

[Home](#)   [Editorial Board](#)   [Contact Us](#)   [Instruction](#)   [Archives](#)   [Reviewer](#)

Texas Journal of Science - ISSN : 0040-4403



ISI Indexed Journal



Impact Factor: 0.113

### Popular Articles ▾

Managerial accounting in an early 19th century German-American religious commune

Beam-column welded RHS connections

Buckling of laminated skew plates

The architecture of transportation systems

#### **About Us**

Texas Journal of Science ISSN : 0040-4403 is a peer reviewed, international scientific journal dedicated for rapid publication of high quality original research articles as well as review articles in the all areas of basic and applied sciences.

Texas Journal of Sciences is devoted to the rapid publication of original and significant research in... [Read More »](#)

### Submission Form

Click on [Paper Template](#) to prepare your article format for sending.

**Submission :**

[Home](#)[Editorial Board](#)[Contact Us](#)[Instruction](#)[Archives](#)[Reviewer](#)

**Texas Journal of Science - ISSN : 0040-4403**

**Editorial Board ( Texas Journal of Science )**

Nitin Aggarwal, University of Wisconsin, USA,  
Yong Chen, University of Michigan, USA,  
Jianmin Gong, Virginia Polytechnic Institute, USA,  
Ventseslav Valev, Institute of Mathematics and Informatics, Bulgaria,  
Jurgen Schulte Mnting(Statistical Consultant), Freiburg, Germany  
Daniel Gamelin (University of Washington, USA)  
Michael Krische (University of Texas at Austin, USA)  
Chi-Kit AU, University of Waikato, New Zealand,  
Umar Ruhi,University of Ottawa, Canada,  
Mingqing Xu, Harvard Medical School, USA,  
Xiaohua Ye, Agiltron Inc., USA,  
Yudong Zhang, Columbia University, Research Scientist, USA,  
Dr Adrian Devine, University of Ulster, Northern Ireland, UK  
Ms Siobhan Drummond, University of the West of Scotland, UK  
Dr Sean Duffy, Letterkenny Institute of Technology, Ireland  
Dimitrios Zeugolis, Investigator, Network of Excellence, Ireland,  
Zheng Li Wu, Assistant Professor, The University of Hong Kong,  
Professor Aurelio Mauri, IULM University, Italy  
Dr Cathy Matheson, Queen Margaret University, Scotland, UK  
Dr Darcy McCormack, Australian Catholic University in Melbourne, Australia  
Dr Una McMahon-Beattie, University of Ulster, Northern Ireland, UK  
H. Fai Poon, Sr. Scientist, Sigma-Aldrich, MO,  
Xiong Yu, Assistant Professor, Case Western Reserve University, OH,  
Sabah Mushatat, Chair, University of Wolverhampton, UK,  
Faisal Arain, Chair, Southern Alberta Institute of Technology, Canada,



# Multi-Objective Resource Allocation Problem with Simulated Annealing Algorithm

Mansour Ghalehnovi<sup>1</sup> (Corresponding author), Mohammad Nabi Zobeydi<sup>2</sup>

<sup>1</sup>Department of Civil Engineering, Ferdowsi University of Mashhad, Mashhad, Iran  
Email: ghalehnovi@ferdowsi.um.ac.ir

<sup>2</sup>Department of Civil Engineering, Zahedan Branch, Islamic Azad University, Zahedan, Iran

## Abstract:

This paper consider on multi-objective resource allocation problem (MORAP). To solve a hypothetical MORA problem, simulated annealing algorithm was used. Results showed that simulated annealing is very efficient to solve this problem. At the end of paper we provide results of running the algorithm in various size problems and we conclude simulated annealing can be an appropriate choice to solve a MORAP.

