

*Full Length Research Paper*

# Cooperative learning and academic hardiness on students' mathematical performance with different levels of mathematics anxiety

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The paper has shown the relationship between mathematics anxiety, mathematics performance and academic hardiness in high school students in term of students learning method (Cooperative learning vs. traditional learning). For students who are working in small math cooperative groups, researchers have found that they can develop problem solving. The main aim of this study is to show that how much learning method could be helpful for learner with high math anxiety. The sample comprised 263 (134 males and 129 females) college students were tested on Mathematics Anxiety Rating Scale, Academic Hardiness Scale and Mathematics examination. Results obtained indicated that students work together with low or high mathematics anxiety had better performance in mathematics score. Also, results have revealed that mathematics anxiety has significant negative correlation with mathematics performance and academic hardiness. It is also found that the gender differences in mathematics anxiety are significant, whereas no significant differences are detected between boys and girls in mathematics performance and academic hardiness. In addition, the result of the study showed that students who work together with low or high mathematics anxiety had better performance in mathematics score.

**Keywords:** Mathematics anxiety, gender difference, academic hardiness, cooperative learning.

## INTRODUCTION

According to the Researches on cooperative learning (CL), cooperative learning and working in the small-group is so useful for academic and social students' ability (Gillies, 2002). CL has its roots in the theories of social interdependence, cognitive development, and behavioral learning. Some research provides exceptionally strong evidence that CL result in greater effort to achieve, more positive relationships, and greater psychological health than competitive or individualistic learning efforts (Johnson, Johnson, and Holubec, 1994). CL may produce positive effects on student achievement (Cohen, 1986; Davidson, 1989; Devries and Slavin, 1978; Johnson and Johnson, 1989; Okebukola, 1985; Reid, 1992; Slavin, 1990) and mathematics performance (Ross, 1995; Whicker, Nunnery, and Bol, 1997). So mathematics educators are shifting away from traditional classrooms to reform oriented mathematics classrooms that focus on

students actively engaged in mathematical discourse in cooperative settings (NCTM, 2000). A traditional classroom is typically institutionally sponsored, classroom-based, and highly structured. While some teachers use informal one-to-one study groups to bolster skills, other more formal structures include designated student roles and specific steps for completing long-term assignments. There is no one "right way" to develop CL, and teachers may choose models and methods that match their particular teaching styles, students, and lesson content. But cooperative environment requires a different role of the teacher. The teacher needs to provide time for the students to discuss the problem, search for methods to solve it, and learn how to evaluate the solution. The class is then divided into groups to accomplish their assigned tasks. Studies of students in CL groups indicate that two elements enhance student achievement. One is group goals; Group members should be interdependent, working to accomplish a common product. Relying on the skills of one group member or allowing one or two to dominate the activity

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does not result in greater understanding for all. The most prominent methods of CL have been developed by scholars and researchers alike (Slavin, 1991; Johnson and Johnson, 1989; Cohen, 1994). So CL may improve learners' mathematical performance and problem solving. Polya (1949) and others (e.g., Branca, 1980) maintain that problem solving is the goal of mathematics learning. More recently the NCTM reiterated its call for problem solving to form an integral part of the mathematics curriculum (NCTM, 2003).

According to the importance of math problem solving, the present study was carried out by the authors to study mathematical problem solving in term of math anxiety and academic hardiness. In this study the effect of math anxiety and academic hardiness on mathematical performance according to the learning method will be discussed. It seems to be more beneficial to describe the mathematical performance, Math Anxiety and Academic Hardiness before introducing research framework.

### **Mathematics anxiety and Mathematical problem solving**

Psychological literature provides a number of conceptualizations of mathematics anxiety. Many students already experience mathematics anxiety. Reported consequences of being anxious toward mathematics include the avoidance of mathematics and the decline in mathematics achievement (Karimi et al., 2009). This kind of 'anxiety' is first detected in the late 1950s. Researchers noticed undergraduate college students reacting emotionally to arithmetic and mathematics. Although the reaction is appeared to be similar to test anxiety in general, they found that mathematics anxiety is a potential factor prose (Dreger and Aiken, 1957). Richardson and Suinn (1972); have defined mathematics anxiety in terms of its (debilitating) effect on mathematical performance. They have observed that the feeling of tension and anxiety interfere with manipulation and solving mathematical problems in a wide variety of ordinary life and academic situations. It also involves feelings of tension and anxiety that interfere with the manipulation of numbers and solving the mathematical problems in a wide variety of ordinary life and academic situations (Suinn, 1988). In recent years, the study of math anxiety and mathematical cognition are two areas that have begun to become covered in research and theory. In fact, relationships between cognitive and affective components of math anxiety have been analyzed (e.g., Ashcraft and Faust, 1994; Bessant, 1995; Faust et al., 1996; Ashcraft and Krik, 2001). A negative relationship between math anxiety and math achievement has been found across all grade levels, k-college. In fact, math anxiety is often associated with relatively low performance in mathematical activity (e.g., Betz, 1978; Hembree, 1990; Bessant, 1995; Jackson and Leffingwell, 1999; Ma, 1999; Mark and Woodard, 2004,

