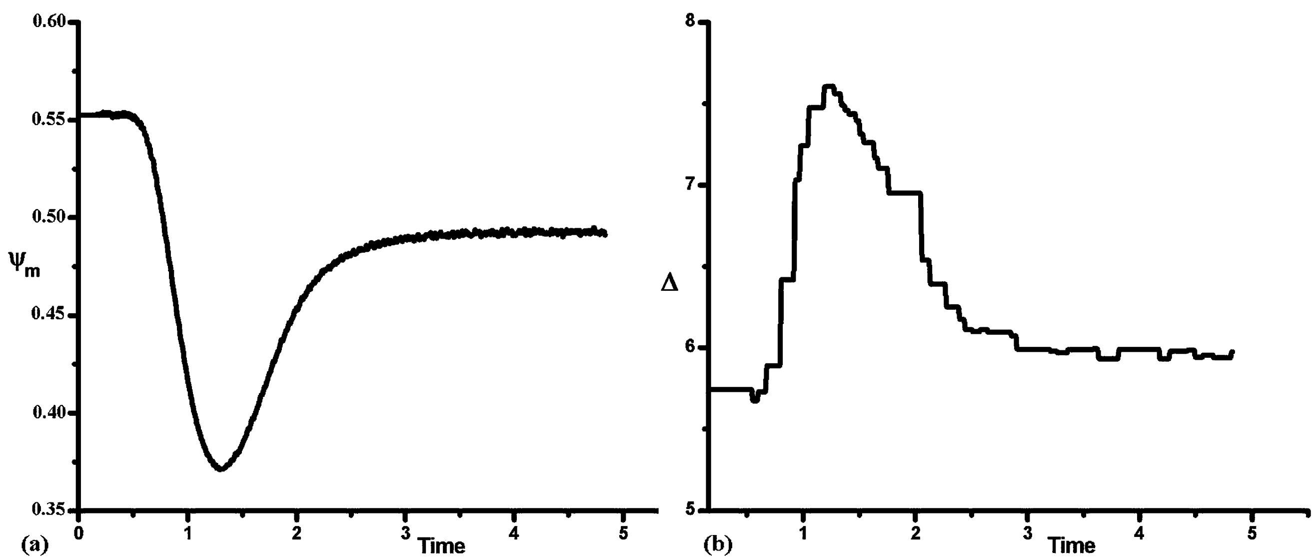

Fig. 4 (**a**) Soliton peak as a function of time during the interaction with subtractive perturbation $\mu = 0.6 - 0.1e^{-0.2\xi^2}$. (**b**) Evolution of the soliton width during the interaction with subtractive perturbation

dows of initial conditions in the border of creating a rarefactive ($a < 0$) or a compressive ($a > 0$) solitary wave. Initial values $\alpha = 0.2$, $\delta = 20°$, $\gamma = 1.0$, $\Omega_{ci} = 0.06$, $\nu = 0.04$ and $\mu = 0.31$ creates a rarefactive solitary wave, while the same values of the parameters but with $\mu = 0.32$ produce a compressive soliton. Therefore we expect that a perturbation of the form $\mu = 0.31 + 0.01e^{-.02\xi^2}$ flips the solitary wave from $\psi_m < 0$ to a $\psi_m > 0$. Surprisingly simulations indicate that such this peak reverting does not occurred. Figure 5 presents the evolution of the soliton height during the interaction with the perturbation $\mu = 0.31 + 0.01e^{-.02\xi^2}$. Absolute value of the peak height decreases but it does not reache the zero value and therefore it does not flip to a positive value. Figure 6a presents the soliton profile after the interaction. This figure shows that the radiated energy is very large. Figure 6b presents the trajectory of the soliton during the interaction. This figure demonstrate noticeable decrease in the soliton velocity after the interaction.

5 Discussion

Simulations strongly prove that created Solitary solutions of the KdV equation in dusty plasmas are very stable. A solitary wave adjusts itself with medium disorders and impurities by emerging some amounts of energy radiation. Indeed in the most of our simulations the solitary wave remains stable after the interaction with local perturbations. For stronger perturbations, our simulations fail to solve problem properly because of numerical errors due to radiations, precision and initial boundary conditions.

