Low-Complexity QRD-based Detection Scheme for Full Rate QOSTBC with Four-Transmit Antenna

Mohammad Towliat and Hossein Zamiri-Jafarian
Ferdowsi University of Mashhad
Mashhad, Iran
motowliat@gmail.com

Abstract— **A new low-complexity Maximum Likelihood (ML) based detection scheme is presented in this paper for full-rate Quasi Orthogonal Space Time Block Code (QOSTBC) in Multi-Input Multi-Output (MIMO) systems with four-transmit antenna. The proposed detection scheme is developed by using both lower and upper triangular versions of QR decomposition of channel matrix, without need of utilizing interference cancellation (IC) method. Simulation results show that the symbol error rate performance of the new proposed detection scheme is the same as that of the QRD-IC-based detection method but with less complexity.**

Keywords- 4×4 ***QOSTBC; QR decomposition; low complexity ML detection;***

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) wireless communication, by utilizing multiple antennas at both transmit and received sides, can potentially achieve much more performance with high data rate [1]. Orthogonal Space-Time Block Code (OSTBC) used in the MIMO transmission system, in addition to improve bit error rate, decreases the complexity of the optimal (Maximum Likelihood (ML)) detection scheme [2]. However, there is no complex OSTBC with more than two transmit antennas that achieves both full-rate and full-diversity.

By compromising on the complexity of the ML detection, Quasi-Orthogonal Space-Time Block Code (QOSTBC) has been introduced that achieves full-rate and full-diversity [3]. Although the ML detector of the OSTBC with tow-transmit antenna searches independently for each symbol, the ML detector of the QOSTBC with four-transmit antenna searches pair of symbols jointly. Thus, the complexity of the ML detection of the QOSTBC becomes significant when the high-order symbol size is used.

QR decomposition technique along with interference cancelation (IC) scheme has been used to decrease the detection complexity of the QOSTBC [4]-[5]. In the QRD-IC-based receiver, the partial ML detection method, which is a suboptimal detector, is utilized by canceling interference. Moreover, since real and imaginary parts in Jafarkhani QOSTBC [3] are independent, other way to decrease the detection complexity is to implement ML detection for real and imaginary parts of symbols independently [6]-[7].

In this paper we propose a new partial ML detection of full-rate full-diversity 4×4 QOSTBC based on using both lower and upper triangular versions of the QR decomposition (LU-QRD) simultaneously. The complexity of the new proposed method is less than that of the QRD-IC-based method; whereas the performances of both methods are the same. The remainder of this paper is organized as follows: Section II describes the challenges of 4×4 QOSTBC optimal detection. In Section III, the system model of 4×4 QOSTBC based on real and imaginary parts of received signal is presented. The conventional QRD-IC-based method for 4×4 QOSTBC detection is introduced in Section IV. The new LU-QRD method of 4×4 QOSTBC detection is proposed in Section V. Section VI provides the simulation results of symbol error rate performances of the optimal, the QRD-IC-based and the new proposed LU-QRD methods. Finally, conclusions are presented in Section VII.

II. PROBLEM DEFINITION

Consider $\mathbf{X}$ as an STBC matrix for N transmitter antennas; the size of $\mathbf{X}$ is $N\times T$, where T is the time which is spent to transmit this code. If $\mathbf{X}$ is made of K different symbols, then the code rate is defined as $R_X \triangleq \frac{K}{T}$.

When $\mathbf{X}$ is 4×4 Jafarkhani QOSTBC, which is made of the symbols $S \triangleq [s_1,s_2,s_3,s_4]$, we have

$$\mathbf{X}=\begin{bmatrix} s_1 & -s_2^* & -s_3^* & s_4 \\ s_2 & s_1^* & -s_4^* & -s_3 \\ s_3 & -s_4^* & s_1^* & -s_2 \\ s_4 & s_3^* & s_2^* & s_1 \end{bmatrix}. \quad (1)$$

This code is called full-rate due to $R_x = 1$. $\mathbf{X}$ is transmitted through the MIMO channel with M receiver antennas. Therefore the received $M\times T$ matrix, is given by

$$\mathbf{Y}=\mathbf{HX}+\mathbf{W}. \quad (2)$$

In (2), the $\mathbf{H}$ and $\mathbf{W}$ matrixes are $M\times N$ MIMO channel and $M\times T$ zero mean white Gaussian noise, with power N_0, respectively. The ML detection of vector S is obtained by