Alamolhodaei, 2009). Highly math anxious students are characterized by a strong tendency to avoid math, especially in females (Bessant, 1995; Ashcraft, 2002) Mathematics anxiety is the outcome of low self-esteem and fear of failure. It causes problems for processing the incoming information as well as the previously learned information for problem solving. Such students tend to avoid mathematics whenever or wherever possible (Daane and Tina, 1986).

### **Academic hardiness**

Kobasa's theory of psychological hardiness (Kobasa, 1979; Ouellette, 1993) provides a useful framework for understanding why certain students are more willing to engage in more challenging academic coursework than others. Hardiness theory posits that three cognitive appraisal processes serve to buffer the deleterious effects of stressful life situations. These cognitive processes are: commitment (i.e., perceiving one's life activities as valuable to self and others), challenge (i.e., perceiving change rather than stability as an expected and normal part of life and viewing change as beneficial to personal development), and control (i.e., perceiving oneself as having personal control over important life events). The quality of students' Academic hardiness is influenced by wide range of environmental factors. The variable is very important for students, institutions of learning, educationists and curriculum practices. Psychological hardiness comprised of three obliquely related attitudes (Kobasa et al., 1982). The three interrelated hardiness attitudes of commitment, control, and challenge enhance the person performance (Maddi and Kobasa, 1979). Moreover it was found that there was a negative relationship between individuals' hardiness scores and mathematics anxiety (Ashcraft, 2002). There is different between boys and girls math experience in school but researches have shown that there was no significant gender difference with respect to academic achievement and general abilities (Lingard et al., 2005; karimi, 2009).

### **Research Framework**

The main aim of the present study is to investigate the effect and relationship between learning method, math anxiety, Academic hardiness and mathematical performance of college students. Thus the main question addressed here is: Can learning method improves the negative effect of high math anxiety on mathematical performance? In an attempt to answer this question the following objectives were sought:

The first objective of the study was to discover whether in which group of math anxiety (low or high), students have the highest mathematical problem solving in math score.

The second objective of the study was to find whether in which group of learning method (traditional or CL), students have the highest mathematical performance in math score.

The third objective was to find in which groups of learning method, students have lowest math anxiety.

The fourth objective was to find in which groups of learning method, students have better academic hardiness.

And the last objective of the study was to find significant differences between boys and girls in math anxiety, mathematics performance and academic hardiness.

## METHOD

### Participants

The sample group of the present study comprises 263 college students who were selected from nine classes of three different universities of Khorasan Razavi Province. For this purpose, random multistage stratified sampling design was used.

### Procedures

The research instruments were Mathematics Anxiety Rating Scale (MARS), Academic Hardiness Scale (AHS) and the average score of two math exams (final math exam and a fixed designed math exam for all participants)

### Mathematics Anxiety Rating Scale (MARS)

This questionnaire which has been recently developed in the Faculty of Mathematical sciences of Ferdowsi University of Mashhad. It consists of 32 items, and each item presented an anxiety arousing situation. The students decided the degree of anxiety and abstraction anxiety aroused using a five rating scale ranging from very much to not at all (5–1). Psychometric properties of this scale are computed by researchers. Cronbach's alpha, the degree of internal consistency of mathematics attention test items for this study was estimated to be 0.94. Students who scored above the sample mean were labeled as high math anxiety and those who scored less than the sample mean, as low math anxiety one. Table 1 shows this distribution.

### The students' math anxiety distribution over the sample

### Academic Hardiness Scale (AHS)

This Scale created by Benishek and Lopez (2001); with

18-item self-report instrument on a four-response Likert scale. This instrument was designed to gather information about student attitudes regarding academic success. The four response options range from 1 = completely false to 4 = completely true. The psychometric properties of this scale has shown that internal consistency alpha coefficient was computed 0.86 (Benishek and Lopez, 2001).

