Direct active and reactive power control of single-phase grid-tie converters

M. Monfared¹  M. Sanatkar¹  S. Golestan²

¹Department of Electrical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
²Department of Electrical Engineering, Abadan Branch, Islamic Azad University, Abadan, Iran
E-mail: m.monfared@um.ac.ir

Abstract: In this study, a simple digital power control technique for single-phase grid-tie converters is proposed. The suggested technique is based on the application of dead-beat control theory to the instantaneous powers in the virtual two-axis reference frame. A voltage estimation scheme is added to the proposed direct power control algorithm that allows grid voltage sensorless operation. The simulation and experimental results confirm that the proposed control strategy provides fast, accurate and decoupled power control with a lower alternating current distortion.

1 Introduction

In recent years, with ever increasing energy demand, and environmental pollution caused by fossil fuels, distributed generation and renewable energy sources (RES) are attracting special attention. Usually, a power electronic converter is required to transfer the electricity generated from these energy sources to the power grid. A single-phase voltage source converter (VSC) is the most widely used solution for connecting a low power RES to the single-phase grid [1].

To regulate the power exchange with the grid, and at the same time, reduce the harmonic components in the alternating current (AC) side current, various control strategies have been proposed, such as current hysteresis control (CHC) [2–5], voltage-oriented control (VOC) [6–10] and proportional-resonant (PR)-based control [11]. The CHC is one of the easiest control strategies, in which, the AC current is kept within the limits of a hysteresis band. Despite the advantages of simplicity, robustness, good stability, automatic current limiting and high dynamic response, the switching frequency is variable and depends on the sampling frequency and the system and load parameters. Variable switching frequency makes it difficult to design the converter power stage and the smoothing filter [2–5]. In VOC, the errors between the active and reactive components of the line current and the reference values are fed to proportional-integral (PI) controllers in the synchronous reference frame, which generate the reference voltage for the converter. This voltage is then applied to the converter using a voltage modulator. Fortunately, because of the internal current control loops, high dynamics and static performance are guaranteed. Also, the advanced modulation techniques can be used to reduce the losses and improve power quality [1, 6–10]. Therefore VOC has found a lot of industrial applications, today. The simplified block diagram of VOC for a single-phase VSC is shown in Fig. 1.

As one can see in Fig. 1, in VOC, the electrical signals are all transformed to the synchronous reference frame, where the quantities are DC and, as a consequence a zero steady-state error is ensured. This transformation needs at least two orthogonal signals, hence a fictitious phase must be generated. To generate the orthogonal component from a single-phase quantity, different techniques can be used, such as applying a 90° phase shift [12], Hilbert transformation [13], using an all-pass filter [14] and using a second-order generalised integrator (SOGI) [15]. The SOGI, an advanced and popular orthogonal signal generation technique is utilised in the structure illustrated in Fig. 1. PR-based control is another technique to control single-phase converters and the CHC method is implemented in the stationary reference frame. Unlike the PI controller, the PR controller provides a very high gain at the fundamental frequency of the AC system (resonant frequency). Hence, this method is able to track sinusoidal signals with a zero steady-state error [11]. Despite its simplicity, PR-based control has several major drawbacks, such as exponentially decaying response to step changes, and great sensitivity and possibility of instability to the phase shift of current sensors [6].

This paper presents a digital control strategy for single-phase grid-connected converters based on the dead-beat control theory and direct control of fictitious instantaneous powers. It is easy to implement, does not need any PI controllers and has excellent dynamic response. Besides, a voltage estimation scheme is added to the proposed direct power control (DPC) algorithm which allows grid voltage sensorless operation. The proposed control scheme is compared with a well-known control strategy, such as the VOC through extensive simulation and experiments under
various operating conditions. The results confirm the superiority of the proposed control strategy in providing a fast, accurate and decoupled power control with a lower AC current distortion.

