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Transverse vibration of fluid conveying carbon nanotubes embedded in two-parameter elastic medium

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In this study, based on continuum mechanics, an Euler–Bernoulli model has been applied to analyze the free transverse vibration of fluid conveying Single-Walled Carbon Nanotubes (SWCNTs), embedded in two-parameter elastic mediums such as; the Pasternak and visco-elastic mediums. By exerting the transfer matrix method (TMM), the vibration analysis of a fluid conveying SWCNT is developed. The effects of fluid flow velocity, Winkler, visco and Pasternak-elastic mediums are studied on the resonance frequencies. The results indicate that, the resonant frequencies at which the structural instability of nanotubes emerge are significantly influenced by their surrounding elastic medium. Each elastic medium is found to have special characteristics, resulting in different frequency behaviors for each SWCNT. Finally, the effects of geometry are studied on the SWCNT's resonance frequencies. Interesting results are obtained, which are helpful in the modeling or designing process of a carbon nanotube.

1. Introduction

In recent years, extensive research has shown that carbon nanotubes (CNTs) with hollow cylindrical geometry exhibit exceptional mechanical properties and thermal stability [1-3]. These tubes are used for a variety of technological and biomedical applications including nano-containers for gas storage and nano-pipes conveying fluids [4-6]. According to the literature, controlling the experiments at the nano-scale is difficult, and the molecular dynamics simulations remain expensive, complex and time consuming, especially for large-sized atomic systems.

The effects of the fluid flow velocity were shown to be significant on the dynamic behaviour of carbon nanotubes, both for the pinned–pinned and clamped–clamped nanotubes [7]. Recently, Wang and Ni [8] modified the results given in Ref. [7]. More recently, they studied the vibration characteristics of CNTs conveying viscous fluid by using the continuum beam model [9]. Subsequently, the vibration characteristics of fluid conveying CNTs with supported ends [10-16]; CNTs with inviscous [8,11] or viscous fluid [9,12]; CNTs modelled as classical Euler-Bernoulli beam [12-15] or classical Timoshenko beam [10,12] were also examined. Though, there are still many unknown aspects to be studied on this matter.

Most of the researches have concentrated on the instability of CNTs resting on elastic medium. The aim of this study is to present a framework for investigating the vibration analysis of single-walled carbon nanotubes (SWCNT) embedded in two-parameter elastic mediums such as pasternak-elastic medium and visco-elastic medium. The system can be modelled as the classical Euler-Bernoulli beam and the role of internal moving fluid is characterized by two parameters, the steady flow velocity and the mass density of fluid.

In this analysis, the linear, one-dimensional vibration of a fluid conveying SWCNT is investigated. The vibration analysis of the system is accomplished by using the transfer matrix method. The first few frequencies and critical velocities of the fluid are determined for two boundary conditions. The effects of the Winkler modulus, the shear factor of the Pasternak-elastic medium and the damping factor of the visco-elastic medium are elucidated on the resonance frequencies. Furthermore, SWCNTs with different aspect ratios (length/diameter) are also investigated. The results of the present analysis are compared with numerical solutions from the literature.

2. Mechanical modelling of fluid conveying SWCNTs

Figures 1, 2 show a SWCNT, as a hollow cylindrical tube in a two-parameter elastic medium.

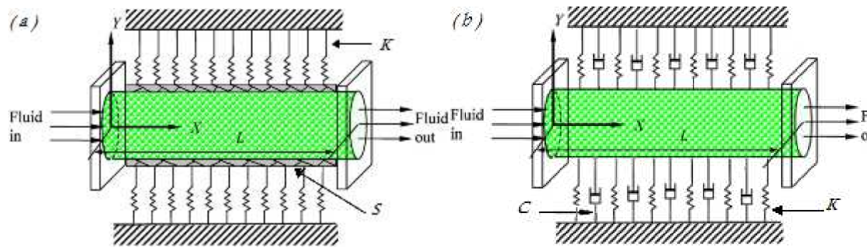

Figure 1.A fluid-conveying SWCNT (a) embedded in Pasternak-elastic medium, (b) embedded in visco-elastic medium.