**Keywords:** Simulated annealing algorithm, Multi-objective resource allocation problem, Project management

## 1- Introduction

Recently, some researchers have adopted computational optimization techniques, such as genetic algorithms and simulated annealing to solve TCTP. Feng et al. and Chua et al. proposed models using genetic algorithms and the Pareto front approach to solve construction time-cost trade-off problems. These models mainly focus on deterministic situations. However, during project implementation, many uncertain variables dynamically affect activity durations, and the costs could also change accordingly. Examples of these variables are weather, space congestion, productivity level, etc. To solve problems of this kind, PERT has been developed to deal with uncertainty in the project completion time. PERT does not take into account the time-cost trade-off. Therefore, combining the aforementioned concepts to develop a time-cost trade-off model under uncertainty would be beneficial to scheduling engineers in forecasting a more realistic project completion time and cost. In this paper, we develop a multi-objective model for the time-cost trade-off problem in PERT networks, using a genetic algorithm. It is assumed that the activity durations are independent random variables with generalized Erlang distributions. It is also assumed that the amount of resource allocated to each activity is controllable, where the mean duration of each activity is a non-increasing function of this control variable. The direct cost of each activity is also assumed to be a non-decreasing function of the amount of resource allocated to it. The problem is formulated as a multi-objective optimal control problem, where the objective functions are the project direct cost (to be minimized), the mean of the project completion time (min), its variance (min) and the probability that the project completion time does not exceed a given level (max). Then, we apply the goal attainment technique,

which is a variation of the goal programming technique, to solve this multi-objective problem.

For the problem concerned in this paper, as a general purpose solution method for non-linear programming problems, in order to consider the nonlinearity of problems and to cope with large-scale problems, we apply the revised GENOCOP V, developed by Suzuki which is a direct extension of the genetic algorithm for numerical optimizations of constrained problems (GENOCOP), proposed by Koziel and Michalewicz. Three factorial experiments are performed to identify appropriate genetic algorithm parameters that produce the best results within a given execution time in the three typical cases with different configurations. Moreover, an experiment in randomized block design is conducted to study the effects of three different methods of solving this problem, including the SA, on the objective function value and on the computational time.

## 2- SA algorithm for numerical optimizations of constrained problems

The simulated annealing algorithm is derived from the field of statistical mechanics. It follows a slow cooling process called 'annealing' to estimate the ground state energy of a matter (Van Laarhoven and Aarts, 1987). Metropolis and his colleagues developed an algorithm based on annealing principle to simulate a solid to thermal equilibrium (Luke, 2002). Krikpatrick, Gelatt, and Vecchi (1983) and Cerny (1985) successfully illustrated the application of this algorithm to optimize a combinatorial problem. The state of the solid, its energy and temperature were represented by the search space, the cost function and the control parameter of a combinatorial problem respectively. Simulated annealing performs better than any local optimization method and yields a solution close to global optimum (Fleischer, 1995). It is mainly attributed to the occasional acceptance of the higher cost function,

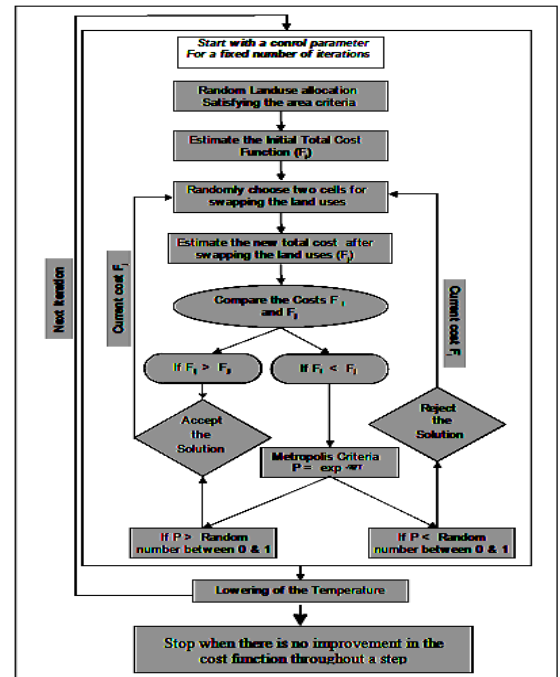


which enables to escape from being trapped at the local minimum. The acceptance of deteriorated cost function is probabilistically determined by the Metropolis Criterion ( $P$ ) as given by equation (1).

