Thermal Vibration of Nanobeams Based on Exact Non-local Stress Theory

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Abstract

Free vibration of nanobeams including the effect of temperature change is investigated based on the exact nonlocal stress theory. By applying the variational principle, the higher order governing equation with the corresponding higher order boundary conditions is derived. The generalized differential quadrature method (GDQM) is applied to determine the fundamental frequency of nanobeam. The influences of temperature change, nonlocal parameter and slenderness ratio on the frequency of pinned-pinned and clamped-clamped nanobeams are explored and discussed in detail. Results show that with increasing the temperature change, the frequency increases at low or room temperature, while decreases at high temperature. It is observed that the frequency increases as the nonlocal parameter increases. In addition, the nonlocal effect is dependent on the temperature change. Furthermore, the thermal effect on the frequency is significant for the nanobeam with large slenderness ratio.

Keywords: nanobeam; thermal vibration; exact nonlocal stress theory.

1. Introduction

In recent years, nanobeams and carbon nanotubes (CNTs) have attracted worldwide attention due to their potential applications in nanoelectromechanical systems (NEMS). Thus, establishing an accurate model of nanobeams is a key issue for successful NEMS design. Unfortunately, the classical continuum theories are deemed to fail for these nanostructures, because the length dimensions at nano scale are often sufficiently small such that call the applicability of classical continuum theories into the question. Consequently, the classical continuum models need to be extended to consider the nanoscale effects. This can be achieved through the nonlocal elasticity theory proposed by Eringen [1] which considers the size-dependent effect. According to this theory, the stress state at a reference point is considered as a function of strain states of all points in the body. This theory involves the information about the long-range forces between atoms and introduces the internal length scale in the constitutive equation as a material parameter. Such nonlocal theory is called “partial nonlocal theory", because the equation of motion is established by direct replacing the nonlocal quantities into the classical governing equation.

A great deal of studies indicates that the mechanical properties of CNTs are related to the temperature change. The effect of temperature change on the natural frequencies of CNTs has been investigated by many researchers such as Wang et al. [2], Murmu and Pradhan [3], Zhen and Fang [4] and Chang [5]. It was concluded that with increasing the temperature change, the natural frequency increased at room or low temperature while decreased at high temperature.

According to Lim [6], the partial nonlocal beam model is not in the equilibrium state. Lim reported that in the partial nonlocal beam model, the existence of nonlocal effects reduced the stiffness of the nanostructures which is in contrary to the results of experiments, molecular dynamic simulation and strain gradient theory. He derived an exact nonlocal beam model based on variational principle in which a higher order governing equation with the corresponding higher order boundary conditions was presented and indicated that the stiffness was enhanced with increasing the nonlocal effects.

In this paper, thermal vibration of nanobeams is studied using the exact nonlocal beam model. The rest of paper is followed as: In section 2, the higher order governing equation along with the associated higher order boundary conditions are derived based on the variational principle. In section 3, the results are discussed in details. Finally, the paper concludes with a brief conclusion.

2. Analytical model

Consider a uniform nanobeam of length *L*, cross-section area *A*, Young's modulus *E*, mass per unit length *m* and inertia moment of cross-section *I*. The Cartesian coordinate system is established where the *x*-axis is the longitudinal axis, the *y*-axis is the neutral axis and the *z*-axis is the lateral axis.

According to the exact nonlocal stress theory, the nonlinear constitutive relation is expressed by [6]

$$\bar{\sigma}_{xx} = -\sum_{n=1}^{\infty} \tau^{2(n-1)} \bar{z} \frac{\mathrm{d}^2 \bar{w}}{\mathrm{d}\bar{x}^2} = \sum_{n=1}^{\infty} \tau^{2(n-1)} \varepsilon_{xx}^{\langle 2(n-1) \rangle} . \tag{1}$$

in which the dimensionless parameters are defined as

$$\bar{\sigma}_{xx} = \frac{\sigma_{xx}}{L}, \quad \tau = \frac{e_0 a}{L}, \quad \bar{w} = \frac{w}{L}, \quad \bar{x} = \frac{x}{L}, \quad \bar{z} = \frac{z}{L}. \tag{2}$$

where σ_{xx} is the nonlocal normal stress, ε_{xx} is the normal strain, $e_0 a$ is the nonlocal parameter and w is the transverse displacement.