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$$
\begin{aligned}
\hat{S} &= \arg\min_{S\in\varsigma}\{\,\|\,\mathbf{Y}-\mathbf{HX}\,\|^2\,\} \\
&= \arg\min_{S\in\varsigma}\{-2\,\mathrm{Re}(Tr[\mathbf{HXY}^H])+Tr[\,\mathbf{HXX}^H\,\mathbf{H}^H\,]\}
\end{aligned} \tag{3}
$$

where $\hat{S}$ and ς are ML estimation of S and the collection of all possible scenarios of S , respectively. Note that $Tr(.)$, $\|.\|^2$ and $(.)^H$ present trace, Frobenius norm and the transposed complex conjugate operations, respectively. The number of ς elements is related to constellation size (C_s) of used modulation.

$$
\mathrm{size}(\varsigma) = C_s^4. \tag{4}
$$

Thus when C_s rises, the size of ς increases significantly. One can show if $\mathbf{X}$ is 4×4 orthogonal (i.e. $\mathbf{XX}^H = \gamma.\mathbf{I}$, where γ and $\mathbf{I}$ are constant scalar and identity matrix, respectively), the detection for each elements of S can be done separately. Hence the total search to detection of the all S symbols in complex domain, is $4C_s$. While in Jafarkhani QOSTBC, $\mathbf{XX}^H$ is given by

$$
\mathbf{XX}^H = \begin{bmatrix} \alpha & 0 & 0 & -\beta \\ 0 & \alpha & \beta & 0 \\ 0 & \beta & \alpha & 0 \\ -\beta & 0 & 0 & \alpha \end{bmatrix} \tag{5}
$$

where $\alpha = \|S\|^2$ and $\beta = 2\,\mathrm{Re}\{s_3 s_2^*\} - 2\,\mathrm{Re}(s_4 s_1^*)$. Therefore (s_1, s_4) and (s_2, s_3) have to detect jointly, and the number of the total search in complex domain becomes $2C_s^2$. Thus, by increasing the constellation size, the computational load of the QOSTBC becomes significant.

III. SYSTEM MODEL

In a MIMO system with four-transmit antenna, the received vector in m-th receiver antenna is given by

$$
Y_m = h_m \mathbf{X} + W_m \tag{6}
$$

where Y_m , h_m and W_m are 1×4 received, 1×4 cannel and 1×4 noise vectors, respectively. When *r* and *i* subscripts are used for real and imaginary parts of the related vector or matrix, respectively, the received vector Y_m can be expressed as [4].

$$
[Y_{mr}\;\; Y_{mi}] = [h_{mr}\;\; h_{mi}]\begin{bmatrix} \mathbf{X}_r & \mathbf{X}_i \\ -\mathbf{X}_i & \mathbf{X}_r \end{bmatrix} + \begin{bmatrix} W_{mr} & W_{mi} \end{bmatrix}. \tag{7}
$$