Many studies show that, the classic Euler-elastic beam offers a simple and reliable model for an overall mechanical deformation of CNTs. By neglecting gravity effects and axial loads, the governing equation of motion of a fluid conveying SWCNT based on the Euler-Bernoulli theory can be expressed as:

$$EI\frac{\partial^4 W(X,T)}{\partial X^4}+(MV^2+P_0A_f-F^*)\frac{\partial^2 W(X,T)}{\partial X^2}+2MV\frac{\partial^2 W(X,T)}{\partial X\,\partial T}+(M+m)\frac{\partial^2 W(X,T)}{\partial T^2}+q=0 \quad (1)$$

where EI is the flexural rigidity of the SWCNT, M is the mass of fluid per unit length, flowing with a steady flow velocity V, m is the mass of the SWCNT per unit axial length (which is equal to the cross-sectional area of SWCNT itself multiplied by mass density of SWCNT), A_f is the area of the inner most cross-section of the SWCNT, and $W(X,t)$ is the transverse deflection of the SWCNT. The parameter X is the axial coordinate and T is time. F^* and P are externally imposed tension and pressurization effects, respectively. As explained in Ref. [24] for elastic tubes, regardless of the details of the wall-fluid interaction and the viscosity of the fluid, tension (F^*) and pressure drop (P_0A_f) cancel each other entirely and vanish from the equation of motion. In Eq.(1) for the Pasternak-elastic medium $q=KW(X,T)-S\frac{\partial^2 W(X,T)}{\partial X^2}$, and for the visco-elastic medium $q=KW(X,T)+C\frac{\partial W(X,T)}{\partial T}$, where K is the winkler modulus, S is the shear factor of the pasternak-elastic medium and C is the damping factor of the visco-elastic medium. For a self-excited vibration, the solution of Eq. (1) can be written as:

$$W(X,T)=\overline{w}(X)e^{i\omega t} \quad (2)$$

in which ω is the complex circular frequency. Defining the following dimensionless parameters:

$$w=\frac{\overline{w}}{L},\quad x=\frac{X}{L},\quad k=\frac{KL^4}{EI},\quad s=\frac{SL^2}{EI},\quad c=\frac{CL^2}{\sqrt{EI(M+m)}},\quad v=VL\sqrt{\frac{M}{EI}},\quad \mu=\frac{M}{(M+m)},$$
$$\beta=\frac{\omega L^2}{\sqrt{EI/(M+m)}}. \quad (3)$$

where L is the SWCNT's length. For the SWCNT embedded in a two parameter elastic medium Eq. (1) may be written in the dimensionless form as:

$$\frac{\partial^4 w}{\partial x^4} + v^2 \frac{\partial^2 w(x,t)}{\partial x^2} + 2iv\beta\sqrt{\mu}\,\frac{\partial^2 w(x,t)}{\partial x\,\partial t} + \frac{\partial^2 w(x,t)}{\partial t^2} + q^* = 0\,. \tag{4}$$

For the Pasternak-elastic medium $q^* = kw(x,t) - s\dfrac{\partial^2 w(x,t)}{\partial x^2}$ and for the visco-elastic medium $q^* = kw(x,t) + c\dfrac{\partial w(x,t)}{\partial t}$. In this paper, two types of boundary conditions are considered for the SWCNT:

$$w(0,t) = \frac{\partial w(0,t)}{\partial x} = w(1,t) = \frac{\partial w(1,t)}{\partial x} = 0 \tag{5}$$

for clamed–clamped ends, and:

$$w(0,t) = \frac{\partial w(0,t)}{\partial x} = w(1,t) = \frac{\partial^2 w(1,t)}{\partial x^2} = 0 \tag{6}$$

for clamed–free ends.

