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Analytical modeling of bending effect on the torsional response of electrostatically actuated micromirrors

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ABSTRACT

This paper presents analytical solutions for the nonlinear problem of electrostatically actuated torsional micromirrors considering the bending of the torsional beams. First the energy method is used for finding the equilibrium equations. Then the explicit function theorem is utilized for finding the equations governing the instability mode of the mirror. These equations are then solved using Homotopy Perturbation Method (HPM) for the especial case of $\alpha = 0$ where α is a small nondimensional geometrical parameter defining the starting point of the underneath electrodes. Then straight forward perturbation method is applied for finding the pull-in angle and pull-in displacement of the micromirror for the general case of $\alpha \neq 0$. The presented results which were in excellent agreement with numerical simulations show that neglecting the bending effect in electrostatic torsion micro actuators can lead to several hundred percent of overestimation of the stability limits of the device. In order to study the voltage-angle and voltage-displacement behavior of the micromirror, equilibrium equations are solved using HPM. Presented results are in good agreement with numerical simulations and also with experimental findings.

1. Introduction

Recently, lots of progress has been experienced in the technology of testing and fabricating new devices in N/MEMS. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them even more attractive [1,2].

Emerging role of N/MEMS in optical systems caused the creation of a new class of MEMS devices known as MicroOptoElectroMechanical Systems (MOEMS). As examples of MOEMS, micromirrors and electrostatic torsional micro actuators can be mentioned. Four types of micromirrors have been reported in the literature, deformable micromirror [3], movable micromirror [4], piston micromirror [5], and torsional micromirror [6] where the torsional micromirror is the most interesting among them [7]. Torsional micromirrors have found wide applications in the small dimensional systems. For example they are the essential element of spatial light modulators [8], digital projection displays [6] and optical crossbar swiches. Due to the diverse application of micromirrors in N/MEMS technologies, many researchers tried to investigate micromirrors behavior. Toshiyoshi and Fujita [9] developed a new type of compact micromirror used as an optical switch by silicon micromachining technique and investigated the voltage-rotation behavior of the micromirror. Degani et al. [10] studied the pull-in in electrostatic torsion actuators using polynomial algebraic approach. Behavior of their fabricated micromirrors were in good agreement with their presented model. Zhang et al. [11] presented normalized equations governing the micromirrors voltage dependent behavior. Their presented model matched well with their experimental measurements. Degani and Nemirovsky [12] presented a new approach for the direct calculations of the pull-in parameters of electrostatic actuators using a lumped two degree of freedom pull-in model. Huang et al. [7] presented a general theoretical model using the coupling effect between the torsion and bending which characterizes the static properties of the electrostatic torsional micromirror, especially its pull-in behavior. Their presented model shows that bending effect in torsional micro-actuators has undeniable effects on the statical behavior of the micromirrors.

In modeling the micromirror's behavior, primary simulation tools approach the pull-In state by iteratively adjusting the voltage applied across the actuator electrodes. The convergence rate of this scheme gradually deteriorates as the pull-in state is approached.

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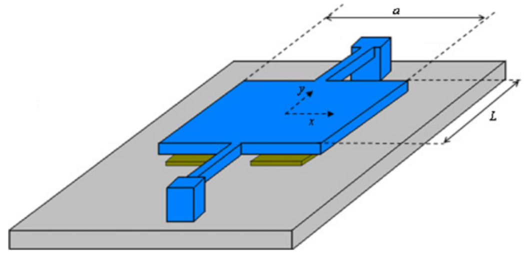

Fig. 1. Schematic view of a torsional micromirror.

The convergence of this method is inconsistent and requires many mesh and accuracy refinements to assure reliable predictions. As a result, the design procedure of electrostatically actuated MEMS devices can be time-consuming [13]. Degani et al. [13] presented a novel displacement iteration pull-in extraction (DIPIE) scheme for the problem of electrostatic torsion micro-actuators. They showed that their presented method converges 100 times faster than the voltage iteration scheme. They [14] presented experimental and theoretical study on the effect of various geometrical parameters on the electromechanical response and pull-in parameters of torsion micro actuators. They also proposed a novel rapid solver for extracting the pull-in parameters of the torsion actuator. Their proposed solver was based on Newton-Raphson scheme and their presented DIPIE algorithm.

On the other hand perturbational based methods have been widely used for modeling nonlinear problems in the diverse field of science and technology including N/MEMS. For example Younis and Nayfeh [15] investigate the response of a resonant microbeam to an electric actuation using the multiple-scale perturbation method. Abdel-Rahman and Nayfeh [16] used the same method to model secondary resonances in electrically actuated microbeams. Since perturbation methods are based upon the assumption that there is a small parameter in the equations, they have some limitations in problems without involvement of small parameters. In order to overcome this limitation a new perturbational based method, namely Homotopy Perturbation Method (HPM) was developed by He et al. [17]. His new method takes full advantages of the traditional perturbation methods and homotopy techniques. HPM has also been used for solving the nonlinear problems encountered in N/MEMS. For example, Moeenfard et al. [18] used HPM to model the nonlinear vibrational behavior of Timoshenko micobeams. Mojahedi et al. [19] applied the HPM method to simulate the static response of nano-switches to electrostatic actuation and intermolecular surface forces.