$$P = \exp^{-\frac{(F_i - F_j)}{T}} \quad (1)$$

where  $F_i$  is the cost function (energy) at current state (configuration)  $i$ ;  $F_j$  is the cost function of the system at new state  $j$  and  $T$  is the control parameter (temperature). The probability of acceptance of higher cost function,  $F_j$  is found by comparing the value of the Metropolis Criterion ( $P$ ) to the uniformly distributed random number ( $R$ ) between 0 and 1. If the value of  $P$  is greater than  $R$ , the higher cost function  $F_j$  and new configuration  $j$  is accepted otherwise it is rejected. This procedure has successfully delivered acceptable and efficient solutions for a wide range of combinatorial optimization problems such as the traveling salesman problem, circuit design (Krikpatrick, Gelatt, and Vecchi, 1983), graph partitioning (Johnson, Aragon, McGeoch, and Schevon, 1989), job shop scheduling (Van Laarhoven, Aarts, and Lenstra, 1992), harvesting scheduling (Lockwood and Moore, 1992) and the land allocation problem (Aerts, 2002). The implementation of simulated annealing requires the definition of the following parameters specific to a problem; the search space, the new solution generation mechanism, the cost function and a cooling schedule including the initial control parameter or temperature, the decrease rate, number of iterations per control parameter stage and the stopping rule (Pirlot, 1996; Sunderman, 1996). In a MOLAA problem, the search space is the initial solution generated by random allocation of land uses satisfying the area requirement for each land use alternative. The summation of the cost of each land unit with respective land use at the initial solution  $i$  equal to the cost function  $F_i$  and is subjected to minimization. A new solution  $j$  is generated by random selection of two land units and exchanging the land uses between them. The new cost function  $F_j$  is compared with previous cost function  $F_i$ . All new solutions with improvement in the cost function ( $F_j < F_i$ ) are always accepted. In case of a solution with higher cost function ( $F_j > F_i$ ), the acceptance is probabilistically determined by the Metropolis Criterion. The algorithm starts at a high value of the initial control parameter and is decreased by a specified rate after completion of a stated number of swaps. The algorithm is stopped at the point when all swaps are unable to reduce the cost function throughout a control parameter stage. The flowchart of simulated annealing for a MOLAA problem is shown in figure 1.

Figure 1 Flow Chart of the Simulated Annealing



### 3. Computational experiments

To investigate the performance of the proposed SA algorithm method (revised GENOCOP V) for the time-cost trade-off problem in PERT networks, we consider 3 typical small, medium and large cases with different configurations. Cases I–III are shown in Figs. 2–4, respectively. Tables 2–4 show the characteristics of the activities in Cases I–III, respectively. The structure of functions (different linear and non-linear forms) and also the distributions (generalized Erlang with different parameters) are selected so as to represent a wide variety of problems encountered in the time-cost trade-off problem in PERT networks. In real cases, these functions can be estimated using linear or non-linear regression. The given threshold value ( $u$ ) in Cases I, II and III is equal to 25, 3 and 8, respectively. The cost unit is in thousand dollars and the time is in months. The objective is to obtain the optimal allocated resource quantities using the SA.

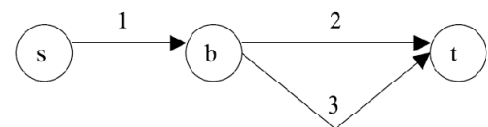


Fig. 2. Case I.

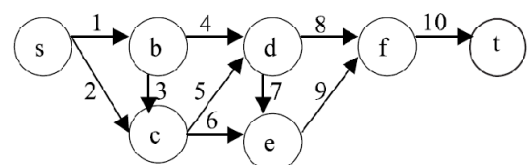


Fig. 3. Case III.



Table 2  
Characteristics of the activities in Case I

a	Distribution	Parameters	$d_a(x_a)$	$g_a(x_a)$	$L_a$	$U_a$
1	Exponential	$\lambda_1$	$3x_1^2 + 2$	$24 - 5x_1$	1	4
2	Exponential	$\lambda_2$	$2x_2 + 1$	$20 - 3x_2$	1	6
3	Generalized Erlang	$(\lambda_{31}, \lambda_{32})$	$x_3$	$15 - 2x_3$	1	6

Table 3  
Characteristics of the exponential activities in Case II

A	$d_a(x_a)$	$g_a(x_a)$	$L_a$	$U_a$
1	$2x_1$	$0.7 - 0.1x_1$	1	5
2	$3x_2 + 1$	$1.5 - 0.2x_2$	1	6
3	$x_3 + 2$	$1 - 0.1x_3$	1	9
4	$x_4$	$1.5 - 0.3x_4$	1	4
5	$3x_5 + 4$	$0.9 - 0.1x_5$	1	5
6	$x_6 + 3$	$1.1 - 0.1x_6$	1	6

