# Procrustes – Based Shape Prior For Parametric Active Contours

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***Abstract*** **– A novel method of parametric active contours with geometric shape prior is presented in this paper. The main idea of the method consists in minimizing an energy function that includes additional information on a shape reference called a prototype. Prior shape knowledge is introduced through a complete family of Euclidean invariants, computed from the similarity between shape of evolving contour and the prototype. This similarity is measured by full Procrustes distance. This extra knowledge enhances the model robustness to noise, occlusion and complex background. We use genetic algorithm to minimize energy function of this new type of snake that we call it Procrustes snake. The variational formulation of the proposed approach is described in details. We obtain promising results with synthetic and real images which show the power of our method for segmentation tasks.**

***Index Terms*** **– Parametric active contours, Snake, Shape prior, Procrustes shape analysis, Genetic algorithm.**

## I. INTRODUCTION

An important problem in image analysis is object segmentation. Snake was proposed by Kass *et al.* [1] for solving this problem. Snake is a deformable contour on image plane that deforms to seek minimum value of its energy function. This energy function was defined so cleverly that takes its minimum value when is fitted on a closed boundary of an object in image plane. Hence, snake converts the segmentation problem to minimization of an energy function. So far different algorithms are used for solving this minimization problem. Calculus of variations (gradient descent) and random search algorithms are two main procedures. Main drawbacks of gradient descent minimization are trapping in local minimum due to noise and pseudo-edges, and numerical instability. In an effort for overcome these difficulties, many researchers have been used random search algorithms for solving this minimization problem. Simulated annealing [2], genetic algorithm [3] and particle swarm optimizer [4] are more convenient algorithms for this purpose. These algorithms are capable of detecting global minimum while escaping local ones.

Another critical problem is how to add shape prior information to energy function of snake that drives it toward boundary of desired object in image. During the last two decades, several approaches incorporating shape prior information have been presented. First, Duncan and Staib [5] propose to determine the parameters of a Gaussian probability distribution that associates the object boundaries to a range of shapes. If the prior is not available, a uniform distribution is used. Prior distributions can be estimated from a sample shape by decomposing the model parameters and collecting statistics. The optimization problem is then performed by the maximum a posteriori using Bayesian rule. Zhong & al. present an affine-invariant deformable contour [6] in a Bayesian framework. They introduce a new internal energy to define the global and local shape deformation of the contours between the shape domain and the image domain. Diffusion-snakes presented in [7] use a modified Mumford-Shah functional to allow the incorporation of statistical shape prior in a single energy function. A variational method is then used to minimize the snake energy.

For adding shape prior information to energy function of snake, we use full Procrustes distance [8]. This distance measures the similarity of two shapes, independent of their position, scale and rotation (Euclidean Transformations) in image plane. Consequently, we propose a modified energy function with shape prior information that the boundary of desired object in image is its global minimum. Noise, occlusion and complex background can not change location of this global minimum severely. Then we minimize this modified energy function by genetic algorithm, to obtain a global minimum for it. So by this method, we have found the boundary of desired object in image independent of its position, size and orientation in image plane.

This paper organized as follows: section 2 describes Procrustes shape analysis. In section 3 we add an extra energy term to usual energy function of parametric active contours based on full Procrustes distance. In section 4 we minimize this modified energy function by genetic algorithm. Experimental results are presented and discussed in section 5. Conclusion is presented in section 6.

## II. PROCRUSTES SHAPE ANALYSIS

Procrustes shape analysis is a particularly popular method in direction statistics and is intended to cope with 2-D shapes. A shape in 2-D space can be described by a vector of $n$ complex numbers:

$$\mathbf{z} = [z_1, z_2, ..., z_n]^T$$
$$z_i = x_i + j \times y_i \quad i = 1,2,...,n \tag{1}$$