By doing some manipulations, one can represent (7) as

$$
\begin{bmatrix} y_{mr,1} \\ y_{mr,2} \\ y_{mr,3} \\ y_{mr,4} \\ y_{mi,1} \\ y_{mi,2} \\ y_{mi,3} \\ y_{mi,4} \end{bmatrix}
=
\begin{bmatrix}
h_{mr,1} & h_{mr,2} & h_{mr,3} & h_{mr,4} & -h_{mi,1} & -h_{mi,2} & -h_{mi,3} & -h_{mi,4} \\
h_{mr,2} & -h_{mr,1} & h_{mr,4} & -h_{mr,3} & h_{mi,2} & -h_{mi,1} & h_{mi,4} & -h_{mi,3} \\
h_{mr,3} & h_{mr,4} & -h_{mr,1} & -h_{mr,2} & h_{mi,3} & h_{mi,4} & -h_{mi,1} & -h_{mi,2} \\
h_{mr,4} & -h_{mr,3} & -h_{mr,2} & h_{mr,1} & -h_{mi,4} & h_{mi,3} & h_{mi,2} & -h_{mi,1} \\
h_{mi,1} & h_{mi,2} & h_{mi,3} & h_{mi,4} & h_{mr,1} & h_{mr,2} & h_{mr,3} & h_{mr,4} \\
h_{mi,2} & -h_{mi,1} & h_{mi,4} & -h_{mi,3} & -h_{mr,2} & h_{mr,1} & -h_{mr,4} & h_{mr,3} \\
h_{mi,3} & h_{mi,4} & -h_{mi,1} & -h_{mi,2} & -h_{mr,3} & -h_{mr,4} & h_{mr,1} & h_{mr,2} \\
h_{mi,4} & -h_{mi,3} & -h_{mi,2} & h_{mi,1} & h_{mr,4} & -h_{mr,3} & -h_{mr,2} & h_{mr,1}
\end{bmatrix}
\begin{bmatrix} s_{r1} \\ s_{r2} \\ s_{r3} \\ s_{r4} \\ s_{i1} \\ s_{i2} \\ s_{i3} \\ s_{i4} \end{bmatrix}
+
\begin{bmatrix} w_{mr,1} \\ w_{mr,2} \\ w_{mr,3} \\ w_{mr,4} \\ w_{mi,1} \\ w_{mi,2} \\ w_{mi,3} \\ w_{mi,4} \end{bmatrix} \tag{8}
$$

where the subscript l ($l = 1,2,3,4$), means the l-th element of the related vector. For more facility, (8) can be written as

$$
\bar{Y}_m = \bar{\mathbf{H}}_m \bar{S} + \bar{W}_m. \tag{9}
$$

Therefore in MIMO system with M receiver antennas, by defining $\bar{Y} = [\bar{Y}_1^T, \ldots, \bar{Y}_M^T]^T$, $\bar{\mathbf{H}} = [\bar{\mathbf{H}}_1^T, \ldots, \bar{\mathbf{H}}_M^T]^T$ and $\bar{W} = [\bar{W}_1^T, \ldots, \bar{W}_M^T]^T$, the Eq. (2) can be given in real domain as

$$
\bar{Y} = \bar{\mathbf{H}}\bar{S} + \bar{W}. \tag{10}
$$

The QR decomposition of $\bar{\mathbf{H}}$ can be given by

$$
\bar{\mathbf{H}} = \mathbf{QR} \tag{11}
$$

where $\mathbf{Q}$ and $\mathbf{R}$ are $8M\times 8$ unitary and 8×8 lower triangular matrixes, respectively. Thus by multiplying both sides of (10) by $\mathbf{Q}^T$, where $(.)^T$ denotes transpose of the matrix, we have

$$
P = \mathbf{R}\bar{S} + \tilde{W} \tag{12}
$$

where $P = \mathbf{Q}^T \bar{Y}$ and $\tilde{W} = \mathbf{Q}^T \bar{W}$. Due to the unitary property of $\mathbf{Q}$ ($\mathbf{Q}^T\mathbf{Q} = I$) the covariance matrix of $\tilde{W}$ remains $\frac{N_0}{2}\mathbf{I}$. The Eq. (12) can be given by

$$
\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{bmatrix}
=
\begin{bmatrix}
a & & & & & & & \\
 & -a & & & & & & \\
 & b & c & & & \mathbf{0} & & \\
b & & & -c & & & & \\
 & & & & -a & & & \\
 & & \mathbf{0} & & & a & & \\
 & & & & & b & c & \\
 & & & & b & & & -c
\end{bmatrix}
\begin{bmatrix} s_{r1} \\ s_{r2} \\ s_{r3} \\ s_{r4} \\ s_{i1} \\ s_{i2} \\ s_{i3} \\ s_{i4} \end{bmatrix}
+
\begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}_3 \\ \tilde{w}_4 \\ \tilde{w}_5 \\ \tilde{w}_6 \\ \tilde{w}_7 \\ \tilde{w}_8 \end{bmatrix}. \tag{13}
$$

where a , b and c are scalars that are related to $\bar{\mathbf{H}}$ entries. Scalar a is presented as

$$
a = \sqrt{\|\mathbf{H}\|^2 - \frac{z^2}{\|\mathbf{H}\|^2}} \tag{14}
$$

Where $z = \sum_{m=1}^{M} 2(h_{mr1}h_{mr4} - h_{mr2}h_{mr3} + h_{mi1}h_{mi4} - h_{mi2}h_{mi3})$.