2 Single-phase active and reactive powers in virtual two-axis reference frame

According to the variables defined in Fig. 1, the active and reactive power expressions for the single-phase VSC can be written as

\[ \begin{align*}
P &= \frac{1}{2} V_m I_{m1} \cos (\varphi_1) \\
Q &= \frac{1}{2} V_m I_{m1} \sin (\varphi_1)
\end{align*} \]  

(1)

where \( V_m \) and \( I_{m1} \) are the peak values of the grid voltage and the fundamental component of the grid current, respectively, and \( \varphi_1 \) is the displacement power factor angle. It is a common practice to transform the multiphase electrical machine and power electronic converter systems into the two-axis stationary (\( \alpha \beta \)) or rotary (\( dq \)) reference frames. These transformations bring significant simplicity and ease of analysis, especially when determining the instantaneous active and reactive powers in three-phase systems. Hence, the essence of this paper is to create a virtual two-phase system from an ordinary single-phase signal. Then, the application of instantaneous power theory develops novel methods for control and analysis of single-phase power-conditioning systems. In the following, at first, the idea of a virtual two-phase equivalent system is discussed, and then the relations to calculate the single-phase powers in the virtual two-axis reference frame system are introduced.

2.1 Fictitious phase

As mentioned before, to generate the secondary orthogonal phase, which is necessary for realising a virtual two-phase system, various methods have been proposed. In this work, the fictitious phase is obtained using the SOGI [15]. Fig. 2 illustrates the basic scheme of the SOGI structure, in which \( k \) is the damping factor, and \( \omega \) is the fundamental angular frequency.

An outstanding feature of SOGI is that, depending on the selected damping factor \( k \), it provides some kind of filtering and can improve the performance under distorted grid voltages.

From Fig. 2, the characteristic transfer function of SOGI can be obtained as

\[ \begin{align*}
x_a(s) &= \frac{k \cos \omega}{s^2 + k \cos \omega + \omega^2} \\
x_\beta(s) &= \frac{k \omega^2}{s^2 + k \cos \omega + \omega^2}
\end{align*} \]  

(2)

Applying (2) to the grid voltage \( (v) \), as well as the current drawn from grid \( (i) \), and without considering the harmonic voltages, the following orthogonal two-phase system can be constructed.

\[ \begin{align*}
v_a &= V_m \sin (\omega t) \\
v_\beta &= -V_m \cos (\omega t)
\end{align*} \]  

(3)

\[ \begin{align*}
i_a &= I_{m1} \sin (\omega t - \varphi_1) + \sum_{n=3,5,\ldots} i_{an} \\
i_\beta &= -I_{m1} \cos (\omega t - \varphi_1) + \sum_{n=3,5,\ldots} i_{bn}
\end{align*} \]  

(4)

In (4), \( i_{an} \) and \( i_{bn} \) are the \( n \)th order harmonic current components.

2.2 Active and reactive powers

Analogous to a three-phase system, the instantaneous active and reactive powers in the \( \alpha \beta \) reference frame can be defined as

\[ \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} v_a & v_\beta \\ v_\beta & -v_a \end{pmatrix} \begin{pmatrix} i_a \\ i_\beta \end{pmatrix} \]  

(5)

From (3), (4) and (5), and after some simple manipulations we obtain

\[ \begin{align*}
p &= V_m I_{m1} \cos (\varphi_1) + V_m \sum_{n=3,5,\ldots} (i_{an} \sin (\omega t) - i_{bn} \cos (\omega t)) \\
q &= V_m I_{m1} \sin (\varphi_1) + V_m \sum_{n=3,5,\ldots} (-i_{an} \cos (\omega t) - i_{bn} \sin (\omega t))
\end{align*} \]  

(6)

that can be rewritten as

\[ \begin{align*}
p &= 2P + V_m \sum_{n=3,5,\ldots} (i_{an} \sin (\omega t) - i_{bn} \cos (\omega t)) \\
q &= 2Q + V_m \sum_{n=3,5,\ldots} (-i_{an} \cos (\omega t) - i_{bn} \sin (\omega t))
\end{align*} \]  

(7)

Assuming that \( \langle p \rangle \) and \( \langle q \rangle \) are the mean values of \( p \) and \( q \), respectively, which are obtained by using an ideal low-pass
filter (LPF), then

\[
\begin{align*}
\bar{P} &= \frac{<p>}{2} \\
\bar{Q} &= \frac{<q>}{2}
\end{align*}
\]

In fact, a pulse width modulation (PWM) scheme transfers the current ripples to the switching frequency that can be easily eliminated from the instantaneous powers defined by (7) by an LPF with a cut-off frequency lower than the switching frequency. Equation (8) shows that the values of filtered fictitious instantaneous powers calculated for the virtual two-phase system are twice the values of the actual single-phase powers. Hence, the filtered fictitious powers are in direct relation with the actual powers and can be used in order to control the actual active and reactive powers.