Small and finite perturbations in a plasma medium change the DAWs identifications. In the other words, solitary wave characters are not as the same as its initial state after the interaction with a small local perturbation, as Figs. 1–6 present. We couldn't find an exact description for solitary wave parameters as functions of medium characters after the interaction.

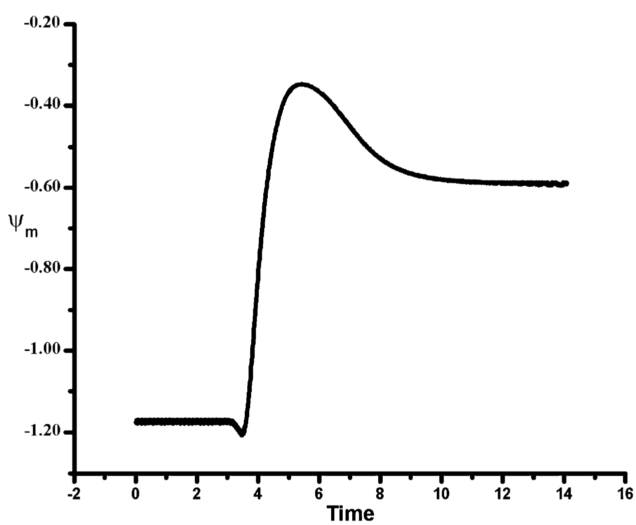

Fig. 5 Soliton peak as a function of time during the interaction with perturbation $\mu = 0.31 + 0.01e^{-0.2\xi^2}$

Perturbation can be added to the different parameters of the medium. Reported simulations in previous section have been done with adding a perturbation on density ratio $\mu = \frac{n_{e0}}{n_{i0}}$. Some simulations also have been done with a perturbation added on the temperature ratio $\alpha = \frac{T_{e0}}{T_{i0}}$, directional cosine of the wave vector in the direction of the external magnetic field (l_z), ion cyclotron frequency $\Omega_{ci} = \frac{|q|B}{m_i}$. These variables can be referred to the physical parameters of the medium. Small spatial changes in the electron density (or ion density) creates a perturbation on μ. If electron thermal energy (or ion thermal energy) in a small volume of space becomes different with its average background value, it can be modeled as a perturbation on α. Small changes in the magnitude of the external magnetic field of its direction can be affected on the parameters Ω_{ci} and l_z respectively.

Simulations show that a perturbation which is able to change a rarefactive soliton to a compressive one (or vice versa) theoretically, will not do this in practice. Note that real situation is more complicated. In fact we have to consider the effects of emerged energy radiation too. In order to have such transition the parameter 'A' must change its sign. Therefore we have a situation with $A = 0$. But for $A = 0$, soliton height ψ_m goes to infinity and therefore presented solution (10) is not valid.

We have added perturbation terms to the above parameters. The magnitude of impression on the solitary wave characters because of added perturbation in several parameters are different. Perturbation on μ has strongest effects while local changes in temperature causes smaller effects. But general behaviour of a solitary wave during the interaction with perturbations on different parameters are almost the same.

