Adaptive Interval Type-2 Fuzzy Sliding Mode Control For a Class of Uncertain Nonlinear Systems

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ABSTRACT. A stable adaptive interval type-2 fuzzy sliding mode control for a class of nonlinear systems is investigated. The method is based on interval type-2 fuzzy logic system (IT2 FLS) to approximate unknown nonlinear functions. The use of this type-2 control requires an additional operation which is the type reduction, in comparing with typ-1 control. The proposed controller use singleton fuzzifier, product inference engine, center of sets type reduction and average defuzzifier. Based on the Lyapunov method, we design the adaptive interval type-2 fuzzy sliding mode control with mathematical proofs for stability and convergence of the closed-loop system. Two nonlinear system simulation examples are presented to verify the effectiveness of the (IT2 FLS) that is introduced.

1. Introduction

The control of nonlinear systems has been one of the important research topics; and therefore, many approaches have been proposed to date [1, 2].Generally there are two kinds of uncertainties, one caused by lake of information about structure and parameters of a system and the other caused by internal and external disturbances. One of the methods, which address uncertainty in plant, dynamics, is sliding mode control (SMC) which is a robust nonlinear discontinuous feedback control technique with the drawback of chattering. The chattering is the main drawback of SMC, which can excite undesirable high-frequency dynamics. In order to reduce chattering phenomenon, a small boundary layer is introduced around the sliding surface for better control accuracy [3]. However, the state trajectory of the resulting system may no longer converge to the sliding surface [4]. SMC also requires the general structure of the plant and parameter uncertainties to remain within known intervals. In the recent years some techniques have been emerged for control of nonlinear systems especially techniques based on fuzzy logic systems (FLS) [5-7]. It has been proven that fuzzy logic can approximate any nonlinear function to any desired accuracy because of the universal approximation theorem [8]. Fuzzy logic provides an important tool for utilization of human expert knowledge in complement to mathematical knowledge. This is mainly due to the possibility of making use of fuzzy knowledge-based control to deal with systems whose dynamics are not so well understood and whose models can not be so conveniently established [4]. Among the FLS which can handle uncertainty in plant dynamics is adaptive control. The objective of adaptive control is to introduce an adaptation law that adjusts the parameters of the controller against system uncertainties and disturbances. Many recent researches have utilized an adaptive fuzzy sliding mode control approach (AFSMC) to handle uncertainties and disturbances, such as in [9-12] which they use type-1 FLS. Quite often, the information that is used to construct the rules in a FLS is uncertain [13, 14]. Type-1 FLSs are unable to directly handle rule uncertainties, since their membership functions are type-1 fuzzy sets. On the other hand, type-2 FLSs involved in this paper whose antecedent or consequent membership functions are type-2 fuzzy sets that can handle rule uncertainties. In this paper we introduce an adaptive interval type-2 fuzzy sliding mode control for a class of uncertain SISO nonlinear systems. The proposed controller use advantage of IT2 FLS and adaptive sliding mode controller. Two nonlinear system simulations are presented to verify the effectiveness of the proposed method.

2. TYPE-2 FUZZY LOGIC SYSTEMS

A type-2 fuzzy set in universal set X is denote as $\tilde{A}$ which is characterized by a type-2 membership function in (1, 2)

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x)\,/x \qquad (1)$$

$$\mu_{\tilde{A}}(x) = \int_{u\in J_x} f_x(u)/u \quad , \quad J_x \in [0,1] \qquad (2)$$

Where $\mu_{\tilde{A}}(x)$ is called a secondary membership function (MF) or a vertical slice and $f_x(u)$ is called secondary grade. The domain of a secondary MF is called the primary membership of x noted J_x where $J_x\epsilon[0,1]$ for $\forall\, x\epsilon X$; u is a fuzzy set in [0,1] , rather than a crisp point in [0,1]. When $f_x(u) = 1$ for $\forall\, u\epsilon J_x$, then secondary membership functions are interval sets and $\tilde{A}$ in (1, 2) can be rewrite as

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x)\,/x = \int_{x\epsilon X}[\int_{u\epsilon J_x} 1/u]/x \quad , \quad J_x\epsilon[0,1] \qquad (3)$$