### Mathematical Performance

For measuring students' math performance, the average of two math exam was count. One of the score was about their final exam and another one was a fixed designed math exam that was taken from all participants. Some examples of the math exam are presented below:

Consider the function:

$$f(x) = \begin{cases} a + bx & \text{if } x > 2 \\ 3 & \text{if } x = 2 \\ b - ax^2 & \text{if } x < 2 \end{cases}$$

Determine the values of constants a and b so that  $\lim_{x \rightarrow 2} f(x)$  exists and is equal to f(2)

Integrate

$$\int (\ln x)^3 dx$$

Differentiate

$$y = 7x(\cos x)^{\frac{x}{2}}$$

### Learning method (CL vs. traditional)

For learning method, participants were divided in two classes. First class studied their lessons cooperatively and second class studied their lessons traditionally.

## RESULTS

According to the hypothesis of the current investigation, the analysis of data is divided into two parts: At first part the relationship between three variables is evaluated and in the second part, the gender differences in three variables are analyzed.

The correlations between levels of mathematics anxiety, mathematics performance, and academic hardiness are presented in the correlation matrix Table 2.

In this study, there are significant relationships between mathematics Anxiety, mathematics performance and academic hardiness.

**Table 1.** The students' Anxiety distribution over the sample

Group	Low	High
<b>Total</b>	n = 148	115
<b>N = 263</b>	56.3%	43.7%

**Table 2.** Means, standard deviations and correlation matrix of mathematics anxiety, Mathematics performance and academic hardiness

	Mean	SD	Mathematics anxiety	Academic hardiness	Mathematics performance
Mathematics anxiety	66.14	7.01	1		
Academic hardiness	52.70	6.05	-.27*	1	
Mathematics performance	11.61	4.59	-.71 **	.14 *	1

N: 263 M= mean. SD= standard deviation., \*\* Correlation is significant at the 0.01 level, p <.01 (2-tailed)., \* Correlation is significant at the 0.05 level, p <.05 (2-tailed)

**Table 3.** Mean, SD and significant difference of Math Anxiety Groups, Learning Method Groups, Academic hardiness and Mathematical Performance

			Mean	SD	P-Value
Math Performance	Math anxiety	Low (Group1)	12.89	4.47	P<.01
		High (Group2)	10.09	4.29	
Math Performance	Learning Method	Traditional (Group1)	10.05	3.89	P<.01
		Cooperative (Group2)	13.50	4.18	
Math anxiety	Learning Method	Traditional (Group1)	80.59	14.16	P<.01
		Cooperative (Group2)	62.58	17.59	
Academic Hardiness	Learning Method	Traditional (Group1)	87.93	13.16	P<.05
		Cooperative (Group2)	91.42	12.56	

In addition, negative significant correlations are found between mathematic anxiety and mathematics performance ( $r = -.71, p <.01$ ) while significant correlation between mathematics anxiety with academic hardiness is detected ( $r = -.27, P <.05$ ). Moreover, a significant correlation between mathematics performance, and academic hardiness is detected ( $r = .14, p <.05$ ).

As shown in Table 3, independent sample T-Test found significant difference between two groups of Math anxiety and students' mathematical performance ( $P <.01$ ). According to second objective, significant difference between two groups of the learning Method and math performance was obtained ( $P <.01$ ). Independent sample T-Test found significant differences between students' Math anxiety who study in cooperative learning and other student in traditional math class ( $P <.01$ ). For the last objective of the study

independent sample T-Test found significantly difference between students' academic hardiness and two groups of learning method (traditional or cooperative) ( $P <.05$ ).

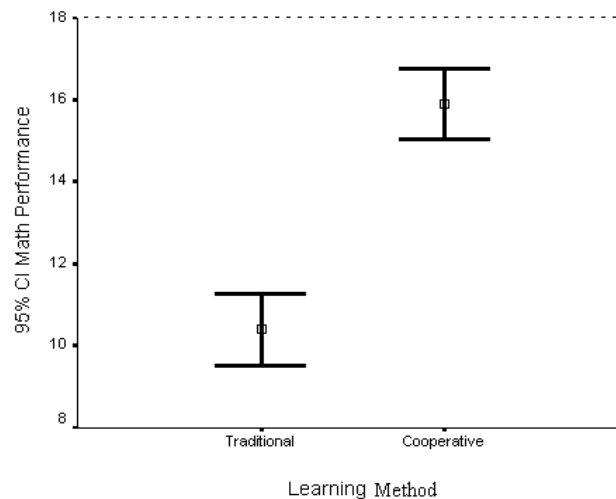
Table 4 displays the scores of males (boys) and females (girls) in three variables. The results of two independent t- tests are described in Table 2.

