BAYES ESTIMATE OF THE PARAMETERS OF GUMBEL'S BIVARIATE EXPONENTIAL DISTRIBUTIONS

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ABSTRACT

For the Bivariate exponential distributions of type I and II are given by Gumbel (1960) we have obtained the Bayes estimate for the parameters \( \theta \) and \( \alpha \) respectively. In this case we do not get the difficulty of exponential integrals. At the end, we have given the tables for the bias and mean squared error of the Bayes estimators.

1. INTRODUCTION

Bivariate exponential distributions can be designed for the life testing of a two component system, which functions even on failure of one of the component. Two battery cells supporting an electronic device, the paired organs of human body are some examples where a bivariate exponential distribution can be applied.

As Lindley (1961) remarks, since they avoid integration over the sample space and the resulting distributional problems, Bayesian methods are often easy to apply than the usual ones, but they do bring in their wake the choice of the prior distribution. Savage (1954) believes that no prior distribution is more correct than any other in the sense that the prior distribution reflects only the beliefs of the person making the inference or decision. According to Gumbel (1960) the correlation coefficient of the variables \( X \) and \( Y \) of type I distribution is never positive and lies in the interval \(-0.5\) to 0.

Barnett (1985) had given the estimate of the parameter \( \theta \) based on coefficient of correlation. Serious difficulties were encountered in the estimation of \( \theta \). Dixit and Karadkar (1999) considered the maximum likelihood and method of moments to estimate \( \theta \). Here, we consider the prior distribution of \( \theta \) as a Beta distribution since \( \theta \) lies between 0 and 1.

2. JOINT DISTRIBUTION OF \( (X_1, Y_1, X_2, Y_2, \ldots, X_n, Y_n) \) FOR GUMBEL'S TYPE I DISTRIBUTION

\[
f(x, y, \theta) = \prod_{i=1}^{n} f(x_i, y_i, \theta)
\]
\[
\prod_{i=1}^{n} \left[ (1 + \theta x_i)(1 + \theta y_i) - \theta \right] = e^{-\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i - \theta \sum_{i=1}^{n} x_i y_i} = P e^{-\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i - \theta \sum_{i=1}^{n} x_i y_i},
\]

where

\[
P = \prod_{i=1}^{n} [(1 + \theta x_i)(1 + \theta y_i) - \theta].
\]

Let \( A_i = x_i + y_i - 1 \) and \( B_i = x_i y_i, i = 1, 2, \ldots, n \).

Therefore,

\[
P = \prod_{i=1}^{n} (1 + A_i \theta + B_i \theta^2).
\]

Case 1: Let \( n = 2 \), \( C_{1,1} = A_1 \) and \( C_{1,2} = B_1 \). Then

\[
P = (1 + A_1 \theta + B_1 \theta^2)(1 + A_2 \theta + B_2 \theta^2)
= (1 + C_{1,1} \theta + C_{1,2} \theta^2)(1 + A_2 \theta + B_2 \theta^2)
= 1 + (C_{1,1} + A_2) \theta + (B_2 + C_{1,1} A_2 + C_{1,2}) \theta^2
+ (C_{1,1} B_2 + C_{1,2} A_2) \theta^3 + C_{1,2} B_2 \theta^4
= 1 + C_{2,1} \theta + C_{2,2} \theta^2 + C_{2,3} \theta^3 + C_{2,4} \theta^4.
\]

This can be written as:

\[
(C_{2,0} \ C_{2,1} \ C_{2,2} \ C_{2,3} \ C_{2,4}) \theta = \theta' - \begin{pmatrix}
1 & 0 & 0 \\
C_{1,1} & 1 & 0 \\
C_{1,2} & C_{1,1} & 1 \\
0 & C_{1,2} & C_{1,1} \\
0 & 0 & C_{1,2}
\end{pmatrix}
\begin{pmatrix}
1 \\
A_2 \\
B_2
\end{pmatrix},
\]

where
Bayes estimate of the parameters of Gumbel's bivariate exponential distributions

\[ \theta = \begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \theta^3 \\ \theta^4 \end{pmatrix} \]