Interval type-2 fuzzy sets (IT2 FSs) illustrate a uniform uncertainty at the primary membership of x. Many researchers use this type of type-2 fuzzy sets because of their simplicity of calculation especially in the type reduction [17-19].An IT2 FS $\tilde{A}$ is described by its lower and upper MFs, $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$, respectively . The Footprint of uncertainty for an IT2 FS is described in terms of these MFs, as

$$\text{FOU}\ (\tilde{A}) = \cup_{x\epsilon X}[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)] \qquad (4)$$

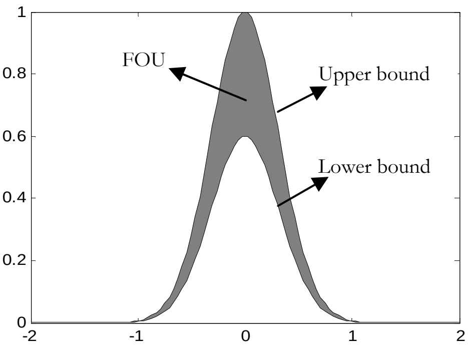

Fig.1. Interval type-2 fuzzy MF with footprint of uncertainty

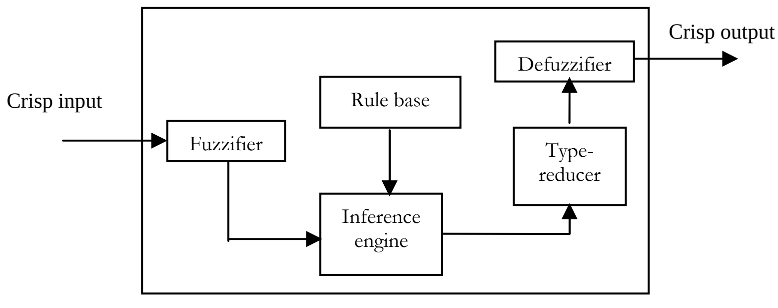

Fig.2. Structure of an interval type-2 fuzzy logic system

Fig.1 illustrates a type-2 fuzzy MF with its FOU, upper and lower bound. Generally, basic structure of an IT2 FLS consists of fuzzifier; fuzzy rule base; fuzzy inference engine; type reducer and defuzzifier. An IT2 FLS is a mapping $f: R^p \to R^1$. After fuzzification, fuzzy inference, type reduction and defuzzification, a crisp output can be obtained. Fig.2 shows the general structure of an IT2 FLS, and in following sections we will introduce each block in the IT2 FLS.

Key words and phrases: Adaptive interval type-2 fuzzy sliding mode, Uncertain nonlinear systems, Type-1 and interval type-2 fuzzy set, Interval type-2 fuzzy logic system (IT2 FLS), Type reducer.

2.1. Fuzzifier

The fuzzifier maps a crisp input into fuzzy sets, these fuzzy sets, in general, can be type-1 or type-2 fuzzy input sets $\tilde{A}$. However, in this paper we use only singleton fuzzifier which output of it will be a single point of a nonzero membership.

2.2. Rule base

The i^{th} rule in the IT2 FLS can be written as bellow

R^l : If x_1 is $\tilde{F}_1^i$ and x_2 is $\tilde{F}_2^i$ and … x_p is $\tilde{F}_p^i$ then y is $\tilde{G}^i$ l=1, 2, …, M (5)

Where x_n's and y are the input and output of the IT2 FLS respectively and $\tilde{F}_n^i$ and $\tilde{G}^i$ are the antecedent and the consequent sets respectively, which both can be IT2 FSs.