$(x_i, y_i)$ are the Cartesian coordinates of the *ith* landmark of shape, vector $\boldsymbol{z}$ is called a configuration (We will represent vectors by using bold letters). Fig. 1 shows a prototype and its *n* landmarks. For two shapes $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$, if their configurations are equal through a combination of translation, scaling and rotation, *i.e.*:

$$\begin{aligned} &\boldsymbol{z}_1 = \alpha 1_n + \beta \times \boldsymbol{z}_2 \qquad \alpha, \beta \in C \\ &\alpha = \alpha_R + j \times \alpha_I \\ &\beta = |\beta| e^{j\angle\beta} \end{aligned} \tag{2}$$

We may consider $\boldsymbol{z}_1$, $\boldsymbol{z}_2$ represent the same shape. In (2) $1_n$ is a $n \times 1$ vector with entries 1, $\alpha_R \times 1_n$ translates $\boldsymbol{z}_2$ by $\alpha_R$ units in the horizontal axis direction and $j \times \alpha_I \times 1_n$ translates $\boldsymbol{z}_2$ by $\alpha_I$ units in the vertical axis direction, $|\beta|$ and $\angle\beta$ scale and rotate $\boldsymbol{z}_2$, respectively. It is very convenient to center shapes by defining the centered configuration $\boldsymbol{u} = [u_1, u_2, ..., u_n]^T$, $u_i = z_i - \bar{z}$, $\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$. The full Procrustes distance between two configurations $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ can be defined as [8] (we suppose that correspond points on two contours have similar indices in two configurations):

$$d_F^2(\boldsymbol{u}_1, \boldsymbol{u}_2) = \min_{\alpha,\beta} \left\| \frac{\boldsymbol{u}_1}{\|\boldsymbol{u}_1\|} - \alpha 1_n - \beta \frac{\boldsymbol{u}_2}{\|\boldsymbol{u}_2\|} \right\|^2 \tag{3}$$

Minimizing the above objective function with respect to $\alpha$ and $\beta$ we have: $\alpha = 0$, $\beta = \frac{\boldsymbol{u}_2^* \boldsymbol{u}_1}{\|\boldsymbol{u}_1\| \times \|\boldsymbol{u}_2\|}$. Where superscript * represents the complex conjugation transpose. Substituting $\alpha$ and $\beta$ in (3), we have:

$$d_F^2(\boldsymbol{u}_1, \boldsymbol{u}_2) = 1 - \frac{\boldsymbol{u}_1^* \boldsymbol{u}_2 \boldsymbol{u}_2^* \boldsymbol{u}_1}{\boldsymbol{u}_1^* \boldsymbol{u}_1 \boldsymbol{u}_2^* \boldsymbol{u}_2} = 1 - \frac{|\boldsymbol{u}_1^* \boldsymbol{u}_2|^2}{\|\boldsymbol{u}_1\|^2 \|\boldsymbol{u}_2\|^2} \tag{4}$$

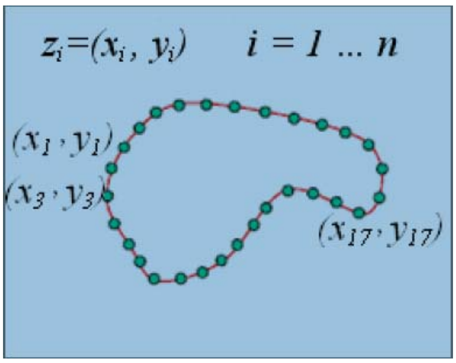



Figure 1 - A prototype and its *n* landmarks.

Based on Cauchy-Schwarz inequality ($|\boldsymbol{u}_1^* \boldsymbol{u}_2|^2 \le \|\boldsymbol{u}_1\|^2 \|\boldsymbol{u}_2\|^2$) we can show that $0 \le d_F(\boldsymbol{u}_1, \boldsymbol{u}_2) \le 1$. When $d_F(\boldsymbol{u}_1, \boldsymbol{u}_2) = 0$, two configurations $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ represent the same shape and when $d_F(\boldsymbol{u}_1, \boldsymbol{u}_2) = 1$, two configurations $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ represent two shapes that have no resemblance to each other. We conclude that smaller $d_F(\boldsymbol{u}_1, \boldsymbol{u}_2)$ means that two configurations $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ represent two shapes that have more resemblance to each other. Consequently, full Procrustes distance measures the degree of resemblance of two shapes independent of their position, scale and rotation (Euclidean transformations) in image plane. For using full Procrustes distance in snake energy, we need a mean shape (prototype) of desired object, to measure resemblance of evolving contour with it. Given a set of *k* sample shapes of an object $(\boldsymbol{u}_1, \boldsymbol{u}_2, ..., \boldsymbol{u}_k)$, we can find their mean by finding $\boldsymbol{u}$ that minimizes the objective function in (5) [8]:

$$\begin{aligned} \hat{\boldsymbol{u}} &= \arg \inf_{\boldsymbol{u}} \sum_{i=1}^{k} d_F^2(\boldsymbol{u}, \boldsymbol{u}_i) = \\ &\arg \inf_{\boldsymbol{u},\alpha_i,\beta_i} \sum_{i=1}^{k} \left\| \frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} - \alpha_i 1_n - \beta_i \frac{\boldsymbol{u}_i}{\|\boldsymbol{u}_i\|} \right\|^2 \end{aligned} \tag{5}$$

Hence, $\boldsymbol{u}$ is the shape that all sample shapes can be fitted to closely by selecting appropriate values for $\alpha_i$ and $\beta_i$ ($\alpha_i = 0$, $\beta_i = \frac{\boldsymbol{u}_i^* \boldsymbol{u}}{\|\boldsymbol{u}_i\| \times \|\boldsymbol{u}\|}$). From (4), (5) we have:

$$\begin{aligned} \hat{\boldsymbol{u}} &= \arg \inf_{\boldsymbol{u}} \sum_{i=1}^{k} 1 - \frac{\boldsymbol{u}^* \boldsymbol{u}_i \boldsymbol{u}_i^* \boldsymbol{u}}{\boldsymbol{u}_i^* \boldsymbol{u}_i \boldsymbol{u}^* \boldsymbol{u}} = \\ &\arg \inf_{\boldsymbol{u}} (k - (\boldsymbol{u}^* S \boldsymbol{u}) / (\boldsymbol{u}^* \boldsymbol{u})) \end{aligned} \tag{6}$$

Where *k* is a constant that represents number of sample shapes and:

$$S = \sum_{i=1}^{k} (\boldsymbol{u}_i \boldsymbol{u}_i^*) / (\boldsymbol{u}_i^* \boldsymbol{u}_i) \tag{7}$$

From (6) we conclude that:

$$\hat{\boldsymbol{u}} = \arg \sup_{\boldsymbol{u}, \|\boldsymbol{u}\|=1} \boldsymbol{u}^* S \boldsymbol{u} \tag{8}$$

If we suppose that $\lambda_i$ and $v_i$ ($\|v_i\|=1$) are corresponding eigenvalues and eigenvectors of matrix S, respectively, we will have:

$$\begin{aligned} &S v_i = \lambda_i v_i \Rightarrow v_i^* S v_i = \lambda_i \qquad i = 1,2,...,n \\ &(v_i^* S v_i)_{\max} = (\lambda_i)_{\max} \end{aligned} \tag{9}$$

Comparing (8) and (9), we conclude that the Procrustes mean shape $\hat{\boldsymbol{u}}$ is the dominant eigenvector of S, *i.e.*, the eigenvector that corresponds to the greatest eigenvalue of S.