IV. QRD-IC-BASED METHOD

To decrease the complexity of the QOSTBC detection, a method based on the QR decomposition of channel matrix along with interference cancelation (called QRD-IC-based method) has been proposed in [5]. To develop the QRD-IC-based method, Eq. (13) can be broken to four independent equations as

$$\begin{bmatrix} p_1 \\ p_4 \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & -c \end{bmatrix} \begin{bmatrix} s_{r1} \\ s_{r4} \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_4 \end{bmatrix}, \begin{bmatrix} p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} -a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} s_{r2} \\ s_{r3} \end{bmatrix} + \begin{bmatrix} \tilde{w}_2 \\ \tilde{w}_3 \end{bmatrix}$$
$$\begin{bmatrix} p_5 \\ p_8 \end{bmatrix} = \begin{bmatrix} -a & 0 \\ b & -c \end{bmatrix} \begin{bmatrix} s_{i1} \\ s_{i4} \end{bmatrix} + \begin{bmatrix} \tilde{w}_5 \\ \tilde{w}_8 \end{bmatrix}, \begin{bmatrix} p_6 \\ p_7 \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} s_{i2} \\ s_{i3} \end{bmatrix} + \begin{bmatrix} \tilde{w}_6 \\ \tilde{w}_7 \end{bmatrix}. \quad (15)$$

The QRD-IC-based method in QOSTBC detection has two steps as follows [5]:

1- Partial diversity detection of s_{r1}, s_{r2}, s_{i1} and s_{i2} without interference, from (15).

2- Remove the interference of the estimated symbols in step-1, then detect s_{r4}, s_{r3}, s_{i4} and s_{i3}, from (15).

For example the QRD-IC-based algorithm for (s_{r1}, s_{r4}) detection is given by

$$\text{1- } \hat{s}_{r1} = \arg\min_{s_{r1}\in\varsigma_r} \{|p_1 - a s_{r1}|^2\}$$
$$\text{2- } \hat{s}_{r4} = \arg\min_{s_{r4}\in\varsigma_r} \{|p_4 - b\hat{s}_{r1} + c s_{r4}|^2\} \quad (16)$$

where ς_r presents the collection of all possible scenarios for real parts of symbols. (s_{r2}, s_{r3}), (s_{i1}, s_{i4}) and (s_{i2}, s_{i3}) can be detected in the similar procedure. Note that the detection has been done in a real domain for both steps, thus the detection scheme becomes more simple.

As it can be seen in this method, in contrast to optimal ML detection one, the computational load does not increment in an non-linear manner by increasing the size of constellation C_s. However, because of using partial diversity ML detection in step-1, the error probability of this method becomes more than that of the optimal ML detection method.

V. PROPOSED METHOD

In this section we propose a new method to detect transmitted symbols in (13), without need of using interference cancellation. The idea is to use the lower and upper triangular versions of QR decomposition of $\bar{\mathbf{H}}$ simultaneously; thus we call it LU-QRD detection method. If the $\mathbf{Q}'\mathbf{R}'$ present upper triangular version of QR decomposition of $\bar{\mathbf{H}}$, from (10) we have

$$P' = \mathbf{R}'\bar{S} + \tilde{W}' \quad (17)$$

where $P' = \mathbf{Q}'^T\,\bar{Y}$ and $\tilde{W}' = \mathbf{Q}'^T\,\bar{W}$. Since $\mathbf{Q}'$ is a unitary matrix, the elements of $\tilde{W}'$ still remain independent. Eq. (17) can be written as