2.3. Inference engine

The inference engine combines rules and maps input vector $\underline{x}=(x_1,x_2,\ldots,x_n)^T$ to an output scalar y. by performing input and antecedent operations, the firing set will be obtain as following

$$F^i(\underline{x})=\prod_{j=1}^{n}\mu_{\tilde{F}_j^i}(x_j) \quad (6)$$

Whereas, in this paper only IT2 FSs are used and the meet operation is implemented by the product t-norm, the firing set will be as following

$$F^i(\underline{x})=[\underline{f}^i(\underline{x}),\overline{f}^i(\underline{x})] \quad (7)$$

$$\underline{f}^i(\underline{x})=\underline{\mu}_{\tilde{F}_1^i}*\underline{\mu}_{\tilde{F}_2^i}*\ldots*\underline{\mu}_{\tilde{F}_n^i} \quad (8)$$

$$\overline{f}^i(\underline{x})=\overline{\mu}_{\tilde{F}_1^i}*\overline{\mu}_{\tilde{F}_2^i}*\ldots*\overline{\mu}_{\tilde{F}_n^i} \quad (9)$$

Where $\underline{f}^i(x)$ and $\overline{f}^i(\underline{x})$ are the i^{th} lower and upper membership function, respectively. The firing set $F^i(X)$ is combined with the consequent of the i^{th} rule using the product t-norm to derive the fired output consequent sets.

2.4. Type reduction and defuzzification

Type reduction was proposed by Karnik and Mendel [14, 20]. This block is the main difference between type-1 and type-2 fuzzy logic systems. Due to output of the inference engine is type-2 fuzzy set, it must be type-reduced before defuzzifier can be used to generate a crisp output. Five different type reduction methods are described in [20] which are based on computing the centroid of an IT2 FS. Center of sets (COS) type reduction is most widely used, which is adopt in this paper, and can be expressed as

$$Y_{cos}=(Y^1,\ldots,Y^M,F^1,\ldots,F^M)=[y_l,y_r]=\int_{y^1}\cdots\int_{y^M}\int_{f^1}\cdots\int_{f^M}1\Big/\frac{\sum_{i=1}^{M}f^iy^i}{\sum_{i=1}^{M}f^i} \quad (10)$$

Where $f^i\epsilon F^i=[\underline{f}^i(X),\overline{f}^i(X)]$, $y^i\epsilon Y^i=[y_l^i,y_r^i]$.

Whereas, in this paper we use IT2 FS then Y_{cos} is the interval set determined with its left end point, y_l, and its right end point y_r.We defuzzify the set obtained from type reducer by using the average of y_l and y_r [17], therefore crisp output is :

$$y(x)=(y_l(x)+y_r(x))/2 \quad (11)$$

In general, there are no closed form formulate for y_l and y_r; however Mendel and Karnik developed two algorithm for calculating these two end points in [21]. If we use singleton fuzzifier, product inference engine and COS type reducer y_l and y_r can expressed as

$$y_l=\frac{\sum_{i=1}^{M}f^iy_l^i}{\sum_{i=1}^{M}f^i}=\theta_l^T\xi_l \quad (12)$$

Where $\theta_l^i=y_l^i$ and $\theta_l=[\theta_l^1,\ldots,\theta_l^M]^T$, $\xi_l^i=\frac{f^i}{\sum_{i=1}^{M}f^i}$ and $\xi_l=[\xi_l^1,\ldots,\xi_l^M]^T$

$$y_r=\frac{\sum_{i=1}^{M}f^iy_r^i}{\sum_{i=1}^{M}f^i}=\theta_r^T\xi_r \quad (13)$$

Where $\theta_r^i=y_r^i$ and $\theta_r=[\theta_r^1,\ldots,\theta_r^M]^T$, $\xi_r^i=\frac{f^i}{\sum_{i=1}^{M}f^i}$ and $\xi_r=[\xi_r^1,\ldots,\xi_r^M]^T$. Now we briefly provide the computation procedure for y_l.without loss of generality, suppose θ_l^i are arranged in ascending order i.e. $\theta_l^1\le\theta_l^2\le\cdots\le\theta_l^M$. After calculating θ_l and θ_r with the algorithm COS in [21], we have the following steps :

Step1: compute the y_l in (12) by initially setting $f_l^i=(\underline{f}^i+\overline{f}^i)/2$ for i=1,2,…,M where $\underline{f}^i$and $\overline{f}^i$ are computed by (8),(9).

Step2: find $\mathcal{K}$ (1≤$\mathcal{K}$≤M-1) such that $\theta_l^{\mathcal{K}}\le y_l\le\theta_l^{\mathcal{K}+1}$.