3 Proposed DPC

The proposed DPC is based on dead-beat control. In discrete-time control theory, dead-beat control is considered because of its high dynamics. The main idea of this method is to find the input signal, which when applied to the system, will make the output error zero (or minimum) in the least possible time.

In the proposed DPC, at each sampling period, the reference voltage for the converter is determined, such that by applying it, the power errors at the next sampling period will be driven to zero. For this purpose, the active and reactive powers at the next sampling period are predicted, and used as the reference values for the current sampling period. The active and reactive powers at the next sampling instant \((k + 1)\) are predicted from the virtual two-phase converter model.

To derive the converter model, we start with the voltage equation of the grid-connected single-phase converter of Fig. 1.

\[
v - v_c = L \frac{di}{dt} + Ri
\]

where \(v_c\) and \(v\) are the converter and grid voltages, respectively, \(i\) is the converter current, and \(L\) and \(R\) are the inductance and equivalent resistance of the smoothing filter. This equation can be rewritten in the \(a\beta\) reference frame as follows

\[
\bar{v}_{a\beta} - \bar{v}_{c,a\beta} = L \frac{d\bar{i}_{a\beta}}{dt} + R\bar{i}_{a\beta}
\]

Using (11), (10) can be transferred to the synchronous reference frame as shown in (12)

\[
\bar{x}_{a\beta} - \bar{x}_{c,a\beta} = \bar{x}_{d\beta} e^{jwt}
\]

\[
\bar{v}_{dq} - \bar{v}_{c,dq} = L \frac{d\bar{i}_{dq}}{dt} + R\bar{i}_{dq} + jL\omega \bar{i}_{dq}
\]

By expanding (12) and using a discrete first-order approximation, the \((k + 1)\)th current sample is obtained as

\[
\begin{align*}
i_d(k + 1) &= \frac{T_S}{L} (v_d(k) - v_{cd}(k)) + \left(1 - \frac{T_S R}{L}\right)i_d(k) + T_S \omega i_q(k) \\
i_q(k + 1) &= \frac{T_S}{L} (v_q(k) - v_{cq}(k)) + \left(1 - \frac{T_S R}{L}\right)i_q(k) - T_S \omega i_d(k)
\end{align*}
\]

where \(T_S\) is the sampling period.

On the other hand, the active and reactive powers in the synchronous reference frame are calculated from (14)

\[
\begin{align*}
p(k + 1) &= \left(v_d(k + 1) i_d(k + 1) - v_q(k + 1) i_q(k + 1)\right) \\
q(k + 1) &= \left(v_d(k + 1) i_q(k + 1) + v_q(k + 1) i_d(k + 1)\right)
\end{align*}
\]

Assuming a sinusoidal voltage, the grid voltages \(v_d\) and \(v_q\) can be considered constant during a small sampling period, that is \(v_{d0}(k + 1) = v_{d0}(k)\). Also, by using a PLL, the \(d\)-axis can be aligned with the fictitious rotating voltage vector \(v_a + jv_{\beta}\) and as a consequence \(v_q = 0\). Based on these two assumptions, and replacing the currents from (13) into (14), the simplified relations for the fictitious active and reactive powers will be obtained as

\[
\begin{align*}
p(k + 1) &= \left(1 - \frac{T_S R}{L}\right) p(k) - T_S \omega q(k) + \frac{T_S}{L}(v_d^2(k)) \\
q(k + 1) &= \left(1 - \frac{T_S R}{L}\right) q(k) + T_S \omega p(k) + \frac{T_S}{L}(v_d(k)v_{cq}(k))
\end{align*}
\]