3. Solutions by the Transfer Matrix Method

In this section, the transfer matrix method is used to find the solutions of Eq. (4). Details of the transfer matrix procedure are presented in Ref. [17] and are directly utilized here. In order to solve the equation of motion, consider the following solution:

$$w(x,t) = \mathrm{Re}\left[\sum_{n=1}^{4} Ae^{\,i\alpha x}\right] \tag{7}$$

where A is a constant. Substitution of Eq. (7) into Eq. (4) gives the following characteristic equation:

$$\alpha^4 - v^2\alpha^2 - 2v\beta\sqrt{\mu}\,\alpha - \beta^2 + k + s\alpha^2 = 0\,. \tag{8}$$

for the Pasternak-elastic medium, and:

$$\alpha^4 - v^2\alpha^2 - 2v\beta\sqrt{\mu}\,\alpha - \beta^2 + k + i\beta c = 0\,. \tag{9}$$

for the Visco-elstic medium.

In Eqs. (8) and (9), four complex roots α_n (n=1, 2, 3 and 4) are functions of ω. The transverse shear force $\hat{f}_y$, rotation angle $\hat{\varphi}_z$ and bending moment $\hat{M}_z$ can be represented as:

$$\hat{f}_y = -\frac{\partial \hat{M}_z}{\partial x},\ \hat{\varphi}_z(x,t) = \frac{\partial w}{\partial x},\ \hat{M}_z(x,t) = EI\frac{\partial \hat{\varphi}_z}{\partial x}\,. \tag{10}$$

Now, Eq. (7) can be expanded in terms of α_n and A_n (n=1, 2, 3 and 4) as follows:

$$w(x,t) = \mathrm{Re}\ \{A_1 e^{i\alpha_1 x} + A_2 e^{i\alpha_2 x} + A_3 e^{i\alpha_3 x} + A_4 e^{i\alpha_4 x}\}\,. \tag{11}$$

By substituting Eq. (11) into Eq. (10), four equations are yield in terms of the constant values A_n. These four equations can be written in a matrix form for the Pasternak-elastic medium as:

$$\begin{Bmatrix} w \\ \hat{\varphi}_z \\ \hat{M}_z \\ \hat{f}_y \end{Bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \\ S_1 q_1 & S_2 q_2 & S_3 q_3 & S_4 q_4 \\ D_1 q_1 & D_2 q_2 & D_3 q_3 & D_4 q_4 \\ T_1 q_1 & T_2 q_2 & T_3 q_3 & T_4 q_4 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix}. \tag{12}$$

where $q_n = e^{i\alpha_n x}$, $S_n = i\alpha_n$, $D_n = -EI\alpha^2{}_n$, $T_n = iEI\alpha^3{}_n$, (n=1, 2, 3 and 4). (13)

Eq. (12) can be written in short form as:

$$\{Z\} = [Q(x)]\{A\}. \tag{14}$$

Assume for the ith SWCNT's element, at x=0, $Z = Z_{i-1}$ and at x=l, $Z = Z_i$ in which Z is the state vector corresponding to both ends of the CNT and l is the length of the SWCNT's element. $\{A\}$ can be expressed by $\{Z\}_{i-1}$ by letting x=0 in Eq. (14):

$$\{A\} = [Q(0)]^{-1}\{Z\}_{i-1}. \tag{15}$$

By substituting Eq. (15) into Eq. (14), and applying $Z = Z_i$ at x=l, one would obtain:

$$\{Z\}_i = [Q(l)][Q(0)]^{-1}\{Z\}_{i-1}. \tag{16}$$

where $[TM] = [Q(l)][Q(0)]^{-1}$ is the transfer matrix of the CNT's element. Suppose [RL] and [RR] to be the boundary conditions of the left and right side of the SWCNT, respectively. By consideration of just one element for the SWCNT, applying the boundary conditions of the SWCNT in Eq. (16) would cause:

$$[RTM]\{Z\}_0 = \{0\}. \tag{17}$$

where $[RTM] = [R_R][TM][R_L]$ is the reduced size of matrix [TM] due to exerting the boundary conditions. For achieving a non-trivial solution for Eq. (17), one should have:

$$|RTM| = \{0\}. \tag{18}$$

Eq. (18) is called the frequency equation and the components of the $|RTM|$ are the functions of the natural frequencies of the SWCNT.