Step3: compute the y'_l in (12) by $f_l^i=\overline{f}^i$ for $i\le\mathcal{K}$ and $f_l^i=\underline{f}^i$ for $\mathcal{K}<i$.

Step4: if $y'_l\neq y_l$ set $y_l=y'_l$ and go to step2. If $y'_l=y_l$ then stop loop and y_l is equal with y'_l.

The proceeding to compute y_r is similar to compute y_l, only in step2 find $\mathcal{K}$(1≤$\mathcal{K}$≤M-1) such that $\theta_r^{\mathcal{K}}\le y_r\le\theta_r^{\mathcal{K}+1}$ and in step3, compute the y'_r in (13) by $f_r^i=\underline{f}^i$ for $i\le\mathcal{K}$ and $f_r^i=\overline{f}^i$ for $\mathcal{K}<i.\underline{f}^i$.

Now the defuzzified crisp output obtain as

$$y(x)=\frac{y_r+y_l}{2}=\frac{1}{2}(\theta_l^T\xi_l+\theta_r^T\xi_r) \quad (14)$$

3. SLIDING MODE CONTROL

Consider a general class of SISO n'th order nonlinear system as

$$x^{(n)}=f(\underline{x},t)+g(\underline{x},t)u+d(t) \quad (15)$$
$$y=x$$

Where f and g are unknown bounded nonlinear functions , where the bounds need not be known, $d(t)$ is the unknown external disturbance, $u\epsilon R$ and $y\epsilon R$ are input and output of the system, respectively, $\underline{x}=[x,\dot{x},\ldots,x^{(n-1)}]\epsilon R^n$ is the state vector of the system which is assumed to be available for measurement. We assume external disturbance $d(t)$ is bounded by a known constant D, i.e.

$$d(t) \leq D \qquad (16)$$

And assume, System (15) is controllable, $g(\underline{x},t) \neq 0$. Without loss of generality, we assume $g(\underline{x},t) > 0$, i.e. can be negative and the control can be similarly derived. The control objective is to determine a feedback control $u = u(\underline{x})$ such that the state $\underline{x}$ of the system follows the desired state vector $\underline{x_d} = [x_d, \dot{x}_d, \dots, x_d^{(n-1)}]$ in the presence of disturbances and uncertainties, that is the tracking error

$$\underline{e} = \underline{x} - \underline{x_d} = [e, \dot{e}, \dots, e^{(n-1)}]^T \qquad (17)$$

Should converge to zero. Then a sliding surface in the space of the error state can be defined as

$$s(\underline{e}) = \underline{c}^T \underline{e} = e^{(n-1)} + c_{n-1}e^{(n-2)} + \dots + c_1 e \qquad (18)$$

Where $\underline{c} = [c_1, \dots, c_{(n-2)}, c_{(n-1)}, 1]^T \epsilon R^n$ are the coefficients of the Hurwitz polynomial $h(r) = \lambda^{(n-1)} + c_{(n-1)}\lambda^{(n-2)} + \dots + c_1$, i.e. all the roots are in the open left-hand (λ is a Laplace operator). If the initial condition $\underline{e}(0) = 0$ then the tracking problem can be considered as the state error vector remaining on the sliding surface $s(\underline{e}) = 0$ for all $t \geq 0$. A sufficient condition that the system controlled is stable is given in [1] as:

$$\frac{1}{2}\frac{d}{dt}s^2(\underline{e}) \leq -\eta|s| \qquad , \eta > 0 \qquad (19)$$

Where η is a constant design parameter. Then sliding condition of (19) can be rewritten as follow

$$s\dot{s} \leq -\eta|s| \quad \text{Or} \quad \dot{s} \leq -\eta sgn(s) \qquad (20)$$

Where

$$sgn(s) = \begin{cases} 1 & for\ s > 0 \\ 0 & for\ s = 0 \\ -1 & for\ s < 0 \end{cases} \qquad (21)$$

By taking the time derivative of both sides of (18), we obtain:

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^i + x^{(n)} - x_d^{(n)} = \sum_{i=1}^{n-1} c_i e^i + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)} \quad (22)$$

Substituting (22) into (20), sliding condition can be re-expressed as:

$$\left(\sum_{i=1}^{n-1} c_i e^i + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)}\right) \leq -\eta sgn(s) \qquad (23)$$