**Case 2:** Let \( n = i \). Then the matrix is as follows:

\[
\begin{pmatrix}
1 & 0 & 0 \\
C_{i-1,1} & 1 & 0 \\
C_{i-1,2} & C_{i-1,1} & 1 \\
C_{i-1,3} & C_{i-1,2} & C_{i-1,1} \\
\vdots & \vdots & \vdots \\
C_{i-1,2(i-1)} & C_{i-1,2(i-1)-1} & C_{i-1,2(i-1)-2} \\
0 & C_{i-1,2(i-1)} & C_{i-1,2(i-1)-1} \\
0 & 0 & C_{i-1,2(i-1)}
\end{pmatrix} \begin{pmatrix}
1 \\
A_i \\
B_i
\end{pmatrix}, \quad (2.5)
\]

where

\[ \theta = \begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \vdots \\ \theta^{2i} \end{pmatrix} \]

since for \( n \geq 2 \), we have

If \( k = 1 \), then

\[
\begin{cases}
C_{n,0} = 1, \\
C_{n,1} = C_{n-1,1} + A_n, \\
C_{n-1,2n-1} = C_{n-1,2n} = 0.
\end{cases} \quad (2.6)
\]
ii) If \( 2 \leq k \leq 2n \), then
\[
C_{n,k} = C_{n-1,k} + C_{n-1,k-1}A_n + C_{n-1,k-2}B_n.
\] (2.7)

Therefore,
\[
P = \prod_{i=1}^{n} (1 + A_i \theta + B_i \theta^2)
= C_{n,0} + C_{n,1} \theta + C_{n,2} \theta^2 + \ldots + C_{n,k} \theta^k + \ldots + C_{n,2n} \theta^{2n},
\] (2.8)

where \( C' s \) are given in (2.6) and (2.7).

Hence (2.1) will become
\[
f(x, y, \theta) = \sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_x - T_y - T_{xy}\theta},
\] (2.9)

where
\[
T_x = \sum_{i=1}^{n} x_i, \quad T_y = \sum_{i=1}^{n} y_i \quad \text{and} \quad T_{xy} = \sum_{i=1}^{n} x_i y_i.
\]

3. BAYES ESTIMATOR OF \( \theta \)

Let the prior distribution of \( \theta \) be \( g(\theta) \) as
\[
g(\theta) = \frac{\theta^{a-1}(1 - \theta)^{b-1}}{\beta(a,b)}, \quad 0 < \theta < 1, \quad a, b \geq 0,
\] (3.1)

where \( \beta(a,b) \) is the beta function.

Case 1): Assume that \( a = b = 1 \) and \( g(\theta) = 1 \) for \( 0 < \theta < 1 \). Then
\[
h(x, y) = \int_0^1 f(x, y | \theta) g(\theta) d\theta
= \int_0^1 \sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_x - T_y - T_{xy}\theta} d\theta
= \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \int_0^1 \theta^j e^{-T_{xy}\theta} d\theta.
\] (3.2)

We know that
\[ \int_0^1 \theta^j e^{-T_{xy} \theta} d\theta = \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[ 1 - \sum_{r=0}^j \frac{e^{-T_{xy} T_{xy}^r}}{r!} \right]. \]

Therefore,

\[ h(x, y) = \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[ 1 - \sum_{r=0}^j \frac{e^{-T_{xy} T_{xy}^r}}{r!} \right] \]  \hspace{0.5cm} (3.3)

and by using some elementary algebra

\[ h(\theta | x, y) = \frac{\sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_{xy} \theta}}{2n \sum_{j=0}^{2n} \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[ 1 - e^{-T_{xy} \sum_{r=0}^j T_{xy}^r} \right]} \]  \hspace{0.5cm} (3.4)

Thus Bayes estimate is given as

\[ \hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(j+2)}{T_{xy}^{j+2}} \left[ 1 - e^{-T_{xy} \sum_{r=0}^{j+1} T_{xy}^r} \right]}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(j+1)}{T_{xy}^{j+1}} \left[ 1 - e^{-T_{xy} \sum_{r=0}^j T_{xy}^r} \right]} \]  \hspace{0.5cm} (3.5)

**Case 2):** Let \( a > 1 \) and \( b = 1 \). By the same way as it is done in the case 1), we get the Bayes estimate

\[ \hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j+2)}{T_{xy}^{a+j+2}} \left[ 1 - e^{-T_{xy} \sum_{r=0}^{a+j+1} T_{xy}^r} \right]}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a+j+1)}{T_{xy}^{a+j+1}} \left[ 1 - e^{-T_{xy} \sum_{r=0}^a T_{xy}^r} \right]} \]  \hspace{0.5cm} (3.6)