The strain energy of a deformed nanobeam is as follows [6]

$$U = \int_V \left\{ \frac{1}{2} E \varepsilon_{xx}^2 + \frac{1}{2} E \sum_{n=1}^{\infty} (-1)^{n+1} \tau^{2n} \left(\varepsilon_{xx}^{\langle n \rangle} \right)^2 + E \sum_{n=1}^{\infty} \left\{ \tau^{2(n+1)} \sum_{m=1}^{n} \left[(-1)^{m+1} \varepsilon_{xx}^{\langle m \rangle} \varepsilon_{xx}^{\langle 2(n+1)-m \rangle} \right] \right\} \right\} \mathrm{d}V . \tag{3}$$

where V denotes the volume of the nanobeam.

The kinematic energy of the nanobeam is

$$K = \frac{m}{2} \int_0^L \left(\frac{\partial w}{\partial t} \right)^2 \mathrm{d}x = \frac{mL^3}{2T^2} \int_0^1 \left(\frac{\partial \bar{w}}{\partial \bar{t}} \right)^2 \mathrm{d}\bar{x}, \quad \bar{t} = \frac{t}{L^2} \sqrt{\frac{EI}{m}} = \frac{t}{T} . \tag{4}$$

where T is time period.

The thermal effect is considered as an axial force due to thermal expansion given by

$$N_T = \alpha T E A . \tag{5}$$

where T is the temperature change and α is the thermal expansion coefficient.

The work done by this axial force is

$$W=\frac{N_T}{2}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2\mathrm{d}x=\frac{\alpha TEAL}{2}\int_0^1\left(\frac{\partial \overline{w}}{\partial \overline{x}}\right)^2\mathrm{d}\overline{x}\,. \tag{6}$$

To satisfy the equilibrium condition, the variational principle requires that the variation of the energy functional vanishes or

$$\delta F=\delta\left(U-K-W\right)=0\,. \tag{7}$$

By substituting Eqs. (3), (4) and (6) into the above equation, the higher order equation of motion with the nonlocal and thermal effects can be expressed as

$$-\left[\sum_{n=1}^{\infty}\left(2n-3\right)\tau^{2(n-1)}\overline{w}^{\langle 2(n+1)\rangle}\right]+\frac{\alpha TAL^2}{I}\overline{w}^{\langle 2\rangle}+\ddot{\overline{w}}=0\,. \tag{8}$$

and the corresponding higher order boundary conditions are obtained as

$$\begin{aligned}
&\left[\sum_{n=1}^{\infty}\left(2n-3\right)\tau^{2(n-1)}\overline{w}^{\langle 2n+1\rangle}\right]-\frac{\alpha TAL^2}{I}\overline{w}^{\langle 1\rangle}=0 \quad or \quad \overline{w}=0,\\
&-\sum_{n=1}^{\infty}\left(2n-3\right)\tau^{2(n-1)}\overline{w}^{\langle 2n\rangle}=0 \qquad or \quad \overline{w}^{\langle 1\rangle}=0,\\
&\sum_{n=1}^{\infty}\left(2n-1\right)\tau^{2n}\overline{w}^{\langle 2n+1\rangle}=0 \qquad or \quad \overline{w}^{\langle 2\rangle}=0,\\
&\dots
\end{aligned}\,. \tag{9}$$