The control problem is to obtain the optimal control input u^* which guarantees the sliding condition (23). If $f(\underline{x},t), g(\underline{x},t)$ and $d(t)$ are known, we can design the optimal sliding mode control law as below:

i) If $s = 0$

$$u^* \leq \frac{1}{g(\underline{x},t)}\left[-\sum_{i=1}^{n-1} c_i e^i - f(\underline{x},t) - d(t) + x_d^{(n)}\right] \qquad (24)$$

ii) If $s \neq 0$

$$u^* \leq \frac{1}{g(\underline{x},t)}\left[-\sum_{i=1}^{n-1} c_i e^i - f(\underline{x},t) - d(t) + x_d^{(n)} - \eta sgn(s)\right] \qquad (25)$$

Therefore optimal control u^* is

$$u^* = \frac{1}{g(\underline{x},t)}\left[-\sum_{i=1}^{n-1} c_i e^i - f(\underline{x},t) - d(t) + x_d^{(n)} - \eta_\Delta sgn(s)\right] \qquad (26)$$

Where $\eta_\Delta \geq \eta > 0$.

4. Adaptive sliding mode control based on IT2 FLS

The result in (26) is realizable only while $f(\underline{x},t)$, $g(\underline{x},t)$ and $d(t)$ are well known. Since these are unknown, we replace $f(\underline{x},t)$ and $g(\underline{x},t)$ by the IT-2 FLS $\hat{f}(\underline{x}|\theta_f)$ and $\hat{g}(\underline{x}|\theta_g)$ which are in the form (14). The resulting control input is as follow

$$u_I = u = \frac{1}{\hat{g}(\underline{x}|\theta_g)}\left[-\sum_{i=1}^{n-1} c_i e^i - \hat{f}(\underline{x}|\theta_f) + x_d^{(n)} - (\eta_\Delta + D)sgn(s)\right] \qquad (27)$$

Where D obtain from (16) and

$$\hat{f}(\underline{x}|\theta_f) = \frac{\hat{f}_l + \hat{f}_r}{2} = \frac{1}{2}\left(\theta_{fl}^T \xi_l + \theta_{fr}^T \xi_r\right) = \theta_f^T \xi_f \qquad (28)$$

$$\hat{g}(\underline{x}|\theta_g) = \frac{\hat{g}_l + \hat{g}_r}{2} = \frac{1}{2}\left(\theta_{gl}^T \xi_l + \theta_{gr}^T \xi_r\right) = \theta_g^T \xi_g \qquad (29)$$

Theorem1. Consider the nonlinear SISO system (15) and the control input *u* in (27) if the fuzzy based adaptive laws are chosen as

$$\dot{\theta}_{fl} = \gamma_1 s \xi_{fl} \qquad (30)$$

$$\dot{\theta}_{fr} = \gamma_2 s \xi_{fr} \qquad (31)$$

$$\dot{\theta}_{gl} = \gamma_3 s \xi_{gl} u \qquad (32)$$

$$\dot{\theta}_{gr} = \gamma_4 s \xi_{gr} u \qquad (33)$$

The closed loop system signals will be bounded and the tracking error will converge to zero asymptotically.

Proof. Define the optimal parameters of fuzzy systems

$$\theta_f^* = \arg\min_{\theta_f \epsilon \Omega_f}\left[\sup_{x \epsilon R^n} |\hat{f}(\underline{x}|\theta_f) - f(\underline{x},t)|\right] \qquad (34)$$

$$\theta_g^* = \arg\min_{\theta_g \epsilon \Omega_g}\left[\sup_{x \epsilon R^n} |\hat{g}(\underline{x}|\theta_g) - g(\underline{x},t)|\right] \qquad (35)$$

Where Ω_f and Ω_g are constant sets for θ_f and θ_g respectively, and they are defined as $\Omega_f = \{\theta_f \epsilon R^n \,|\, |\theta_f| \leq M_f\}$ and $\Omega_g = \{\theta_g \epsilon R^n \,|\, 0 < |\theta_g| \leq M_g\}$, where M_f and M_g are positive constant. The minimum approximation error is defined as:

$$\omega = [f(\underline{x},t) - \hat{f}(\underline{x}|\theta_f^*)] + [g(\underline{x},t) - \hat{g}(\underline{x}|\theta_g^*)]u \qquad (36)$$