**Case 3):** Assume that \( a \geq 1 \) and \( b > 1 \). Then

\[ h(x, y) = \int_0^1 \sum_{j=0}^{2n} C_{n,j} \theta^j e^{-T_x - T_y} \theta \frac{\theta^{-a-1} (1-\theta)^{b-1}}{\beta(a, b)} d\theta \]

\[ = \sum_{j=0}^{2n} C_{n,j} e^{-T_x - T_y} \theta^{a+j-1} (1-\theta)^{b-1} e^{-T_{xy} \theta} \beta(a, b) \]  \hspace{0.5cm} (3.7)
Consider

\[ I = \int_0^1 \frac{\theta^{a+j-1}(1-\theta)^{b-1}e^{-T_{xy}\theta}}{\beta(a,b)} d\theta \]

\[ = \int_0^1 \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i}{i! \beta(a,b)} \theta^{a+i+j-1}(1-\theta)^{b-1} d\theta \]

\[ = \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i}{i! \beta(a,b)} \int_0^1 \theta^{a+i+j-1}(1-\theta)^{b-1} d\theta. \]

Then

\[ I = \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i \beta(a+i+j,b)}{i! \beta(a,b)}. \] (3.8)

Therefore,

\[ h(x, y) = \sum_{j=0}^{2n} C_{n,j} e^{-T_x-T_y} \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i \beta(a+i+j,b)}{i! \beta(a,b)}. \] (3.9)

and posterior probability density function of \( \theta \) is

\[ h(\theta | x, y) = \frac{\sum_{j=0}^{2n} C_{n,j} \theta^{a+j-1}(1-\theta)^{b-1}e^{-T_{xy}\theta}}{\sum_{j=0}^{2n} \sum_{i=0}^{\infty} (-1)^i \frac{T_{xy}^i \beta(a+i+j,b)}{i! \beta(a,b)}}. \] (3.10)

Hence Bayes estimate for \( a \geq 1 \) and \( b > 1 \) is given as

\[ \hat{\theta}_B = \frac{\sum_{j=0}^{2n} \sum_{i=0}^{\infty} C_{n,j} (-1)^i \frac{T_{xy}^i \beta(a+i+j+1,b)}{i! \beta(a+i+j,b)}}{\sum_{j=0}^{2n} \sum_{i=0}^{\infty} C_{n,j} (-1)^i \frac{T_{xy}^i \beta(a+i+j,b)}{i! \beta(a+i+j,b)}}. \] (3.11)

Consider the confluent hypergeometric function (see Abramowitz and Stegun (1972)) as
\[1 \text{F}_1(c; d; x) = \sum_{r=0}^{\infty} \frac{\Gamma(c + r)}{\Gamma(c)} \frac{\Gamma(d)}{\Gamma(d + r)} \frac{x^r}{r!},\]  
(3.12)

and

\[1 \text{F}_1(c; d; x) = e^x 1 \text{F}_1(d - c; d; -x).\]  
(3.13)

By using the above two equations we get

\[
\hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a + j + 1)}{\Gamma(a + b + j + 1)} 1 \text{F}_1(a + j + 1; a + b + j + 1; -T_{xy})}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a + j)}{\Gamma(a + b + j)} 1 \text{F}_1(a + j; a + b + j; -T_{xy})},
\]  
(3.14)

or

\[
\hat{\theta}_B = \frac{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a + j + 1)}{\Gamma(a + b + j + 1)} 1 \text{F}_1(b; a + b + j + 1; T_{xy})}{\sum_{j=0}^{2n} C_{n,j} \frac{\Gamma(a + j)}{\Gamma(a + b + j)} 1 \text{F}_1(b; a + b + j; T_{xy})}.
\]  
(3.15)

4. **JOINT DISTRIBUTION OF** \((X_1, Y_1, X_2, Y_2, \ldots, X_n, Y_n)** **FOR GUMBEL'S TYPE II DISTRIBUTION**

\[
f(x, y; \theta) = \prod_{i=1}^{n} e^{-x_i - y_i} [1 + \alpha (2e^{-x_i} - 1)(2e^{-y_i} - 1)]
\]

\[= \prod_{i=1}^{n} [1 + \alpha (2e^{-x_i} - 1)(2e^{-y_i} - 1)] e^{-\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i}
\]

\[= Q e^{-\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i},
\]  
(4.1)

where

\[Q = \prod_{i=1}^{n} [1 + \alpha (2e^{-x_i} - 1)(2e^{-y_i} - 1)].\]