In the current paper energy methods are utilized for finding the equations governing the voltage-deflection and voltage-rotation of electrostatically actuated micromirrors and also for finding the equations governing the pull-in state and instability mode (i.e. torsion or bending) of the mirror. Then HPM is used to investigate the instability mode in the special case of $\alpha = 0$ where $\alpha = 0$ is small geometrical parameter defining the size and position of the underneath electrodes. This solution is used in a straight forward perturbation approach for finding the instability mode in the general case of $\alpha \neq 0$. HPM is employed to analytically study the voltage dependent behavior of the mirror. The presented analytical results are in good agreement with the experimental and numerical results.

2. Problem formulation

The micromirror shown in Fig. 1 is considered. When there exist a potential difference between the mirror plate and one of the underneath electrodes, the systems acts like a capacitor. The capacitance of this system can be computed by integrating the capacitance of capacitance of differential capacitors each of them having a capacity of [14]:

$$dC = \frac{\varepsilon L \cdot dx}{h_0 - \delta - x\theta} \tag{1}$$

where ε is the permittivity of the permittivity of the free space, L is the length of the micromirror, h_0 is the initial distance between the mirror and the electrode, δ is the deflection of the supporting torsion beams and θ is the torsion angle of the mirror. Using Eq. (1), one can conclude:

$$C = \int_{a_1/2}^{a_2/2} dC = \frac{\varepsilon L}{\theta_{\max}} \frac{1}{\Theta} \ln\left(\frac{1 - \Delta - \alpha\Theta}{1 - \Delta - \beta\Theta}\right) \tag{2}$$

where $\theta_{\max} = 2h_0/a$ is the maximum rotation angle of the micromirror, a is the width of the micromirror and α, β, $\triangle$ and Θ are nondimensionalized parameters defined as:

$$\alpha = \frac{a_1}{a} \tag{3}$$

$$\beta = \frac{a_2}{a} \tag{4}$$

$$\Delta = \frac{\delta}{h_0} \tag{5}$$

$$\Theta = \frac{\theta}{\theta_{\max}} \tag{6}$$

where a_1 and a_2 are some geometrical parameters defining the size and position of the electrodes as illustrated in Fig. 1. The electrical potential energy stored in the system is:

$$U_{\text{Elec}} = \frac{1}{2} C \cdot V^2 = \frac{\varepsilon L}{2\theta_{\max}} \frac{V^2}{\Theta} \ln\left(\frac{1-\Delta-\alpha\Theta}{1-\Delta-\beta\Theta}\right) \tag{7}$$

where U_{Elec} is the potential electrical energy stored in the system. Furthermore the mechanical strain energy stored in the torsion beams can be calculated as:

$$U_{\text{Mech}} = \frac{1}{2} S_0 \theta^2 + \frac{1}{2} K_0 \delta^2 = \frac{1}{2} S_0 \theta_{\max}^2 \Theta^2 + \frac{1}{2} K_0 h_0^2 \Delta^2 \tag{8}$$

where S_0 and K_0 are the effective torsional and lateral stiffness of the supporting torsion beams respectively and can be calculated as:

$$S_0 = \frac{2GI_{\text{p}}}{l} \tag{9}$$

$$\begin{aligned} P^0: \quad & L_1\left(\Delta_0, \Theta_0\right) = 0 \\ P^0: \quad & L_2\left(\Delta_0, \Theta_0\right) = 0 \end{aligned} \tag{10}$$

in these equations, l is the length of each torsion beam, G and E are the shear modulus of elasticity and young's modulus of the beams material, respectively, I_{p} is the polar moment of inertia of the beams cross section and I_{b} is the second moment of inertia of the inertia of the beams cross section around its neutral axis. For beams with rectangular cross section, I_{p} and I_{b} are [7]:

$$I_{\text{p}} = \frac{1}{12} w t^3 \tag{11}$$

$$I_{\text{p}} = \frac{1}{3} t w^3 - \frac{64}{\pi^5} w^4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tanh \frac{(2n-1)\pi t}{2w} \tag{12}$$

where t and w are the length and width of the beam's cross section, respectively.