Then from substituting (27) and (36) into (22), derivative of sliding surface is

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^i + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)}$$
$$= f(\underline{x},t) - \hat{f}(\underline{x}|\theta_f) + \left(g(\underline{x},t) - \hat{g}(\underline{x}|\theta_g)\right)u + d(t) - (\eta_\Delta + D)sgn(s)$$
$$= \hat{f}(\underline{x}|\theta_f^*) - \hat{f}(\underline{x}|\theta_f) + (\hat{g}(\underline{x}|\theta_g^*) - \hat{g}(\underline{x}|\theta_g))u + d(t) - (\eta_\Delta + D)sgn(s) + \omega$$
$$= (\theta_f^{*T}\xi_f - \theta_f^T\xi_f) + (\theta_g^{*T}\xi_g - \theta_g^T\xi_g)u + d(t) - (\eta_\Delta + D)sgn(s) + \omega$$
$$= \emptyset_f^T\xi_f + \emptyset_g^T\xi_f u + d(t) - (\eta_\Delta + D)sgn(s) + \omega$$
$$= \frac{1}{2}(\emptyset_{fl}^T\xi_{fl} + \emptyset_{fr}^T\xi_{fr}) + \frac{1}{2}(\emptyset_{gl}^T\xi_{gl} + \emptyset_{gr}^T\xi_{gr})u + d(t) - (\eta_\Delta + D)sgn(s) + \omega \qquad (37)$$

Where $\emptyset_f = \theta_f^* - \theta_f$ and $\emptyset_g = \theta_g^* - \theta_g$. Now the Lyapunov function is defined as:

$$V = \frac{1}{2}s^2 + \frac{1}{4\gamma_1}\emptyset_{fl}^T\emptyset_{fl} + \frac{1}{4\gamma_2}\emptyset_{fr}^T\emptyset_{fr} + \frac{1}{4\gamma_3}\emptyset_{gl}^T\emptyset_{gl} + \frac{1}{4\gamma_4}\emptyset_{gr}^T\emptyset_{gr} \qquad (38)$$

Where γ_1 and γ_2 are positive constant. The time derivative of V is:

$$\dot{V} = s\dot{s} + \frac{1}{2\gamma_1}\emptyset_{fl}^T\dot{\emptyset}_{fl} + \frac{1}{2\gamma_2}\emptyset_{fr}^T\dot{\emptyset}_{fr} + \frac{1}{2\gamma_3}\emptyset_{gl}^T\dot{\emptyset}_{gl} + \frac{1}{2\gamma_4}\emptyset_{gr}^T\dot{\emptyset}_{gr}$$
$$= s(\frac{1}{2}(\emptyset_{fl}^T\xi_{fl} + \emptyset_{fr}^T\xi_{fr}) + \frac{1}{2}(\emptyset_{gl}^T\xi_{gl} + \emptyset_{gr}^T\xi_{gr})u + d(t) - (\eta_\Delta + D)sgn(s) + \omega) +$$
$$\frac{1}{2\gamma_1}\emptyset_{fl}^T\dot{\emptyset}_{fl} + \frac{1}{2\gamma_2}\emptyset_{fr}^T\dot{\emptyset}_{fr} + \frac{1}{2\gamma_3}\emptyset_{gl}^T\dot{\emptyset}_{gl} + \frac{1}{2\gamma_4}\emptyset_{gr}^T\dot{\emptyset}_{gr} =$$
$$\frac{1}{2\gamma_1}\emptyset_{fl}^T(\dot{\emptyset}_{fl} + \gamma_1 s\xi_{fl}) + \frac{1}{2\gamma_2}\emptyset_{fr}^T(\dot{\emptyset}_{fr} + \gamma_2 s\xi_{fr}) + \frac{1}{2\gamma_3}\emptyset_{gl}^T(\dot{\emptyset}_{gl} + \gamma_3 s\xi_{gl}u) +$$
$$\frac{1}{2\gamma_4}\emptyset_{gr}^T(\dot{\emptyset}_{gr} + \gamma_4 s\xi_{gr}u) + sd(t) - s(\eta_\Delta + D)sgn(s) + s\omega \qquad (39)$$