4. Results and discussions

Based on the above formulations for a fluid conveying SWCNT, the vibration characteristics of SWCNTs are discussed in detail. The effects of the steady flow velocity, surrounding elastic medium and geometrical properties are studied on the resonant frequencies and the instability of SWCNTs. In order to calculate the natural frequencies, a computer program based on the transfer matrix method (TMM) was written. In the following examples, water is considered as the internal fluid of the SWCNT with a density of 1 g/cm3. The density of the CNT is assumed to be 2.3 g/cm3.

Boundary Condition	*Present Study*	*[7]*	*[8]*
SS-SS (k=1 GPa)	4666	4660	-
SS-SS (k=0)	1193	1190	1190
C-C (k=1 GPa)	5175	5200	-
C-C (k=0)	2385	2380	2390

Table 1 Critical flow velocities obtained by the TMM compared with previous studies.

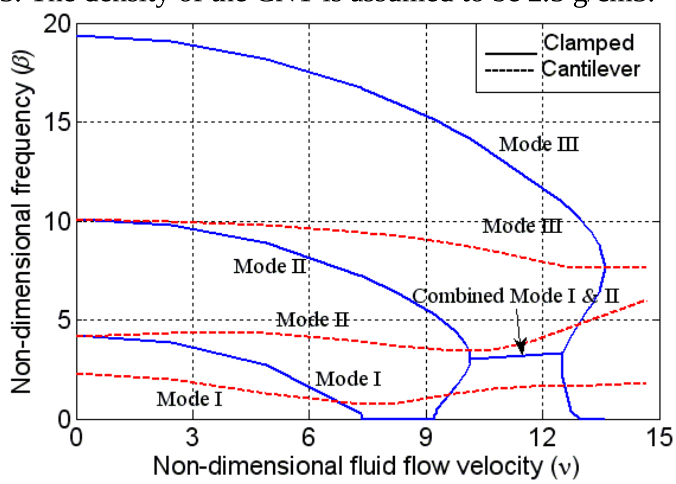

Figure 2 Flow velocity dependence of the lowest three modes of a SWCNT.

Let us first analyze the general frequency behaviour of a fluid conveying SWCNT. Consider two SWCNTs with clamped-clamped and clamped-free (cantilever) boundary conditions, embedded in a Pasternak-elastic medium with R_{out}=3.4nm, L/R_{out}=100, h=0.1nm, E=3.5TPa, k=200 and s=20. In Fig. 2, the non-dimensional resonance frequencies of these two SWCNTs are plotted as a function of the non-dimensional fluid flow velocity. In this figure the first three vibration modes of the CNTs are presented. The following results are obtained from Fig. 2:

1. Resonance frequencies are highly dependent upon the fluid flow velocity. The graph of each mode can be divided into two parts. In the first part of each mode, as the fluid flow velocity increases, resonance frequencies decrease.
   For the clamped SWCNT such decrease continues until the frequency reaches zero at a certain flow velocity. The velocity at which the frequency tends to zero is called the critical flow velocity. For the clamped SWCNT, it is understood by Fig. 2 that a critical flow velocity occurs for each resonance mode. At such velocity, a static structural instability characterized by adjacent neutral equilibrium states takes place (divergence instability). The flow velocity at which the divergence instability takes place is independent of frequency, because it is a static behaviour. Hence, at the critical flow velocity of each mode, the clamped SWCNT becomes unstable and tends to buckle. However it should be mentioned that the frequency equals to zero over a certain velocity domain rather than a single flow velocity. Therefore, instability occurs at a critical velocity domain, rather than a single critical flow velocity.
2. On the other hand, for the cantilever SWCNT, the decrease of the resonance frequency continues until it reaches a minimum. It is interesting to point out that, unlike the clamped SWCNT, the cantilever's frequency does not reach zero. Therefore the cantilever SWCNT does not possess a critical flow velocity, and hence it does not become unstable.
   On the other hand, for cantilever SWCNTs analyzed in [20], divergence instability takes place. However, by comparing the length of the present cantilever SWCNT (which is L=340nm) with the one analyzed in [20] (which is L=4000nm) one can clearly understand such an strange behaviour. The critical force of a column, derived by Euler is given by:
   $$F_{cr} = \frac{\pi^2 EI}{(K_c L)^2} \tag{19}$$
   in which K_c and L are the column's effective length factor and length, respectively. According to Eq. (19) as the length of a cantilever beam reduces, its critical force increases. Such increase of force causes the instability to be produced with more difficulty at higher forces. Therefore, it can be concluded that, for fluid conveying cantilever SWCNTs, which have a small length (similar to Fig. 2), due to high critical forces required, instability does not occur. Consequently, the SWCNT's boundary condition is very influential on its instability.
3. When the flow velocity reaches about $v \approx 9$, the clamped SWCNT regains stability in the first mode. As the flow velocity increases further, at $v \approx 12.5$ the first and second modes combine. In such a case a coupled-mode flutter occurs, producing large vibration amplitudes. At higher flow velocities the flutter takes place between the second and the third mode. This phenomenon repeats itself at higher flow velocities, for different modes.