Know the co-energy of the torsional actuator can be generally written as [12]:

$$U^* = U_{\text{Elec}} - U_{\text{Mech}} \tag{13}$$

or in more detail:

$$U^* = \frac{\varepsilon L}{2\theta_{\max}} \frac{V^2}{\Theta} \ln\left(\frac{1-\Delta-\alpha\Theta}{1-\Delta-\beta\Theta}\right) - \frac{1}{2} S_0 \theta_{\max}^2 \Theta^2 - \frac{1}{2} K_0 h_0^2 \Delta^2 \tag{14}$$

At equilibrium points, the co-energy exhibits a local extremum and so its derivative with respect to Δ and Θ must be zero.

$$\frac{\partial U^*}{\partial \Delta} = 0 \tag{15}$$

$$\frac{\partial U^*}{\partial \Theta} = 0 \tag{16}$$

which leads to the following equilibrium equations.

$$\Xi_1(\Delta, \Theta) = K_b \Delta - \frac{V^2\left((1/1-\Delta-\beta\Theta) - (1/1-\Delta-\alpha\Theta)\right)}{\Theta} = 0 \tag{17}$$

$$\Xi_2(\Delta, \Theta) = K_t \Theta - \frac{V^2\left(((1-\Delta)/(1-\Delta-\beta\Theta)) - ((1-\Delta)/(1-\Delta-\alpha\Theta) + \ln((1-\Delta-\beta\Theta)/(1-\Delta-\alpha\Theta))\right)}{\Theta^2} = 0 \tag{18}$$

where

$$K_b = \frac{2K_0 h^2 \theta_{cr}}{\varepsilon L} \tag{19}$$

$$K_t = \frac{2S_0 \theta_{cr}^3}{\varepsilon L} \tag{20}$$

Using the implicit function theorem and the equilibrium equations, one can easily show that the local maximum is reached when

$$\begin{vmatrix} \dfrac{\partial^2 U^*}{\partial \Delta^2} & \dfrac{\partial^2 U^*}{\partial \Theta \partial \Delta} \\ \dfrac{\partial^2 U^*}{\partial \Delta \partial \Theta} & \dfrac{\partial^2 U^*}{\partial \Theta^2} \end{vmatrix} = 0 \tag{21}$$

Degani and Nemirovsk [12] showed that by eliminating the voltage in Eq. (21) and equilibrium equations, these equations can be further simplified to

$$\frac{\partial U_{\text{Mech}}}{\partial \Delta} \frac{\partial C}{\partial \Theta} - \frac{\partial U_{\text{Mech}}}{\partial \Theta} \frac{\partial C}{\partial \Delta} = 0 \tag{22}$$

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$$\left(\frac{\partial^2 U_{\text{Mech}}}{\partial \Delta^2}\frac{\partial C}{\partial \Delta}-\frac{\partial U_{\text{Mech}}}{\partial \Delta}\frac{\partial^2 C}{\partial \Delta^2}\right)\left(\frac{\partial^2 U_{\text{Mech}}}{\partial \Theta^2}\frac{\partial C}{\partial \Theta}-\frac{\partial U_{\text{Mech}}}{\partial \Theta}\frac{\partial^2 C}{\partial \Theta^2}\right)-\left(\frac{\partial^2 U_{\text{Mech}}}{\partial \Delta\partial\Theta}\frac{\partial C}{\partial \Delta}-\frac{\partial U_{\text{Mech}}}{\partial \Delta}\frac{\partial^2 C}{\partial \Delta\partial\Theta}\right)^2=0 \tag{23}$$

using some algebraic manipulations, Eqs. (22) and (23) can be restated as Eqs. (24) and (25) respectively.

$$f_1\left(\Delta,\Theta,\alpha\right)=\frac{1}{1-\Delta-\alpha\Theta}-\frac{1}{1-\Delta-\beta\Theta}-\frac{\Gamma\Delta}{\Theta^2}\left(\frac{\alpha\Theta}{1-\Delta-\alpha\Theta}-\frac{\beta\Theta}{1-\Delta-\beta\Theta}+\ln\left(\frac{1-\Delta-\alpha\Theta}{1-\Delta-\beta\Theta}\right)\right)=0 \tag{24}$$

$$\begin{aligned}f_2(\Delta,\Theta,\alpha)=&\frac{1}{\Theta^3}\left(\frac{1}{1-\Delta-\beta\Theta}-\frac{1}{1-\Delta-\alpha\Theta}+\frac{\Delta}{(1-\Delta-\alpha\Theta)^2}-\frac{\Delta}{(1-\Delta-\beta\Theta)^2}\right.\\&\left(\frac{3\beta\Theta}{1-\Delta-\beta\Theta}-\frac{3\alpha\Theta}{1-\Delta-\alpha\Theta}+\frac{\alpha^2}{(1-\Delta-\alpha\Theta)^2}-\frac{\beta^2}{(1-\Delta-\beta\Theta)^2}\right)\\&\left.-\frac{\Gamma\Delta^2}{\Theta^4}\left(\frac{1}{1-\Delta-\beta\Theta}-\frac{1}{1-\Delta-\alpha\Theta}+\frac{\alpha\Theta}{(1-\Delta-\beta\Theta)^2}-\frac{\beta\Theta}{(1-\Delta-\beta\Theta)^2}\right)=0\right)^2\end{aligned} \tag{25}$$