Where $\dot{\emptyset}_{fr} = -\dot{\theta}_{fr}$, $\dot{\emptyset}_{fl} = -\dot{\theta}_{fl}$, $\dot{\emptyset}_{gr} = -\dot{\theta}_{gr}$ and $\dot{\emptyset}_{gl} = -\dot{\theta}_{gl}$, substituting (30), (31), (32) and (33) into (39), then we have

$$\dot{V} = sd(t) - s(\eta_\Delta + D)sgn(s) + s\omega = sd(t) - |s|(\eta_\Delta + D) + s\omega \le -|s|(\eta_\Delta) + s\omega \qquad (40)$$

to be based on the approximation theorem [8], it can be anticipated that the term $s\omega$ should be very small if it not equals to zero in the IT-2 FLS, we obtain :

$$\dot{V} = -|s|(\eta_\Delta) \le 0 \qquad (41)$$

Since $\underline{c} = [c_1, \ldots, c_{(n-2)}, c_{(n-1)}, 1]^T$ in which the c_i's are all real and are chosen such that $h(\lambda) = \sum_{i=1}^{n} c_i\lambda^{(i-1)}, c_n = 1$ is a Hurwitz polynomial, we have $\lim_{t\to\infty}|e(t)| = 0$, therefore $\lim_{t\to\infty}|s(e)| = 0$, the proof is completed.

5. Simulation Examples

In this section, we want to apply our proposed adaptive fuzzy controller for two examples. The first example is a regulation problem to let the output of a first order nonlinear system to track a constant trajectory. The second example is to let a second order nonlinear system to track a sin-wave trajectory.

Example 1. Consider a first order system as follow

$$\dot{x} = \frac{1-e^{-x}}{1+e^x} + u(t) \qquad (42)$$

We defined $s = e$, the desired trajectory $x_d = 0$, the initial state $x(0) = 1.5$, step size is 0.02 s and $\eta_\Delta = 0.2$. Choose four member membership over interval [-3,3] as follow

$$\begin{aligned}
\overline{\mu}_{N2} &= \frac{1}{1+\exp(5(x+2))}, & \underline{\mu}_{N2} &= \frac{0.8}{1+\exp(5(x+2))} \\
\overline{\mu}_{N1} &= \exp(-(x+1)^2), & \underline{\mu}_{N1} &= 0.8\exp(-(x+1)^2) \\
\overline{\mu}_{p1} &= \exp(-(x-1)^2), & \underline{\mu}_{p1} &= 0.8\exp(-(x-1)^2) \\
\overline{\mu}_{p2} &= \frac{1}{1+\exp(-5(x-2))}, & \underline{\mu}_{p2} &= \frac{0.8}{1+\exp(-5(x-2))}
\end{aligned} \qquad (43)$$

Where $\overline{\mu}_i$ and $\underline{\mu}_i$ are upper bound and lower bound membership functions, respectively. The initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are chosen uniformly over interval [-2,2] and [-1.6,2.4] , respectively. Let the learning rate $\gamma_1 = \gamma_1 = 40$.

Figs. 3,4 show the system response , and compare adaptive sliding mode control (ASMC) based on type-1 and interval type-2 fuzzy sets. Simulation results show effectiveness of interval type-2 method against type-1 method. From Fig.3 we can see the steady tracking error of type-2 is less than type-1.