To successfully eliminate the power errors at the \((k + 1)\)th sampling instant, the predicted powers by (15) must follow the reference values available at the current sampling instant \(k\), that is

\[
\begin{align*}
p(k + 1) &= p_{ref}(k) \\
q(k + 1) &= q_{ref}(k)
\end{align*}
\]

Fig. 3 Block diagram of the proposed DPC
By replacing (16) into (15) and neglecting the small inductor resistance $R$, the reference values for the converter voltages that satisfy the control law of (16) are computed as

$$
\begin{align*}
\bar{v}_{cd}(k) &= v_d(k) - L \frac{k_p \Delta p(k)}{T_S} - L \frac{k_q \Delta q(k)}{T_S} \\
\bar{v}_{cq}(k) &= \frac{L}{v_d(k)} \left[ \frac{k_p \Delta p(k) + L o q(k)}{T_S} \right]
\end{align*}
$$

(17)

where $\Delta p(k) = p_{ref}(k) - p(k)$ and $\Delta q(k) = q_{ref}(k) - q(k)$, and $k_p$ and $k_Q$ are the proportional gains for active and reactive power controllers. Attention should be paid to the relation between the fictitious-instantaneous powers and the real average powers as defined in (8). After the reference converter voltages calculated from (17) are transformed to the $a\beta$ reference frame, the fictitious signal is discarded and $v_a$ will provide the reference value for the PWM generator. Fig. 3 shows the block diagram of the proposed DPC for the single-phase converter.

### 4 Control delay compensation

Owing to computational time delay, there always exists one sample time delay between the reference voltages calculated by the controller and the converter output voltages. Indeed, the converter reference voltages that are calculated at the $k$th sampling instant are applied to the converter at the beginning of the $(k+1)$th sampling instant. In order to compensate for this delay, a linear model, presented in (18), is utilised to estimate the reference powers at the next sampling instant $(k+1)$th from its present $(k)$th and past $(k-1)$th values.

$$
\begin{align*}
p_{ref}(k) &= 2p_{ref}(k) - p_{ref}(k-1) \\
q_{ref}(k) &= 2q_{ref}(k) - q_{ref}(k-1)
\end{align*}
$$

(18)

In (18), the powers with a ‘*’ are estimates of the reference powers at $(k+1)$ and are used in (17) to calculate the appropriate control signals.

### Table 1 Parameters of the simulated system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>sampling frequency, kHz</td>
<td>5</td>
</tr>
<tr>
<td>switching frequency, kHz</td>
<td>5</td>
</tr>
<tr>
<td>inductance of filter, mH</td>
<td>3.7</td>
</tr>
<tr>
<td>resistance of filter, Ω</td>
<td>0.25</td>
</tr>
<tr>
<td>DC-link capacitor, μF</td>
<td>6000</td>
</tr>
<tr>
<td>AC voltage amplitude, Vrms</td>
<td>70</td>
</tr>
<tr>
<td>AC voltage frequency, Hz</td>
<td>50</td>
</tr>
<tr>
<td>DC-link voltage, V</td>
<td>200</td>
</tr>
</tbody>
</table>

Fig. 4 Simulated waveforms with various step changes of reference powers

- a VOC
- b Proposed DPC

Fig. 5 Simulated grid current harmonic spectrum ($P_{ref} = 500 W$, $Q_{ref} = 0 VAr$)

- a VOC
- b Proposed DPC
5 Grid voltage estimation

The concept of DPC for three-phase converters permits voltage sensorless implementation. This means that the grid voltages are estimated using the measured currents and converter parameters, and the voltage sensors are eliminated. Hence, in accordance with the idea of voltage sensorless implementation of three-phase DPC, in this paper, a voltage estimation scheme is added to the proposed DPC algorithm which allows getting rid of the grid voltage measurement. The relations of the grid voltage estimator are presented in (19) [16].

\[ V_{est}(k) = V_{est}(k-1) + k_v(i(k) - i(k-1)) \cos(\theta_{est}(k-1)) \]
\[ \theta_{est}(k) = \theta_{est}(k-1) + \omega T_s - k_u(i(k) - i(k-1)) \sin(\theta_{est}(k-1)) \]

In the above equations, \( V_{est} \) and \( \theta_{est} \) are the peak and phase angle of the grid voltage, respectively, and, \( k_v \) and \( k_u \) are the proportional gains that are tuned according to the directions of [16] \( (k_v = 2.36, k_u = 0.02) \). Since voltage estimation is performed in each sampling period based on the instantaneous variables, the harmonic components of the voltage waveform can also be successfully estimated.