## III. EMBEDDING SHAPE PRIOR TO ENERGY FUNTON OF SNAKE

A traditional snake is a controlled continuity spline that moves and localizes onto a specified contour under the influence of its energy function minimization [1]. Let a snake be a parametric contour, $v(s) = (x(s), y(s))$, where parameter $s \in [0,1]$. It moves around the image spatial domain to minimize the discretized energy function, defined by "[9], [10]," :

$$E_{image}(v) = w_1(\sum_{i=1}^{n}(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2) + ...$$
$$w_2(\sum_{i=1}^{n}(\overline{d} - [(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2])^2) + ...$$
$$w_3(\sum_{i=1}^{n}(x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2) + ...$$
$$+ w_4(\sum_{i=1}^{n} - |G(x_i, y_i)|^2_{normalized}) \quad (10)$$

That:

$$\overline{d} = (\sum_{i=1}^{n}(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2)/n \quad (11)$$

In the above equation, $v$ is the evolving contour with $n$ points, $(x_i, y_i)$ are the Cartesian coordinates of *ith* point and $(x_0, y_0) = (x_n, y_n)$. $w_i$ s are constant weights that are used to tune the impact of each energy terms.

The first term in above energy function is called first order continuity. This term drives snake points into image plane because it tends to reduce the distance between adjacent points. Hence this term prevents of gaps in contour that are due to noise and pseudoedges. Existence of this term is essential, because when snake lies on homogeneous regions of image, the image energy is negligible and only minimization of this term can move the snake toward boundary of desired object. One important problem of this term is a tendency for points to bunch up on a strong portion of an edge [9]. For solving this problem, we use the second term in energy function. This term encouraging even spacing of points, it tends to keep the distance between each pair of adjacent points equal and prevents of tendency for points to bunch up on a strong portion of an edge ($\overline{d}$ is average distance between adjacent points in contour).

The third term in above energy function is called second order continuity. If the *ith* point of snake pushed toward the middle of two adjacent points, this term will be minimized. Consequently, the shape of evolving contour will remain second order continuity *i. e.* without sharp corners that are usually due to noise and pseudoedges.

The fourth term (image energy) considers the gradient magnitude. It is normalized to measure the relative magnitude as [9]:

$$\left|G(x_i, y_i)\right|_{normalized} = \frac{\min\{|G|\} - |G(x_i, y_i)|}{\max\{|G|\} - \min\{|G|\}} \quad (12)$$

Where min and max denote the minimum and maximum gradients in the $(x_i, y_i)$ local m–neighbourhood, respectively. Note that because the numerator in above equation is always negative, we can minimize this term for locating the largest gradient, which is the edge. Consequently, minimization of this term will move snake points toward edge points in image.

To introduce shape prior information, we add fifth energy term ($E_{shape}$) to traditional energy function that guides the snake toward a given prototype (mean shape $\hat{u}$) independently of its pose, size and orientation in image.

$$E(v) = E_{image}(v) + w_5 E_{shape}(v) \quad (13)$$

That:

$$E_{shape}(v) = d_F^2(\hat{u}, v) \quad (14)$$

In the above equation, $d_F(\hat{u}, v)$ is full Procrustes distance between evolving contour $v$ and mean shape of desired object $(\hat{u})$ that was computed in previous section based on a set of $k$ sample shapes of desired object. We mention that parametrisation of the evolving contour ($v$) should match that of mean shape $\hat{\boldsymbol{u}}$ *i. e.*, correspond points on two contours have similar indices in two configurations. For this purpose, we reparametrize $\boldsymbol{v}$ by changing the starting point (applying circular shift to $\boldsymbol{v}$) and calculate $d_F(\hat{u}, v)$ for each parametrisation. The minimum of $d_F(\hat{u}, v)$ over $n$ possible parametrisation will be replaced in (14).

Note that full Procrustes distance only depends on degree of resemblance of two shapes. Hence, minimization of this term drives the evolving contour to boundary of desired object in image, independent of its pose, size and orientation in image *i.e.*, this term will be minimized if $\boldsymbol{v}$ and $\hat{u}$ represent the shape of a single object. The movement of this type of snake under the influence of full Procrustes distance between evolving contour and the mean shape of desired object inspired us to dub it Procrustes snake.