Table 4  
Characteristics of the exponential activities in Case III

a	$d_a(x_a)$	$g_a(x_a)$	$L_a$	$U_a$
1	$2x_1$	$0.7 - 0.1x_1$	1.5	3
2	$3x_2 + 1$	$1.5 - 0.2x_2$	1.5	3
3	$x_3 + 2$	$1 - 0.1x_3$	1.5	3
4	$x_4$	$1.5 - 0.3x_4$	1.5	3
5	$3x_5 + 4$	$1.3 - 0.2x_5$	1.5	3
6	$x_6 + 3$	$1.1 - 0.1x_6$	1.5	3
7	$2x_7 + 5$	$1.5 - 0.2x_7$	1.5	3
8	$4x_8 + 1$	$1 - 0.2x_8$	1.5	3
9	$5x_9 + 2$	$0.9 - 0.1x_9$	1.5	3
10	$2x_{10} + 3$	$2 - 0.4x_{10}$	1.5	3

Since one month deviation from the mean project completion time is considered to be as important as its variance and also 20 and 5 times as important as one thousand dollars deviation from the project direct cost, and the probability that the project completion time does not exceed the threshold, respectively, then the under-attainment of the goals are considered as ( $c1 = 0.7407$ ,  $c2 = 0.037$ ,  $c3 = 0.037$ ,  $c4 = 0.1853$ ) in all cases. The following b vectors are also considered in the three indicates cases: Case I: ( $b1 = 25$ ,  $b2 = 8$ ,  $b3 = 25$ ,  $b4 = 0.98$ ), Case II: ( $b1 = 40$ ,  $b2 = 1.5$ ,  $b3 = 0.7$ ,  $b4 = 0.95$ ), and Case III: ( $b1 = 65$ ,  $b2 = 5$ ,  $b3 = 3.5$ ,  $b4 = 0.95$ ). However, in this stage the fixed values for b and c are considered in the three cases, but in the next experiments we consider different sets of b and c in each case.

#### 4. Estimates of Activity Times

Activity time is the elapsed time required for an activity. Estimating activity times is probably one of PERT's most critical features. Agribusiness firms are often so closely linked with agricultural production that they become as susceptible to seasonal fluctuations and market variations as producers. Consequently, agribusiness managers are reluctant to commit themselves to a rigid time schedule. Weather conditions, alone, prompt uncertainties and make it difficult for the manager to develop a single time estimate. However, experience has shown that managers are less reluctant if allowed three different estimates, especially when they understand PERT and how the concept of three time estimates is used. PERT, therefore, calls for not one, but three estimates of every activity time and allows the manager an opportunity to express his uncertainty about the possible time range of an activity. All three time estimates assume a static level of resource use. The estimates should be as good as possible because PERT

results depend directly on them. To obtain accurate estimates is not easy. It will require research, collaboration with planning team members, and homework. Simple guesswork is inadequate. If some time estimates are mere guesses, the manager will soon realize that they jeopardize or needlessly extend the project schedule date. Once the estimator realizes that his contributions are a small, but vital component of the PERT system, he will try to steadily improve his estimates. In short, guesswork will not replace intelligently derived estimates.

The person most familiar with the operation and requirements of each activity should submit the three time estimates. These should meet the following criteria:

1) *Optimistic Time* -- the minimum time period in which the activity can be accomplished, i.e., the time it would take to complete it if everything proceeded better than expected. (labeled a.)

2) *Most Likely Time* -- the best estimate of the time period in which the activity can be accomplished, i.e., the estimate submitted if one

(only) had been requested. (labeled m.)

3) *Pessimistic Time* -- the maximum time period it would take to accomplish the activity, i.e., the time required if everything went wrong, excluding major catastrophes. (labeled b.) It is acceptable to state these estimates in days, weeks, or months as long as the measure is used consistently. Once made, activity time estimates are firm and should not be changed without a change in the nature and scope of the activity or in the level of resources allocated to it. The following time relationships must be adhered to:  $a. \leq m. \leq b.$

Time Distribution: When an estimator makes three time estimates for performing an activity, he implies the existence of a distribution of possible activity times which may approximate that shown in Figure 10, where a is the optimistic time, m the most likely time, b the pessimistic time, and  $t_e$  for each distribution, as shown below, the statistical mean or average value of the three time estimates. More simply,  $t_e$  is defined as the average time an activity would require were it repeated many times.