6 Simulation results

In order to verify the validity of the proposed DPC, a digital computer simulation model has been developed in MATLAB/SIMULINK. The steady-state, the transient response and the current harmonic spectrum of the proposed DPC are compared with the VOC. The decoupling feedforward control for the active and reactive powers is also added to the VOC [1, 6–10]. The PI controllers of the VOC are tuned according to the optimal approach suggested in [6].
leading to a 200 Hz bandwidth. The parameters of the converter and control system are given in Table 1.

Fig. 4 shows the responses of the VOC and the proposed DPC to various step changes in the active and reactive reference powers. It can be seen from Fig. 4 that the dynamic response of the proposed DPC is incredibly faster than the VOC. Besides its fast response, a decoupled control of the active and reactive powers is also achieved in the proposed DPC.

The harmonic spectrum of the grid current with 500 W active power at unity power factor is shown in Fig. 5. The total harmonic distortion (THD) of the proposed DPC is slightly better than the VOC.

7 Experimental results

An experimental test bench, shown in Fig. 6, is implemented to confirm the performance of the proposed scheme. The DC side of the converter is connected to a resistive load. In all experiments, the reference active power is generated from a DC-link voltage regulator and the reference reactive power is set arbitrarily. The TMS320F28335 digital signal processor (DSP) is used to implement the control algorithm. An digital-to-analogue (D/A) converter, AD7564, is used to display the measured waveforms. The DSP sends waveforms on the serial peripheral interface and AD7564 serial D/A converts these digital data to analogue signals to be displayed on the oscilloscope.

![Fig. 9 Experimental waveforms with step change of reactive power reference from 100 to 200 VAr (V_{dc} = 200 V)](image)

| a | VOC |
| b | Proposed DPC |

The experimental parameters are chosen, the same as simulations as in Table 1. The experimental results of the VOC and the proposed DPC techniques are compared in Figs. 7–9. Fig. 7 shows the steady-state waveforms, where the DC-link voltage is set to 200 V, which translates to 400 W, and the power factor is regulated at unity. The sinusoidal current waveform approves proper operation of the proposed DPC. Compared with the VOC, the proposed technique provides more precise current control with minimum distortions, even when the grid voltage is distorted. The THD for the proposed DPC and VOC is 3.9 and 4.5%, respectively, which meets the IEEE Std 519 recommendation.

The dynamic performance of the converter with the VOC and the proposed scheme are presented in Figs. 8 and 9. These results show the decoupled power control of the proposed DPC as well as the VOC scheme. From Fig. 9, the performance of the converter controlled by the proposed technique is better than the VOC regarding reactive power tracking speed. When comparing the transient behaviour with the DC-link voltage change, only a slight improvement is obtained with the proposed technique. The general conclusion drawn from the experiments is that the proposed approach is an interesting alternative to the VOC technique for grid integration of single-phase VSCs.

8 Conclusions

In this paper, a DPC technique for the single-phase grid-tie converter has been proposed which is based on the dead-beat strategy. The PI controllers of the VOC are replaced with simple algebraic equations. A fictitious phase is obtained using the SOGI scheme to improve the performance of the converter under distorted grid voltages. The grid voltage sensor is replaced with a voltage estimator. The simulation and experimental results confirm the superiority of the proposed technique in providing more precise control, better regulation performance and at the same time faster dynamic response.

9 References

9 Samercurh, S., Premrudeepreechacharn, S., Kumsuwun, Y., Higuchi, K.: ‘Power control of single-phase voltage source inverter for grid-connected