The investigation on the mean scores of males and females indicates that females scored slightly higher on the mathematics anxiety scale ( $m = 65.37, SD = 7.47$ ) than males ( $m = 62.50, SD = 4.41$ ). There are not significantly differences between males and females on mathematics performance. ( $m = 12.90, SD = 3.89$ ) for males and ( $m = 11.29, SD = 3.18$ ) for females. In Academic hardiness also there is not significantly differences between two gender groups, ( $m = 54.21, SD = 7.34$ ) for males and ( $m = 56.46, SD = 8.61$ ) for females.

**Table 4.** Means, standard deviations and estimated two independent samples t test for boys and girls in three variables

Dependent variable	Sex	Mean	SD	P-Value
Math Anxiety	Male	62.50	4.41	5.32**
	Female	65.37	7.47	
Math Performance	Male	12.90	3.89	.31
	Female	11.29	3.18	
Academic Hardiness	Male	54.21	7.34	2.36
	Female	56.46	8.61	

\*\* Difference is significant at the 0.01 level (2-tailed).



**Figure 1.** Comparing students with high math anxiety In terms Of two groups of Learning Method and Mathematical Performance

Since, the high math anxiety group has worse performance in mathematics, in this part of results students' performance in high math anxiety group has been compared to two groups of learning method.

Independent sample T-Test found significant difference between students' mathematical performance and Learning method in group 1 and 2 ( $P < .05$ ). In other words, students placed in group 2 of learning method have better performance than students another group as shown in figure 1.

## DISCUSSION

The results of this study have revealed significant relationship between mathematics anxiety and mathematical performance. It means that students who have high mathematics anxiety tended to perform lower score in mathematical performance (Baloglu and Koçak, 2006; Jain and Dowson, 2009, Alamolhodaei, 2009). However, students who have low mathematics anxiety tended to perform high score in mathematics.

The findings confirm the pervious findings, which report significant relationships between mathematics anxiety and mathematics performance (Clute, 1984; Hembree, 1990; Lee, 1996; Ashcraft, 2001). According to the result, there is negative significant relationship between mathematics anxiety and academic hardiness. This Finding confirms the previous research which report negative relationship between hardiness and anxiety (Ashcraft, 2002). It could happen because when students with positive attitude toward mathematics are confronted with new situation of mathematics' problem solving feel less anxious and use their math ability with less tension. Also there is a significant positive relationship between academic hardiness and mathematics performance which consequently supports the pervious evidence (Maddi, 1979). Students with positive attitude will be more motivated to learn mathematics and more strongly committed to their math classes than another group. According to this study, a significant gender differences was not found on the math anxiety test; whereas this was significant on the score of math performance and academic hardiness. This finding

supports the previous studies by Lee (1996) and Orenstein (1994). Moreover, based on Zaslavsky (1994), mathematics anxiety is a common problem between all groups, while women and minorities are the most affected ones.

The most important result of this study was that students with high math anxiety had better performance on the mathematical task, when they were working cooperatively. ( $P < .05$ ) It again maybe happened because learners with cooperative method of learning were applying their academic abilities in the best way and had better performance. Moreover, meaningful learning could be accrued when students were working in a group compared to others who were studying alone. In addition, sharing of their conceptual knowledge and procedural knowledge supports their better understanding and doing mathematical tasks. The situation of working group may help students to reduce their math anxiety and more concentration on mathematical concepts. Also they practice to solve their math problem in friendly group and after while they can overcome their math anxiety.

Based upon this study, it could be useful that math teachers to pay more attention to their students' individual differences such as math anxiety and choose the better teaching method that improves students' mathematical performance. Therefore CL is helpful learning method for teachers to help students to overcome their high math anxiety and their stresses.

According to the above results, the performance of students in mathematics could be influenced by mathematics anxiety. It is also reported by the other researchers that reducing of this kind of anxiety leads learners to improve their mathematics scores and academic performance. Mathematics anxiety can be reduced by using some special methods in teaching and psychological intervention, which is applicable in the pedagogical domains and the improvement curriculum practices. Moreover, the correlation between academic hardiness and mathematics performance has revealed that students with hardy attitudes will be more motivated to learn class material and more strongly committed to their classes than those reporting not having hardy attitudes. Therefore, it could be a useful factor in increasing the quality of learning. It is worth mentioned that further research should be conducted on the mathematics anxiety in different stages of academic levels, difference kinds of anxiety, across the other states and different learning method (e.g., heuristically learning, meta learning, discovery learning, e-learning and etc).

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