The first nonlocal terms in Eqs. (8) and (9) are the most important terms reflecting the nonlocal effect. Retaining these terms, the equation of motion is obtained as

$$-\tau^2\overline{w}^{\langle 6\rangle}+\overline{w}^{\langle 4\rangle}+\frac{\alpha TAL^2}{I}\overline{w}^{\langle 2\rangle}+\ddot{\overline{w}}=0\,. \tag{10}$$

and the corresponding boundary conditions are

$$\begin{aligned}
&\tau^2\overline{w}^{\langle 5\rangle}-\overline{w}^{\langle 3\rangle}-\frac{\alpha TAL^2}{I}\overline{w}^{\langle 1\rangle}=0 \quad or \quad \overline{w}=0,\\
&-\tau^2\overline{w}^{\langle 4\rangle}+\overline{w}^{\langle 2\rangle}=0 \qquad or \quad \overline{w}^{\langle 1\rangle}=0,\ .\\
&\tau^2\overline{w}^{\langle 3\rangle}-\overline{w}^{\langle 1\rangle}=0 \qquad or \quad \overline{w}^{\langle 2\rangle}=0.
\end{aligned} \tag{11}$$

Before leaving this section, it should be noticed that the exact nonlocal beam model represented by Eqs. (10) and (11) converts to the classical beam model when becomes zero.

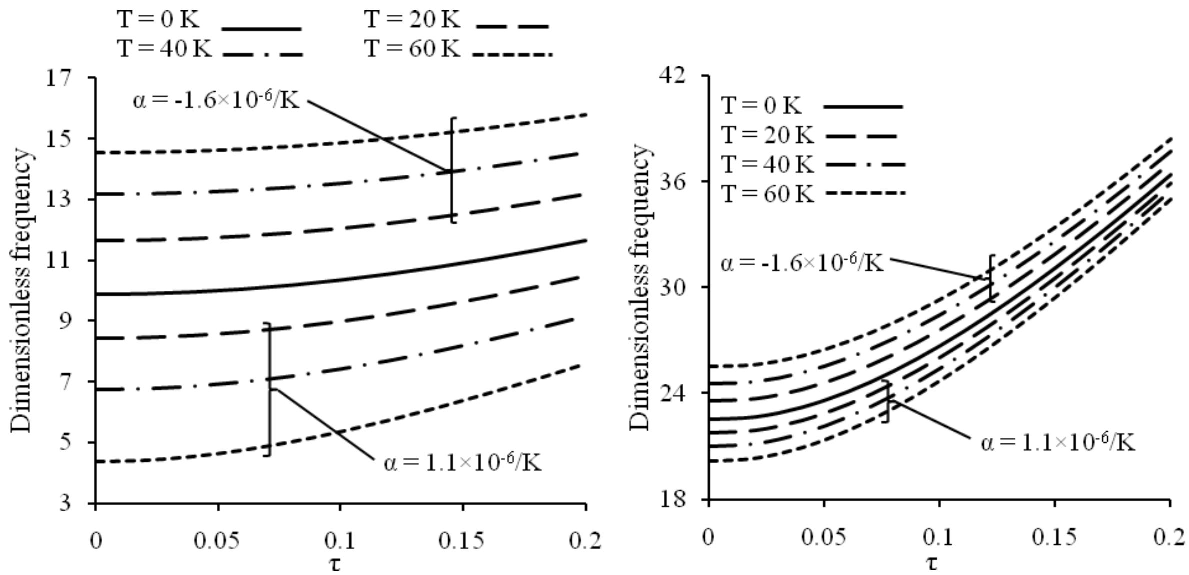
3. Results and discussion

The GDQM is utilized to discritize Eqs. (10) and (11) to obtain the fundamental frequency of a pinned-pinned and clamped-clamped nanobeam. The basis of this numerical solution method is that the different order partial derivative of a function at a given grid point is approximated by a weighted linear sum of function values at all grid points and their derivatives at any necessary grid points [7]. This method has been proposed to solve boundary value problems of more than forth order to overcome the difficulties in implementing the multiple boundary conditions at a single point in the DQM. This method has been revealed to be efficient and accurate for the static and dynamic analysis of nanobeams.