In order to evaluate the validity of the present method, results were compared with those given in the literature. A SWCNT with outermost radius R_{out}=50 nm, thickness h=10 nm and length L=2000 nm was considered. In Table 1 critical flow velocities obtained by the TMM are compared to those given by Yoon et al. [7] and Wang et al. [8]. According to Table 1, compared to other methods the TMM yields results with a maximum error of less than 0.009 %. Therefore the TMM is found to be highly accurate for both simply supported (SS-SS) and clamped (C-C) boundary conditions. In the following sections, all calculations have been made for a clamped-clamped SWCNT with R_{out}=3.4nm, E=3.5TPa, h=0.1nm and L/R_{out}=100.

4.1 Effects of the Winkler elastic medium

It is to note that in Eq. (1), the Winkler modulus *K*, is the coefficient of the *X* term, therefore one would expect it to act similar to a spring. According to the fundamentals of vibration, the resonance frequency of a one-degree freedom system is as follows:

$$\omega = \sqrt{\frac{k_s}{m_s}} \tag{20}$$

in which k_s and m_s are the stiffness and mass of the system, respectively. The additional stiffness supplied by the Winkler elastic medium makes the system stiffer. Thus, when a CNT is supported by a Winkler elastic medium ($K \neq 0$), according to Eq. (20) its resonance frequency increases.
In Fig. (3), the first three modes of the clamped SWCNT are plotted for three values of the non-dimensional Winkler modulus *k*=0, 100 and 200. According to this figure, as the Winkler modulus increases, the resonance frequencies and critical flow velocities increase likewise. However, the Winkler modulus seems to be less influential at higher modes. Moreover the Winkler medium does not have a significant effect on the length of the critical velocity domain.

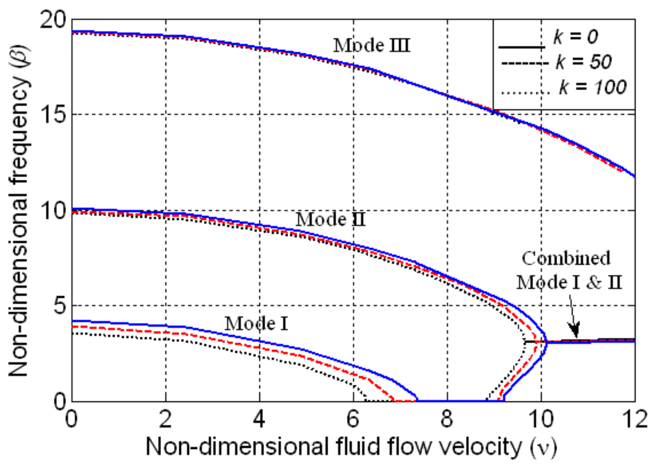

Figure 3 Flow velocity dependence of the lowest three modes of a SWCNT embedded in a Winkler elastic foundation.