## IV. ENERGY FUNCTION MINIMIZATION USING GENETIC ALGORITHM

In previous section we proposed a modified energy function for parametric active contours so that the boundary of desired object in image is its global minimum. In this section, for probing this global minimum in image plane, we use genetic algorithm. Some benefits of GA for solving this minimization problem are [2]: a low order of complexity, the ability to handle arbitrary constraints, operation in discrete space and the ability to escape local minima and finding global minimum. GA starts with a fixed population of candidate solutions and each of the candidates is evaluated with a fitness function that is a measure of the candidate potential as a solution to the problem. The fitness function maps an individual of the population in to a scalar. Genetic operators like selection, crossover and mutation are implemented to simulate the natural evolution. A population, usually presented by a binary string is modified by the probabilistic application of the genetic operators from one generation to the next. The fitter individual has more chance for reproduction in next generation. However, the solutions with lower fitness are not always rejected from the population set to resist the loss of any otherwise useful genetic materials.

Our candidate solutions are discrete contours with $n$ points in image plane. For encoding these contours, we use chromosomes as:

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_1$ | $y_1$ | $x_2$ | $y_2$ | $\cdots$ | $x_n$ | $y_n$ |

That $(x_i, y_i)$ are the Cartesian coordinates of *ith* snake point and their values are integer values in the range of image domain. The fitness of this typical chromosome is computed by substituting it in energy function of Procrustes snake (13).

GA operators such as selection, crossover and mutation have various types. In this paper we use rank selection, uniform crossover with crossover rate $P_C$ and simple mutation with mutation rate $P_M$. In rank selection, first we rank individuals according to their fitness (smaller fitness better rank), then we give chance to individuals for reproduction in next generation proportional to their rank. In uniform crossover, each gene of children comes with equal probability from one of the parents. In simple mutation, each gene with probability $P_M$ will be replaced with a random positive integer in the range of image domain.

## V. EXPRIMENTAL RESULTS

In the first example we perform segmentation of a circle overlapping a rectangle. First we extract the mean shape for a rectangle and a circle with 24 and 23 landmarks, respectively. These mean shapes are depicted in Fig. 2 and Fig. 3.

Fig. 4 depicts the image and initial contour at the border of image. This initial contour evolves under the influence of energy function minimization by GA. Fig. 5(a) depicts the final contour when we do not use shape prior information $(w_5 = 0)$. Fig. 5(b) depicts final contour when we use shape prior information of a circle *i. e.* replacing $\hat{u}$ with mean shape of a circle and $(w_5 = 1)$.

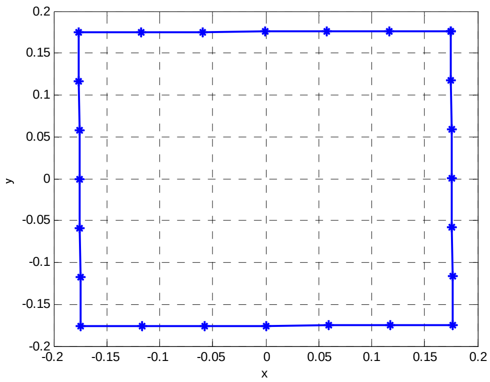


Figure 2 - Mean shape for a rectangle with 24 landmarks.

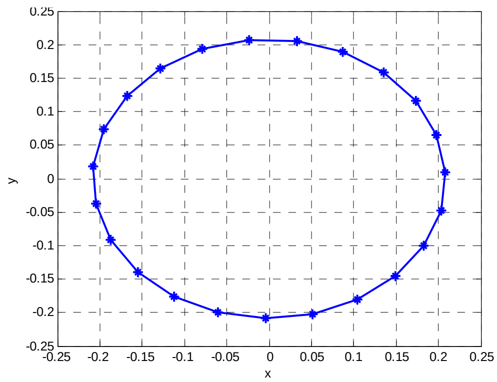


Figure 3 - Mean shape for a circle with 23 landmarks.