The cross-section of the nanobeam is considered to be rectangular with the ratio of thickness to the length $h/L=0.01$. The thermal expansion coefficient of the nanobeam is chosen to be -

1.6×10^{-6}/K for the case of low or room temperature and 1.1×10^{-6}/K for the case of high temperature [2].

The variation of dimensionless frequency with respect to the dimensionless nonlocal parameter for different values of temperature change in low and high temperature conditions are plotted for pinned-pinned and clamped-clamped nanobeams in Figs. 1(a) and 1(b), respectively. From these figures, it is found that with increasing the temperature change the frequency increases at low or room temperature while decreases at high temperature. Moreover, increasing the nonlocal parameter causes higher frequency which means that the existence of the nonlocal effect makes the nanobeam stiffer.



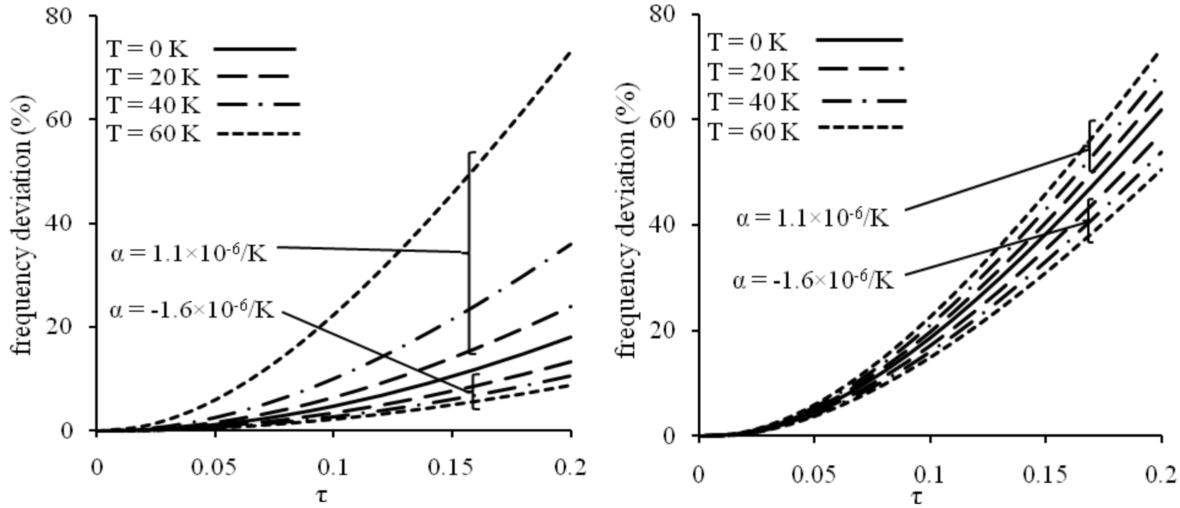
(a) (b)

Figure 1. The dimensionless frequency as a function of dimensionless nonlocal parameter for different values of temperature change in low and high temperature environment for (a) pinned-pinned and (b) clamped-clamped nanobeams.

The nonlocal effect on the frequency of pinned-pinned and clamped-clamped nanobeams is illustrated, respectively, in Figs. 2(a) and 2(b) for various values of temperature change in low and high temperature conditions. The frequency deviation percent is defined by

$$\text{frequency deviation percent} = \frac{\left(\text{frequency}_{\text{Nonlocal}}\text{-frequency}_{\text{Local}}\right)}{\text{frequency}_{\text{Local}}}\times100\,. \qquad (12)$$