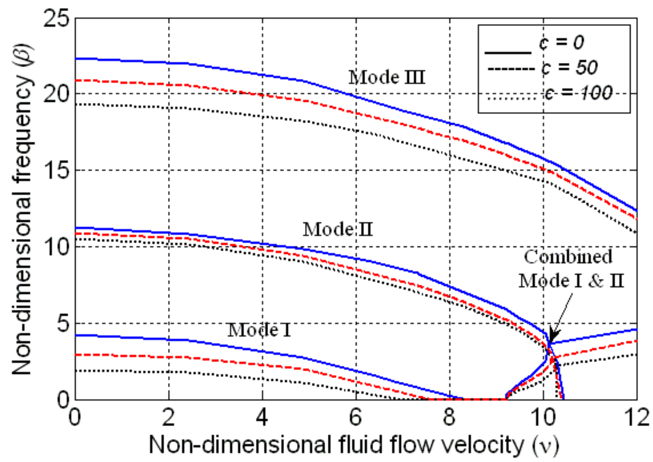

Figure 4 Flow velocity dependence of the lowest three modes of a SWCNT embedded in a visco-elastic foundation.

4.2 Effects of the visco-elastic medium

As discussed earlier, the visco model of the elastic medium should in general contain both the Winkler stiffness term *K* and the damping factor term *C*. It should be stated that in Eq.(1) *C* is the coefficient of the $\frac{\partial X}{\partial T}$ term, therefore one would expect it to act similar to a damper. The damping of the visco-elastic medium along with the viscosity of the fluid, cause the fluid-induced vibration systems to become non-conservative. This means that, when resonant frequencies are not zero, the decaying rate of amplitude could obtain negative or positive values. Fig. (4) shows the first three modes of the clamped SWCNT for a constant value of *k*=200 , and three values of non-dimensional damping factor *c*=0, 50 and 100. It is observed that, as the dimensionless damping factor is increased; the system becomes more damped, resulting in lower resonance frequencies for all modes. Moreover, the presence of a larger damping factor causes the instability of all modes to occur at lower flow velocities, decreasing the lower bound of the critical velocity domain. The upper bound of the critical velocity domain is unaffected by the damping factor. As a result, SWCNTs with different *c* parameters, all regain stability at the same flow velocity, independent of the damping factor. Furthermore, the presence of a damping factor causes the instability to occur at bigger velocity domains, increasing the length of the critical velocity range. Comparison of Figs. (3) and (4) indicates that, in the presence of a visco-elastic medium, the velocity ranges of the coupled mode are converted into a single steady flow velocity. Such an observation was also predicted by [18,19].

4.3 Effects of the Pasternak-elastic medium

The Pasternak-elastic medium also consists of both, the Winkler modulus K and the shear modulus S. in Eq. (1) S is the coefficient of the $\frac{\partial^2 X}{\partial T^2}$ term, however, with a negative sign. Therefore S is expected to have a role similar to the mass of the system, however, with an inverse effect (because of the negative sign). Consequently, the addition of a shear modulus S, is similar to removing some of the system's mass. In Fig. 5 resonance frequencies of the first three modes of the clamped SWCNT are plotted for a constant value of k=200 and different values of non-dimensional shear modulus s=0, 10 and 20. It is observed that, as the shear modulus increases, resonance frequencies increase likewise. According to the above explanation as the shear modulus increases, more mass is removed from the system and therefore according to Eq. (20), resonance frequencies increase. Furthermore the addition of a shear modulus increases the lower bound and upper bounds of the critical velocity domain. Consequently, the instability occurs at higher flow velocities; however, the length of the critical velocity domain stays unchanged. Hence, one should bear in mind that, the inclusion of the Pasternak medium is very crucial, as the simple Winkler medium overpredicts the resonance frequencies of the system.