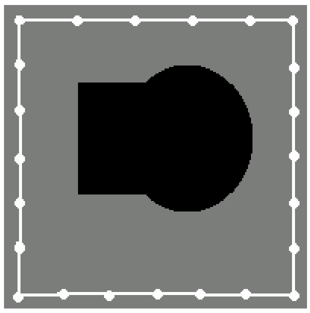
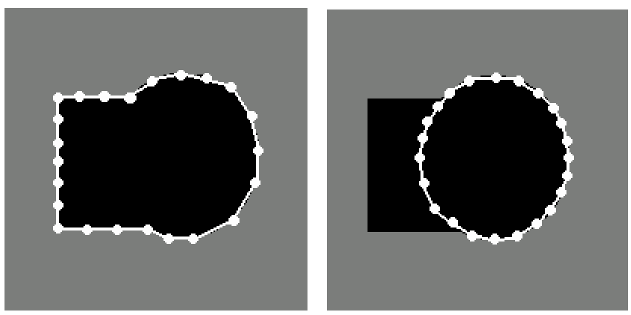


Figure 4 - Synthetic image and initial contour



(a) (b)

Figure 5 – a) Final contour without using of shape prior. b) Final contour with using shape prior information for a circle.

In the second example we show the robustness of the method against heavy noise. We corrupt the previous image with heavy salt and pepper noise, then perform segmentation by adding shape prior information for a rectangle to energy function i. e. replacing $\hat{u}$ with mean shape of a rectangle. Fig. 6 represents that correct segmentation is obtained despite the large amount of noise thanks to using of GA.

In the third example, we use Procrustes snake for ship detection. Fig. 7 depicts mean shape of one type of ship extracted from a set of 30 sample shapes with 31 landmarks. Fig. 8 depicts the initial contour at the border of image, Fig. 9(a) depicts the final contour without shape prior $(w_5 = 0)$ (traditional snake model) and Fig. 9(b) depicts the final contour with adding shape prior for the ship *i. e.* replacing $\hat{u}$ with mean shape of a ship and $(w_5 = 1)$, which is showing that correct segmentation is obtained despite the occlusion.

In the last example we show the robustness of the method against the complex background and its independence of initial contour. Fig. 10 depicts the image and initial contour at the border of image. Fig. 11(a) depicts the middle contour after 100 iterations of GA and Fig. 11(b) depicts the final contour after 147 iterations of GA. We see that boundary of the ship is extracted properly thanks to using of GA and adding shape prior information for the ship into snake energy.

Fig. 12 depicts the snake energy versus iterations. It is obvious that GA conflict with a strong local minimum after 100 iterations (Fig. 11(a)), but escapes it and detects the global minimum of snake energy after 147 iterations. This global minimum corresponds to our desired ship's boundary in image plane (Fig. 11(b)).

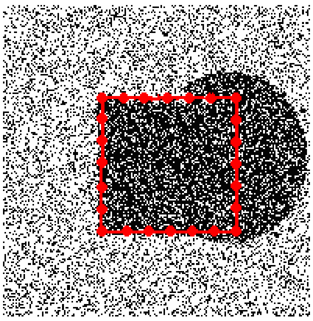


Figure 6 - Final contour in the presence of heavy noise.

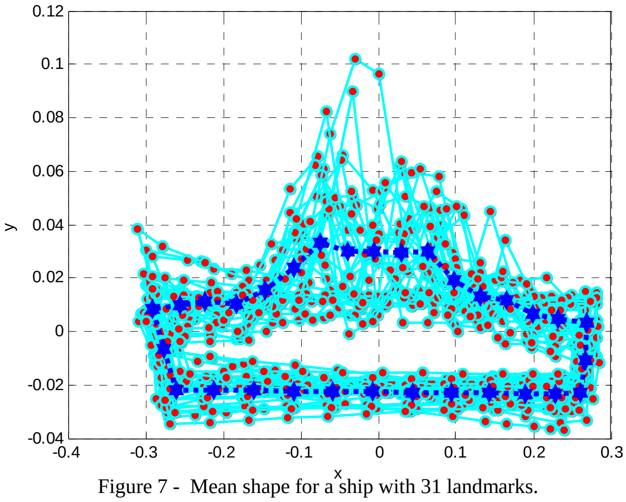



Figure 7 - Mean shape for a ship with 31 landmarks.