Let \(D_i = (2e^{-x_i} - 1)(2e^{-y_i} - 1)\). Then
\[ Q = \prod_{i=1}^{n} (1 + \alpha D_i). \] (4.2)

**Case 1):** Assume that \( n = 2 \) and \( E_{1,1} = D_1 \)

\[ Q = (1 + D_1 \alpha)(1 + D_2 \alpha) = \prod_{i=1}^{2} (1 + D_i \alpha) \]

\[ = (1 + E_{1,1} \alpha)(1 + D_2 \alpha) \]

\[ = 1 + (E_{1,1} + D_2) \alpha + E_{1,1} D_2 \alpha^2 \]

\[ = 1 + E_{2,1} \alpha + E_{2,2} \alpha^2 \]

\[ = E_{2,0} + E_{2,1} \alpha + E_{2,2} \alpha^2. \] (4.3)

Hence,

\[ E_{2,0} = 1, \quad E_{2,1} = E_{1,1} + D_2 \quad \text{and} \quad E_{2,2} = E_{1,1} D_2. \]

This can be written as

\[ (E_{2,0} \quad E_{2,1} \quad E_{2,2}) \alpha = \alpha' \begin{pmatrix} 1 & 0 \\ \alpha & 1 \\ \alpha^2 & \alpha \end{pmatrix} \begin{pmatrix} 1 \\ D_2 \end{pmatrix}, \] (4.4)

where

\[ \alpha = \begin{pmatrix} 1 \\ \alpha \\ \alpha^2 \end{pmatrix}. \]

**Case 2):** Assume that \( n = i \). Then

\[ Q = \prod_{j=1}^{i} (1 + \alpha D_j). \] (4.5)

So the matrix is as follows:
\[ (E_{i,0} \ E_{i,1} \ E_{i,2} \ E_{i,3} \ \cdots \ E_{i,i}) \alpha = \alpha', \]  

where
\[
\alpha = \begin{pmatrix}
1 \\
\alpha \\
\alpha^2 \\
\vdots \\
\alpha^i 
\end{pmatrix}.
\]

Hence for \( n > 2 \)

i) If \( k = 1 \), then
\[
\begin{cases}
E_{n,0} = 1, \\
E_{n,1} = E_{n-1,1} + D_n, \\
E_{n-1,1} = 0.
\end{cases}
\]

(4.7)

ii) If \( 2 \leq k \leq 2n \), then
\[
E_{n,k} = E_{n-1,k} + E_{n-1,k-1}D_n.
\]

(4.8)

Therefore,
\[
Q = \prod_{i=1}^{n} (1 + \alpha D_i)
\]

\[
= E_{n,0} + E_{n,1} \alpha + E_{n,2} \alpha^2 + \cdots + E_{n,n} \alpha^n
\]

\[
= \sum_{j=0}^{n} E_{n,j} \alpha^j,
\]

(4.9)
where $E's$ are given in (4.7) and (4.8). Hence (4.1) will become

$$f(x, y, \alpha) = \sum_{j=0}^{n} E_{n, j}{\alpha}^{j} e^{-T_x - T_y}, \tag{4.10}$$

where

$$T_x = \sum_{i=1}^{n} x_i \quad \text{and} \quad T_y = \sum_{i=1}^{n} y_i.$$

5. **BAYES ESTIMATOR OF $\alpha$**

Let the prior distribution of $\alpha$ be $g(\alpha)$, where

$$g(\alpha) = \frac{1}{2}, \quad -1 \leq \alpha \leq 1. \tag{5.1}$$

The joint distribution of $(X, Y)$ is as follows:

$$h(x, y) = \int_{-1}^{1} \sum_{j=0}^{n} E_{n, j} \frac{\alpha^{j}}{2} e^{-T_x - T_y} d\alpha$$

$$= \sum_{j=0}^{n} E_{n, j} e^{-T_x - T_y} \frac{1}{2} \int_{-1}^{1} \alpha^{j} d\alpha$$

$$= \sum_{j=0}^{n} E_{n, j} e^{-T_x - T_y} \left[ \frac{1 + (-1)^{j+1}}{2(j + 1)} \right]. \tag{5.2}$$