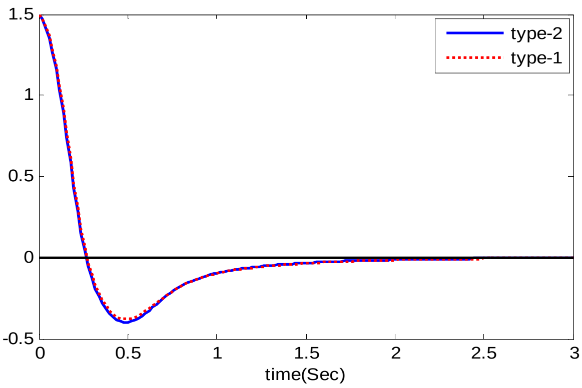

Fig.3 system output for type-1 (dot line) and interval type-2 (solid line)

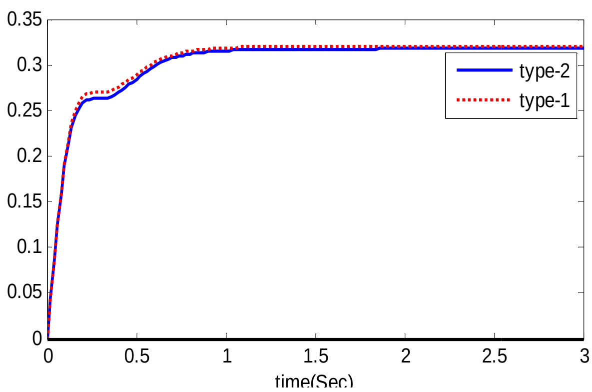

Fig.4 the tracking performance $\int_{t=0}^{3} e^2\,dt$ of type-1 (dash line) and interval type-2 (solid line)

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + u(t) + d(t) \qquad (44)$$
$$d(t) = 5\cos(t) + 0.5\sin(4t)$$

Where $d(t)$ is an unknown disturbance with known bound D=5.5. We defined $s = \dot{e} + 4e$, the desired trajectory is $x_d = \sin(t)$. The membership functions for system states x_1 and x_2 are chosen as in (43), then there are 16 rules to approximate the system function$f$. The initial state $x_1(0) = 2$, $x_2(0) = 2$, step size is 0.02 s and $\eta_\Delta = 0.2$ and the initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are chosen uniformly over interval [-2,2] and [-1.6,2.4] , respectively. Let the learning rate $\gamma_1 = \gamma_1 = 20$. Figs .5,6 ,shows the system response to the input and error signal for adaptive sliding mode control based on type-1 and interval type-2 fuzzy sets. From the simulation results it can be seen that the ASMC based on interval type-2 has better response against the type-1.

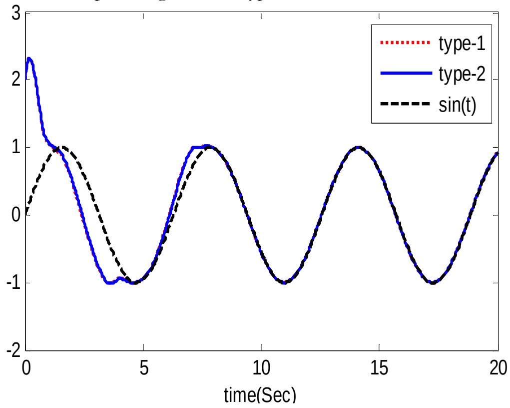

Fig.5 System output for type-1, interval type-2 and desired output.

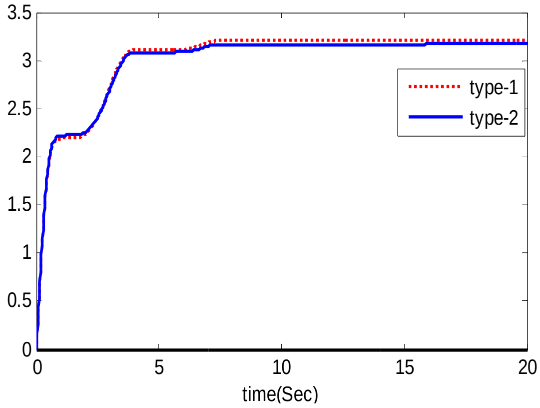

Fig.6 the tracking performance $\int_{t=0}^{20} e^2\, dt$ of type-1 (dot line) and interval type-2 (solid line)

6. Conclusions

In this paper an adaptive interval type-2 fuzzy sliding mode controller for a class of nonlinear systems designed. We introduced the type-2 fuzzy logic system to approximate the unknown nonlinear term whose antecedent and consequent membership functions are type-2 fuzzy sets that can handle rule uncertainties. The simulation results show that the controller achieves good control performance and guarantees the system stability and has good performance against the type-1 method.

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