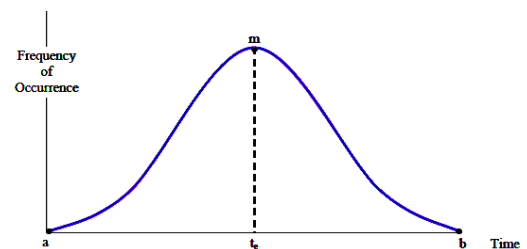


Figure 4: Frequency Distribution of Possible Activity Times

Estimator's judgment and calculations. Once established, their relative positions on the distribution affect the value or the position of  $t_e$ . In fact, once the numerical values of a, m, and b have been determined, an estimate of  $t_e$  is relatively easy to determine. For use in the PERT procedure, an estimate of  $t_e$  is 4



$a + m + b$ , and forms our basis for estimating the *expected* value of the activity performance time.

To further illustrate, let's assume that a planning team member is asked to provide time estimates ( $a$ ,  $m$ , and  $b$ ) for performing three different activities (A, B, and C) which normally fall within his functional area. The curves below represent his best estimates, see Figure 5

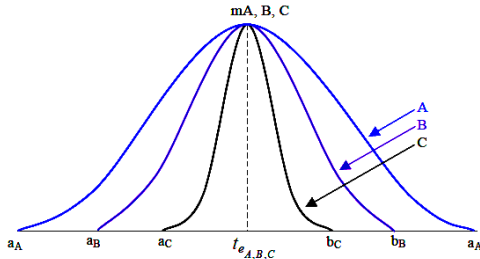


Figure 5: Distribution of Activity Times

## 5. PERT Cost Limitations

The discussion so far might lead one to believe that PERT Cost creates only benefits and no problems. Unfortunately this is not the case. Despite all its attributes, it has some basic problems which managers should recognize:

- a) It may be difficult to adapt PERT Cost activity costs to fiscal accounting practices.
- b) It may become difficult to allocate project overhead costs to the activities when several functional divisions are concerned with simultaneous activities.
- c) Historical information for making activity cost assignments may be lacking for non-repetitive projects.
- d) Firm personnel may lack the sophistication in cost analysis needed to optimize costs.
- e) Since time and cost are directly related, and since inexperienced planners tend to be pessimistic about meeting time schedules, cost estimates are often artificially padded to allow for a margin of safety.
- f) Because PERT deals with so many uncertainties it is difficult to extrapolate knowledge from previous experience. Thus, the assumption that a particular decision will result in a least-cost performance may be incorrect.
- g) It is often too easy to subject cost estimates to precise mathematical measurements; the results giving the user a false sense of security. This does not imply that costs are not predictable for discrete activities or that PERT Cost is not a sound framework for cost control. Nevertheless, the difficulty of obtaining reliable cost estimates certainly limits the system's overall effectiveness.

PERT Cost is not, after all, a complete departure from earlier planning and control techniques. Its elements existed before under other names and systems. Most PERT Cost components, therefore, are evolutionary, i.e., concepts of managerial planning and control have simply been refined and linked within a more compatible and effective system.

## 6. Conclusion

In this paper, we developed a new multi-objective model for the time-cost trade-off problem in PERT networks with generalized Erlang distributions of activity durations, using

a genetic algorithm. It was assumed that the amount of resource allocated to each activity is controllable, where the mean duration of each activity is a non-increasing function of this control variable. The direct cost of each activity was also assumed to be a non-decreasing function of the amount of resource allocated to it. To obtain the optimal resources allocated to the activities, we developed a goal attainment model with four conflicting objectives, minimization of the project direct cost, minimization of the mean of project completion time, minimization of the variance of project completion time and also maximization of the probability that the project completion time does not exceed the given threshold.

The problem considered in this paper has continuous decision variables and involves non-linearity. After the reformulation of the problem, we applied a genetic algorithm for numerical optimizations of constrained problems (revised GENOCOP V) to solve the problem.

This work investigated appropriate levels for genetic algorithm parameters in the three different typical cases with different configurations. It was found that in all cases, the minimum objective function value occurred with low level of population size and high level of generations within a given execution time, based upon regression analysis. We also concluded that the genetic algorithm produces  $z$  and the computational time that are significantly lower than  $z$  and computational time obtained from the discrete-time approximation method with two different levels of  $K$  and  $Dt$ , in all cases. Finally, it is seen that the proposed genetic algorithm method is an efficient method for the time-cost tradeoff problem in PERT networks.