So the posterior distribution of $\alpha$ is

$$h(\alpha | x, y) = \frac{\sum_{j=0}^{n} E_{n, j} \alpha^{j}}{\sum_{j=0}^{n} E_{n, j} \left[ 1 + (-1)^{j} \right] \left[ 0.5(j + 1)^{-1} \right]}, \tag{5.3}$$

and Bayes estimate of $\alpha$ is

$$\hat{\alpha}_B = \frac{\sum_{j=0}^{n} E_{n, j} \left[ 1 - (-1)^{j} \right] (j + 2)^{-1}}{\sum_{j=0}^{n} E_{n, j} \left[ 1 + (-1)^{j} \right] (j + 1)^{-1}}. \tag{5.4}$$
6. CONCLUSION

We therefore conclude that Bayesian methods and consequently Bayesian estimates are advantageous as the difficulty of exponential integrals has been overcome.

Also in order to get an idea of the Bias and the mean square error ($MSE$) of the Bayes estimator of parameters, we have generated a sample of size 5, 10, ..., 50 from the Gumbel's type I and II distributions with $\theta = 0.3, 0.5, 0.7$ and $\alpha = 0.9, -0.6, -0.2, 0.2, 0.6, 0.9$, respectively. We have given Tables based on one thousand independent replication by R software.

Table 6.1: Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type-I distribution for $a = 1$ and $b = 1$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
</tr>
<tr>
<td>5</td>
<td>-0.17450</td>
<td>0.04190</td>
<td>-0.00598</td>
</tr>
<tr>
<td>10</td>
<td>-0.15501</td>
<td>0.04267</td>
<td>-0.00700</td>
</tr>
<tr>
<td>15</td>
<td>-0.13402</td>
<td>0.04122</td>
<td>-0.01302</td>
</tr>
<tr>
<td>20</td>
<td>-0.12555</td>
<td>0.04006</td>
<td>-0.02109</td>
</tr>
<tr>
<td>25</td>
<td>-0.11380</td>
<td>0.03840</td>
<td>-0.00390</td>
</tr>
<tr>
<td>30</td>
<td>-0.09835</td>
<td>0.03682</td>
<td>-0.02129</td>
</tr>
<tr>
<td>35</td>
<td>-0.08402</td>
<td>0.03102</td>
<td>-0.01112</td>
</tr>
<tr>
<td>40</td>
<td>-0.09230</td>
<td>0.03618</td>
<td>-0.02627</td>
</tr>
<tr>
<td>45</td>
<td>-0.08063</td>
<td>0.03104</td>
<td>-0.0276</td>
</tr>
<tr>
<td>50</td>
<td>-0.07315</td>
<td>0.03071</td>
<td>-0.01795</td>
</tr>
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</table>

Tables 6.1, 6.2 and 6.3 show the Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type I distribution for ($a = 1$, $b = 1$), ($a = 2$, $b = 1$) and ($a = 3$, $b = 2$), respectively. We have inserted the Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type II distribution in Tables 6.4 and 6.5 for $\alpha = -0.9, -0.6, -0.2, 0.2, 0.6, 0.9$, respectively.

Tables are shown that the $MSE$ of the Bayes estimators are decreasing when $n$ increases.
Table 6.2: Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type-I distribution for $a = 2$ and $b = 1$

<table>
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<tr>
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<th>0.7</th>
</tr>
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<tbody>
<tr>
<td>$N$</td>
<td>Bias</td>
<td>$MSE$</td>
<td>Bias</td>
</tr>
<tr>
<td>5</td>
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<td>0.17729</td>
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<td>15</td>
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<td>0.14836</td>
<td>-0.21211</td>
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<td>-0.23468</td>
<td>0.07401</td>
<td>-0.15578</td>
</tr>
<tr>
<td>50</td>
<td>-0.23228</td>
<td>0.07328</td>
<td>-0.14550</td>
</tr>
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</table>

Table 6.3: Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type-I distribution for $a = 3$ and $b = 2$

<table>
<thead>
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<th>$\theta$</th>
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<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Bias</td>
<td>$MSE$</td>
<td>Bias</td>
</tr>
<tr>
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<td>0.22440</td>
<td>-0.09516</td>
</tr>
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Table 6.4: Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type II distribution

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Table 6.5: Bias and $MSE$ of Bayes estimator of parameter of Gumbel's type II distribution

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REFERENCES


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