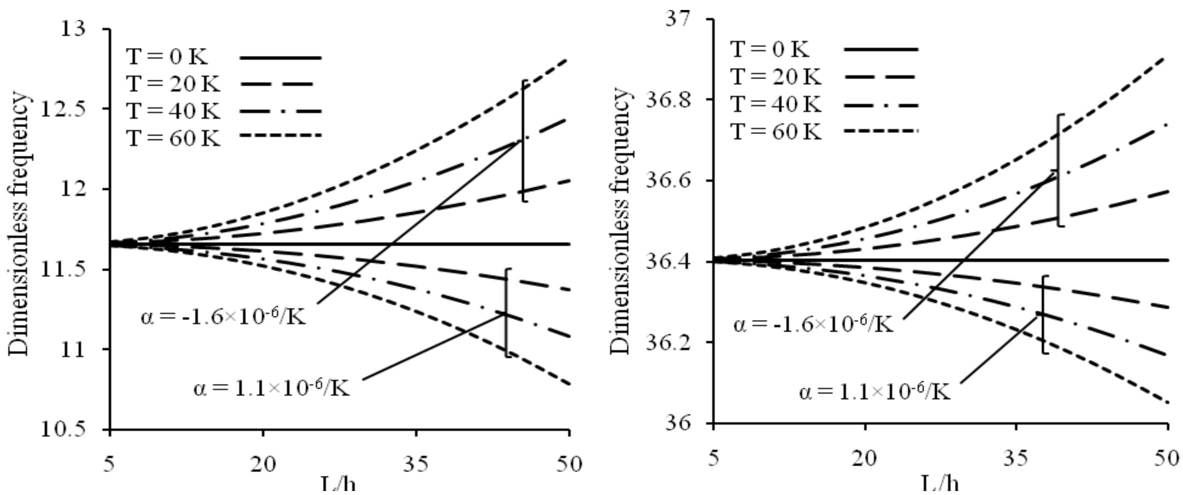
It can be seen that the frequency deviation increases as the nonlocal parameter increases. The effect of nonlocal parameter on the frequency is significantly dependent on the temperature change. For large temperature change, the difference between the local and nonlocal frequencies increases. This indicates that the thermal effect reduces the nonlocal effect at low temperature, whereas enhances the nonlocal effect at high temperature. Likewise, the effect of temperature change on the frequency is more pronounced for the pinned-pinned nanobeam rather than the clamped-clamped nanobeam especially for large nonlocal parameter. However, the nonlocal effect on the frequency for the clamped-clamped nanobeam is stronger compared to the pinned-pinned nanobeam.



(a) (b)

Figure 2. The frequency deviation versus the dimensionless nonlocal parameter for different values of temperature change in low and high temperature environment for a (a) pinned-pinned and (b) clamped-clamped nanobeam.

The dimensionless frequency varying with the slenderness ratio (L/h) for different temperature changes in low and high temperature condition are depicted in Figs. 3(a) and 3(b) for the pinned-pinned and clamped-clamped nanobeams, respectively. The dimensionless nonlocal parameter is taken as $\tau = 0.2$. From these figures, it is found that in the absence of temperature change increasing the slenderness ratio has no effect on the frequency but in the presence of temperature change, with the increase of slenderness ratio the frequency increases at low temperature while decreases at high temperature. Furthermore, the thermal effect on the frequency is slight for the nanobeam with small slenderness ratio while it is significant for the nanobeam with large slenderness ratio.



(a) (b)

Figure 3. The dimensionless frequency with respect to slenderness ratio for different values of temperature change in low and high temperature environment for a (a) pinned-pinned and (b) clamped-clamped nanobeam.

4. Conclusion

Thermo-mechanical vibration of nanobeams was analyzed using the exact nonlocal stress theory. The higher order governing equation and associated higher order boundary conditions were developed based on variational principle. The numerical solutions for thermal vibration of nanobeams were obtained by imploying the GDQM. The effecs of main parameters including the temperature change, the nonlocal parameter and the slenderness ratio on the vibration behavior of pinned-pinned and clamped-clamped nanobeams were elucidated. It was found that increasing the temperature change caused the enhancement and reduction of the frequency, respectively, at low and high temperature environment. Moreover, the nonlocal effect increased the stiffness of the nanobeam. It was indicated that the difference between the local and nonlocal frequencies increased as the temperature change increased. Likewise, the pinned-pinned and clamped-clamped nanobeams were more sensitive to the temperature change and the nonlocal parameter compared to the other one, respectively. It was also shown that increasing the slenderness ratio increased the thermal effect.

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