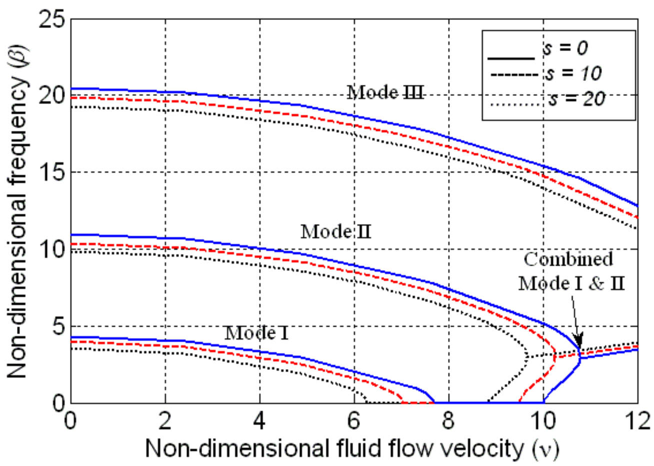

Figure 5 Flow velocity dependence of the lowest three modes of a SWCNT embedded in a Pasternak elastic foundation.

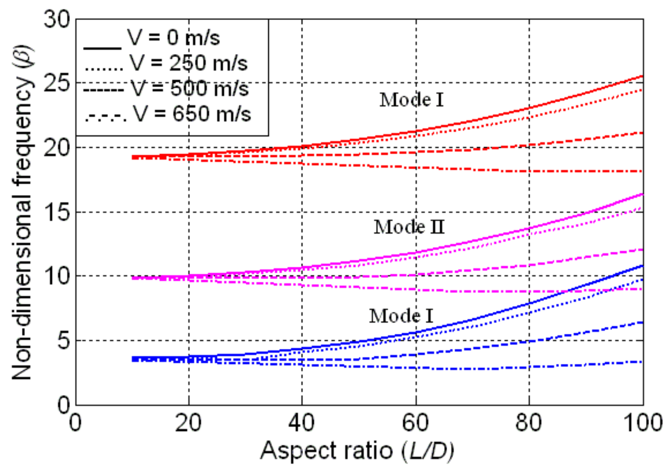

Figure 6 Aspect ratio dependence of a SWCNT with k=200, s=20, h=0.1nm for different flow velocities.

4.4 Effects of the aspect ratio

Now, let us consider the influence of the aspect ratio ($L/2R$) on the SWCNTs frequency behaviour. In Fig. 5 the non-dimensional resonance frequency has been plotted as a function of the aspect ratio, for several fluid flow velocities. According to Fig. 6 for flow velocities smaller than V=650 m/s, the relationship between the frequency and the aspect ratio is increasing. It is interesting that, for a clamped SWCNT operating at V=650 m/s no matter what the radius or the length is, the resonance frequencies for all the three modes stay unchanged. Hence, there exists a steady flow velocity at which the resonance frequency is completely independent of the aspect ratio. Similar behaviours are predicted for SWCNTs with different geometries and boundary conditions. Though it is not shown in Fig. 6, but it is expected that for flow velocities bigger than V=650 m/s as the aspect ratio increases, the frequency decreases until it equals to zero and becomes unstable.

5. Conclusion

The free vibration and instability of fluid conveying SWCNTs were studied, considering the effects of internal moving fluid, boundary conditions, elastic mediums and geometrical changes. The analysis was carried out using the transfer matrix method (TMM). The TMM was found to be efficient and simple to apply. It was observed that the divergence instability of the SWCNT occurred at a certain critical velocity domain. However, the presence of such a domain is very much influenced by the SWCNT's boundary conditions. The variations of the critical flow velocities and resonance frequencies embedded in three different elastic mediums: (1) Winkler, (2) Visco and (3) Pasternak, were also examined. The Winkler and Pasternak-elastic mediums were found to have an increasing effect on the resonance frequency and critical flow velocity. Whereas, the visco-elastic medium decreased the resonance frequency and critical flow velocity. Although, the visco-elastic medium increased the instability domain (critical velocity domain) of the SWCNT, the other two mediums did not have a significant effect on the length. Finally, for constant flow velocities the effect of a varying aspect ratio (L/D) was studied on the resonance frequencies. It was found that for at certain flow velocity the resonance frequency was independent of the aspect ratio.

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