## 7. References

- Aerts, J. C. J. H. (2002) Spatial decision support for resource allocation: Integration of optimization, uncertainty analysis and visualization techniques, PhD Thesis, University of Amsterdam, The Netherlands
- Bojórquez-Tapia, L. A., Ongay-Delhumeau, E. and Ezcurra, E. (1994) "Multivariate approach for suitability assessment and environmental conflict resolution" *Journal of Environmental Management*, 14, 187 - 198.
- Boston, K. and Bettinger, P. (1999) "An Analysis of Monte Carlo Integer Programming, Simulated Annealing and Tabu Search Heuristics for Solving Spatial Harvest Scheduling Problems" *Forest Science*, 45(2), 292 - 301.
- Carver, S. J. (1991) "Integrating multi-criteria evaluation with geographical information systems" *International Journal of Geographical Information Science*, 5(3), 321-339.
- Cerney, V. (1985) "Thermodynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithms", *Journal of Optimization Theory and Application*, 45 41-51.
- Diamond, T. J. and Wright, J. R. (1989) "Efficient land allocation" *Journal of Urban Planning and Development*, 115(2), 81 - 96.
- Eastman, J., R., Kyem, P. A. K., Toledano, J. and Jin, W. (1993) "GIS and decision making" *Explorations in*



Geographic Information Systems Technology, Volume 4,  
UNITAR, Geneva

Eastman, J. R. (2001) "Guide to GIS and image  
processing" Vol.2, Idrisi 32 (2), Clark University, USA

Fleischer, M. (1995) "Simulated annealing: Past, Present,  
and Future" Proceedings of the 1995 Winter Simulation  
Conference ed. C. Alexopoulos, K. Kang, W. R. Liegdon,  
and D. Goldsman

Johnson, D. S. Aragon, C. R., McGeoch, L. A. and  
Schevon, C. (1989) "Optimization by simulated annealing:  
an experimental evaluation; part I, graph partitioning"  
Operations Research, 42, 865-892.

Krikpatrick, S., Gelatt, C. D. and Vecchi (Jr.) M. P. (1983)  
"Optimization by simulated annealing" Science, 220, 671-  
680.

Leahy, S. (2003) Program for Simulated Annealing, The  
ANU, Australia (unpublished).

Luke, B. T. (2002) Simulated Annealing. In Learning from  
the web.net  
<http://www.members.aol.com/btluke/simann1.htm>, cited  
on 9/13/2002

Lockwood, C. and Moore, T. (1992) "Harvesting  
scheduling with spatial constraints: a simulated annealing  
approach" Canadian Journal of Forest Research, 23, 468-  
478.

Malczewski, J., Moreno-Sanchez, R. Bojórquez-Tapia, L.  
A. and Ongay-Delhumeau, E. (1997) "Multicriteria group  
decision-making model for environmental conflict analysis  
in the Cape Region, Mexico", Journal of Environmental  
Planning and Management, 40, 345 - 357.

Pereira, J. M. C. and Duckstein, L. (1993) "A multiple  
criteria decision-making approach to GIS-based and land  
suitability evaluation" International Journal of  
Geographical Information Systems, 7, 407- 424.

Pirlot, M. (1996) "General local search methods" European  
Journal of Operational Research, 92, 493-511

Proctor, W. (1999) "A practical application of multi  
criteria analysis to forest planning in Australia" IUFRO  
international symposium: From theory to practice-gaps and  
solutions in managerial economics and accounting in  
forestry, Prague, Czech Republic

Sundermann, E. (1996) PET image reconstruction using  
simulated annealing  
<http://petexp.rug.ac.be/~erik/research/welcome.html> cited  
on 9/13/2002

Van Laarhoven, P. J. M. and Aarts, E. H. L. (1987)  
Simulated Annealing: Theory and Applications, Dordrecht,  
Holland: D. Reidel Publishing Company.

Van Laarhoven, P. J. M., Aarts, E. H. L. and Lenstra  
(1992) "Job shop scheduling by simulated annealing"  
Operations Research, 40 (1), 113 - 125.

Voogd, H. (1983) Multi criteria evaluation for urban and  
regional planning, London: Pion Limited.

Voudouris, C. (1997) "Guided local search for  
combinatorial optimization problems" PhD Thesis.  
Department of Computer Science. University of Essex,  
UK.