$$\begin{bmatrix} p'_1 \\ p'_2 \\ p'_3 \\ p'_4 \\ p'_5 \\ p'_6 \\ p'_7 \\ p'_8 \end{bmatrix} = \begin{bmatrix} -c & & & b & & & & \\ & c & b & & & & & \\ & & a & & & \mathbf{0} & & \\ & & & -a & & & & \\ & & & & -c & & & b \\ & \mathbf{0} & & & & c & b & \\ & & & & & & -a & \\ & & & & & & & a \end{bmatrix} \begin{bmatrix} s_{r1} \\ s_{r2} \\ s_{r3} \\ s_{r4} \\ s_{i1} \\ s_{i2} \\ s_{i3} \\ s_{i4} \end{bmatrix} + \begin{bmatrix} \tilde{w}'_1 \\ \tilde{w}'_2 \\ \tilde{w}'_3 \\ \tilde{w}'_4 \\ \tilde{w}'_5 \\ \tilde{w}'_6 \\ \tilde{w}'_7 \\ \tilde{w}'_8 \end{bmatrix} \quad (18)$$

It can be shown that the upper triangular matrix, $\mathbf{R}'$, can be determined from the lower triangular matrix $\mathbf{R}$, without extra calculation as

$$r'_{k,l} = r_{8-k+1,\,8-l+1} \quad (19)$$

where $r'_{k,l}$ and $r_{k,l}$ are (k,l)-th entry of $\mathbf{R}'$ and $\mathbf{R}$, respectively for $k,l = 1,\ldots,8$. In addition by defining $\mathbf{Q} = [\mathbf{Q}_1^T,\ldots,\mathbf{Q}_M^T]^T$ and $\mathbf{Q}' = [\mathbf{Q}'^T_1,\ldots,\mathbf{Q}'^T_M]^T$, one can show that

$$q'_{m\;k,l} = f_k \cdot f_l \cdot q_{m\;8-k+1,8-l+1} \quad (20)$$

where $q'_{m\;k,l}$ and $q_{m\;k,l}$ are (k,l)-th entry of $\mathbf{Q}'_m$ and $\mathbf{Q}_m$ respectively, for $k,l = 1,\ldots,8$ and f_k is the k-th element of $f = [1\ \text{-}1\ \text{-}1\ 1\ \text{-}1\ 1\ 1\ \text{-}1]$.

Thus, without extra calculation, upper triangular version of the QR decomposition of $\bar{\mathbf{H}}$ can be determined from its lower version. By considering both lower and upper triangular versions of the QRD of $\bar{\mathbf{H}}$, from (13) and (18), we can obtain the following relations.

$$\begin{bmatrix} p_1 \\ p_2 \\ p'_3 \\ p'_4 \end{bmatrix} = \begin{bmatrix} a & & & \mathbf{0} \\ & -a & & \\ & & a & \\ \mathbf{0} & & & -a \end{bmatrix} \begin{bmatrix} s_{r1} \\ s_{r2} \\ s_{r3} \\ s_{r4} \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}'_3 \\ \tilde{w}'_4 \end{bmatrix}$$
$$\begin{bmatrix} p_5 \\ p_6 \\ p'_7 \\ p'_8 \end{bmatrix} = \begin{bmatrix} -a & & & \mathbf{0} \\ & a & & \\ & & -a & \\ \mathbf{0} & & & a \end{bmatrix} \begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \\ s_{i4} \end{bmatrix} + \begin{bmatrix} \tilde{w}_5 \\ \tilde{w}_6 \\ \tilde{w}'_7 \\ \tilde{w}'_8 \end{bmatrix}. \quad (21)$$

As (21) illustrates, the real and imaginary parts of all symbols can be detected independently based on partial ML method. In the proposed LU-QRD method, it does not need to use the interference cancelation step of the QRD-IC-based method that requires extra computations. Therefore, the computational load of the LU-QRD method is less than that of the QRD-IC-based method. Table 1 illustrates the number of required real domain additions and multiplications for

QRD-IC-based and LU-QRD methods, after QR decomposition process (according to (11)), in a MIMO system with M receiver antennas. As it can be seen, for example when M =1, the number of required additions and multiplications in LU-QRD method are 6.67 % and 10.53 % less than that of the QRD-IC-based, respectively.

Table 1. Computational complexity of the proposed LU-QRD and QRD-IC-based methods.