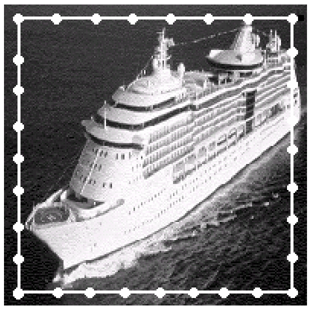
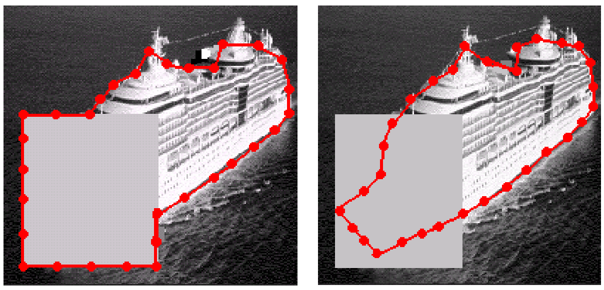


Figure 8 - Image of ship and initial contour.



(a) (b)

Figure 9 - a) Final contour without using of shape prior. b) Final contour with using shape prior information for a ship.

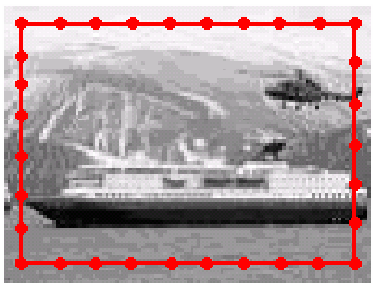
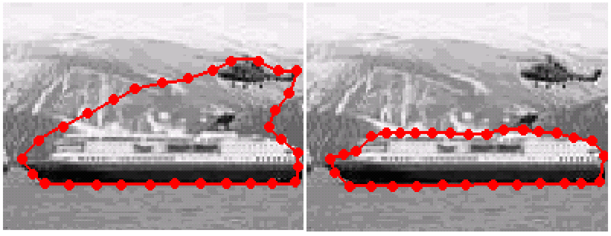


Figure 10 - Image of ship and initial contour.



(a) (b)

Figure 11 – a) Middle contour after 100 iterations of GA. b) Final contour after 147 iterations of GA.

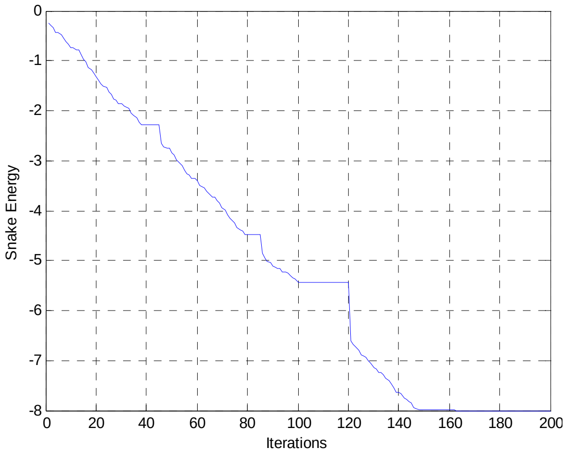



Figure 12 - Snake energy versus iterations of GA.

## VI. CONCLUSION

A new method of parametric active contours based on geometrical shape prior was presented in this paper. It used information from a mean shape of desired object and gray levels to guide the active contour to boundary of desired object. Mean shape was extracted from a set of sample shapes and full Procrustes distance was used for measuring the degree of similarity between evolving contour and mean shape. We used GA for minimization energy function of this new type of parametric active contours. Main drawback of our scheme is due to using of GA because it is a time consuming process. But, some benefits of GA are: a low order of complexity, the ability to handle arbitrary constraints, operation in discrete space and the ability to escape local minima and finding global minimum. By using of suboptimum minimization algorithms such as greedy algorithm, one can implement this minimization problem much faster. For using of suboptimum algorithms, one needs a good initialization contour near the boundary of desired object. The extension of the algorithm to more general geometrical transformations such as affine ones [11] is also an interesting perspective.

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