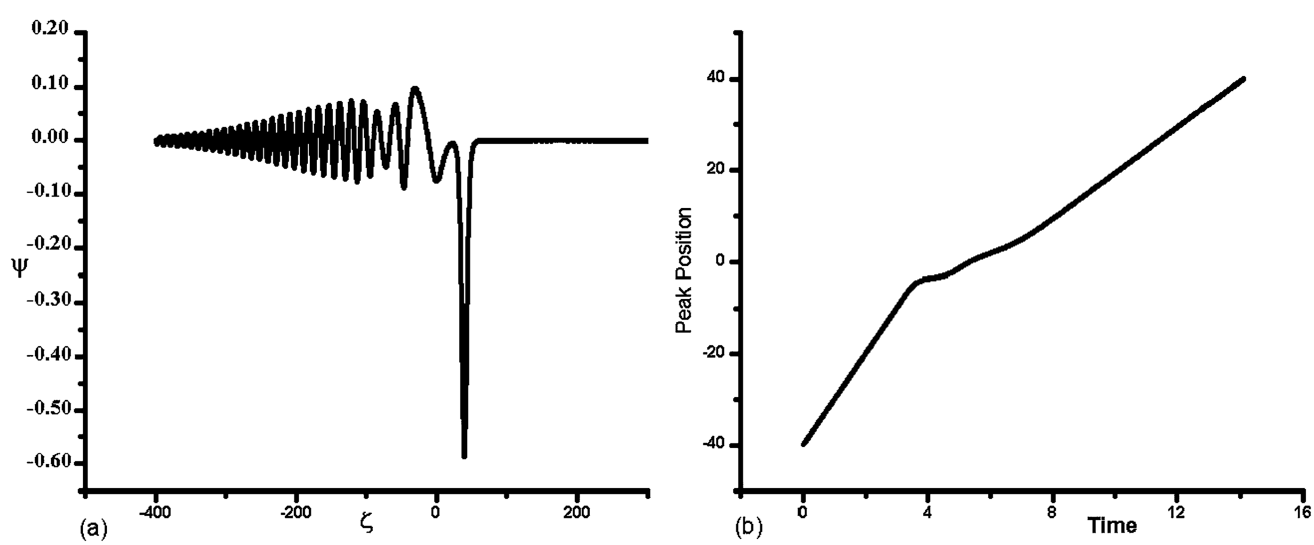

Fig. 6 (**a**) Soliton profile after the interaction with perturbation $\mu = 0.31 + 0.01e^{-0.2\xi^2}$. (**b**) Soliton trajectory during the interaction

6 Conclusion and remarks

Behaviour of a solitary wave in a specified plasma medium during the interaction with small local perturbations due to random disorders or local inhomogeneities has been investigated. It is shown that solitary waves remains stable after the interaction with small perturbations. In general case soliton emerges some amount of energy radiation in backward relative to its velocity and tune itself with new situation. However the perturbation is local but soliton identifications will not get back to its initial values before the interaction. Solitary wave characters can be calculated as functions of medium parameters with a theoretical model, when the medium is completely homogeneous. Because of non reversal effects of such interactions we can not strongly connect soliton parameters to plasma characters in a bubbly medium.

This investigation needs to repeat for other DAW solutions in different plasmas. It is great if one can present better theoretical description for solitary wave characters after the interaction as functions of initial situation before the interaction and perturbation function. Perturbations in the border of creating a compressive or a rarefactive soliton need more investigation. Such this problem may be happen for monotonic and oscillatory shock wave solutions in plasmas. These problems can be studied in further investigations. These studies help us to find better knowledge about the general behaviour of Daws in real plasmas.

References

Anowar, M.G.M., Mamun, A.A.: Phys. Lett. A **372**, 5896–5900 (2008)
Barkan, A., Angelo, N.D., Merlino, R.L.: Planet. Space Sci. **44**, 239 (1996)
Fei, Zhang, Kivshar, Y.S., Vazquez, L.: Phys. Rev. A **46**, 5214 (1992)
Fortov, V.E., Ivlev, A.V., Khrapak, S.A., Khrapak, A.G., Morfill, G.E.: Phys. Rep. **421**, 1 (2005)
Hakimi, E., Javidan, K.: Phys. Rev. E **80**, 016606 (2009)
Homann, A., Melzer, A., Peters, S., Piel, A.: Phys. Rev. E **56**, 7138 (1997)
Javidan, K.: J. Phys. A **39**, 10565 (2006)
Javidan, K.: Phys. Rev. E **78**, 046607 (2008)
Javidan, K.: J. Math. Phys. **51**, 112902 (2010)
Kivshar, Y.S., Fei, Zhang, Vazquez, L.: Phys. Rev. Lett. **67**, 1177 (1991)
Pakzad, H.R., Javidan, K.: Chaos Solitons Fractals **42**, 2904–2913 (2009)
Piette, B.M.A.G., Zakrzewski, W.J.: J. Phys. A, Math. Theor. **40**, 329 (2007)
Rao, N.N., Shukla, P.K., Yu, M.Y.: Planet. Space Sci. **38**, 546 (1990)
Shukla, P.K.: Phys. Plasmas **8**, 1791 (2001)
Tsytovich, V.N., Morfill, G.E., Thomas, H.: Plasma Phys. Rep. **28**, 6232 (2002)