QRD-based method \ Computational load	*Number of required additions*	*Number of required multiplications*
QRD-IC	$56M+4$	$64M+12$
LU-QRD	$56M$	$64M+4$

VI. Simulation Results

In this section, we simulate the MIMO system with four transmitter antennas that sends $\mathbf{X}$, the full-rate 4×4 Jafarkhani QOSTBC, through a fading MIMO channel with 1, 2 and 4 receiver antennas ($M=1,2$ and 4). The symbols of $\mathbf{X}$ are selected form QAM constellation with size $C_s=16$ and 64.

Figure 1 presents the symbol error probability versus Signal to Noise Ratio (SNR) of optimal (complete search), QRD-IC-based and LU-QRD methods for $M=1,2$ and 4 when 16-QAM modulation scheme is used. As it can be seen the performances of the QRD-IC-based method and the LU-QRD method are the same and they become very close to the performance of the optimal one for $M=4$.

Figure 2 shows the symbol error probability of three mentioned detection methods in Figure 1 when 64-QAM modulation scheme is used. As it can be seen, the performances of the QRD-IC-based method and the LU-QRD method are the same in this case as well. Moreover, by increasing the constellation size, the performance of the proposed method (also the QRD-IC-based method) becomes closer to the performance of the optimal method.

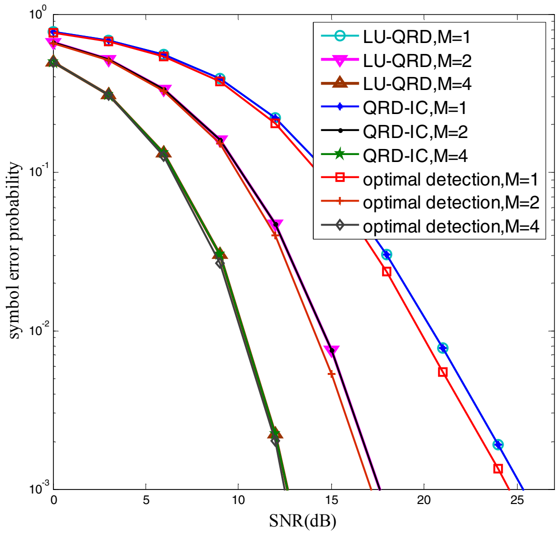

Figure 1. Symbol error probability performnces of the proposed LU-QRD method, QRD-IC-based method and optimal method for 4×4 QOSTBC detection with M=1,2,4 receiver antenas and $C_s=16$.

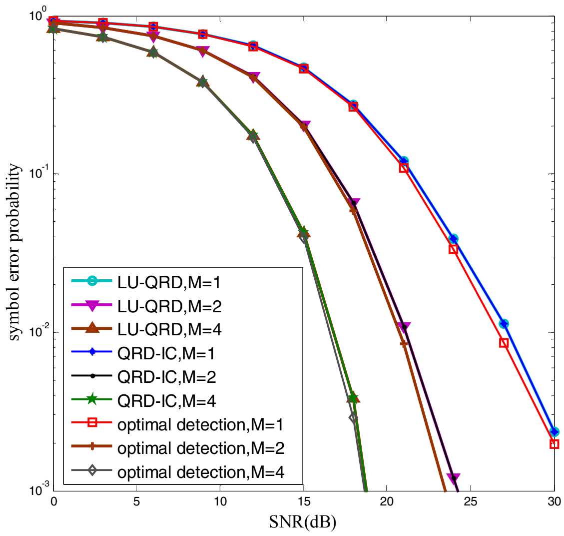

Figure 2. Symbol error probability performnces of the proposed LU-QRD method, QRD-IC-based method and optimal method for 4×4 QOSTBC detection with M=1,2,4 receiver antenas and $C_s=64$.

VII. Conclutions

In this paper we proposed a new low-complexity detection method for 4×4 QOSTBC based on using both lower and upper triangular versions of the QR decomposition of channel matrix simultaneously. The complexity of the new proposed called LU-QRD is less than that of the QR decomposition based on interference cancellation (called QRD-IC-based) method, but the performances of both methods are the same. Moreover, by increasing the number of receive antennas in the MIMO system, although the complexity of the proposed method is much lower than that of the optimal method, its error performance is very close to the performance of the optimal one.

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