where

$$\Gamma=\frac{K_0h_0^2}{\theta_{\max}^2S_0} \tag{26}$$

3. Analytical modeling of pull-in

3.1. Special case of $\alpha = 0$

When $\alpha = 0$, using some algebraic manipulations, equilibrium equations can be more simplified as

$$\frac{1}{\Theta^2}\left(-\Gamma\Delta\left(1-\Delta\right)\left(1-\Delta-\beta\Theta\right)\ln\left(\frac{1-\Delta}{1-\Delta-\beta\Theta}\right)-\beta\Theta\left(-\Gamma\Delta+\Gamma\Delta^2+\Theta^2\right)\right)=0 \tag{27}$$

$$\begin{aligned}&\frac{1}{\Theta^2}\left(-3\beta(2\beta\Theta\Delta-\beta\Theta+3\Delta^2-4\Delta+1)\,(1-\Delta-\beta\Theta)^2\ln\left(\frac{1-\Delta}{1-\Delta-\beta\Theta}\right)-\beta^2\Theta(-3+7\;\beta\Theta-4\beta^2\Theta^2+15\Delta-21\Delta^2\right.\\&\left.-25\beta\Delta\Theta+9\Delta^3+\beta(18+\Gamma\beta)\Theta\Delta^2+8\beta^2\Delta\Theta^2\right)=0\end{aligned} \tag{28}$$

Now, HPM is utilized for solving Eqs. (27) and (28). So, these equations are divided to a linear and a nonlinear part. The linear part of these equation can be simply obtained using a Taylor series expansion of Eqs. (27) and (28) around $(\Delta, \Theta) = (0, 0)$.

$$L_1(\Delta,\Theta)=-\beta\Theta+\frac{1}{2}\Gamma\beta^2\Delta \tag{29}$$

$$L_2(\Delta,\Theta)=\frac{1}{2}\beta^3-\frac{3}{2}\beta^4\Theta-2\beta^3\Delta \tag{30}$$

where $L_1(\Delta,\Theta)$ and $L_2(\Delta,\Theta)$ are the linear parts of Eqs. (27) and (28), respectively. Obviously the nonlinear part of these equations can be obtained by subtracting the linear part from the main equation. In other words

$$N_1(\Delta,\Theta)=\left(\frac{-\Gamma\Delta(1-\Delta)(1-\Delta-\beta\Theta)\ln((1-\Delta)/(1-\Delta-\beta\Theta))-\beta\Theta(-\Gamma\Delta+\Gamma\Delta^2+\Theta^2)}{\Theta^2}\right)+\beta\Theta-\frac{1}{2}\Gamma\beta^2\Delta \tag{31}$$

$$\begin{aligned}N_2(\Delta,\Theta)=&\frac{-3\beta(2\beta\Theta\Delta-\beta\Theta+3\Delta^2-4\Delta+1)(1-\Delta-\beta\Theta)^2}{\Theta^2}\ln\left(\frac{1-\Delta}{1-\Delta-\beta\Theta}\right)\\&-\frac{\beta^2\Theta(-3+7\;\beta\Theta-4\beta^2\Theta^2+15\Delta-21\Delta^2-25\beta\Delta\Theta+9\Delta^3+\beta(18+\Gamma\beta)\Theta\Delta^2+8\beta^2\Delta\Theta^2)}{\Theta^2}-\frac{\beta^3}{2}+\frac{3\beta^4}{2}\Theta+2\beta^3\Delta\end{aligned} \tag{32}$$

where $N_1(\Delta, \Theta)$ and $N_2(\Delta, \Theta)$ are the nonlinear parts of Eqs. (27) and (28), respectively. Now the Homotopy form is constructed.

$$L_1(\Delta,\Theta)+PN_1(\Delta,\Theta)=0 \tag{33}$$

$$L_2(\Delta,\Theta)+PN_2(\Delta,\Theta)=0 \tag{34}$$

in these equations, P is an embedded parameter. When $P = 0$ Eqs. (33) and (34) converts to a linear equation, and when $P = 1$ they convert to the Eqs. (27) and (28), respectively. In the next step, the independent variables Δ and Θ are expanded using the embedded parameter P.

$$\Delta=\Delta_0+P\Delta_1+P^2\Delta_2+\cdots \tag{35}$$

$$\Theta=\Theta_0+P\Theta_1+P^2\Theta_2+\cdots \tag{36}$$

by substituting Eqs. (35) and (36) into Eqs. (33) and (34) and finding the Taylor series expansion of the resulting equation with respect to P, the following equations are obtained.

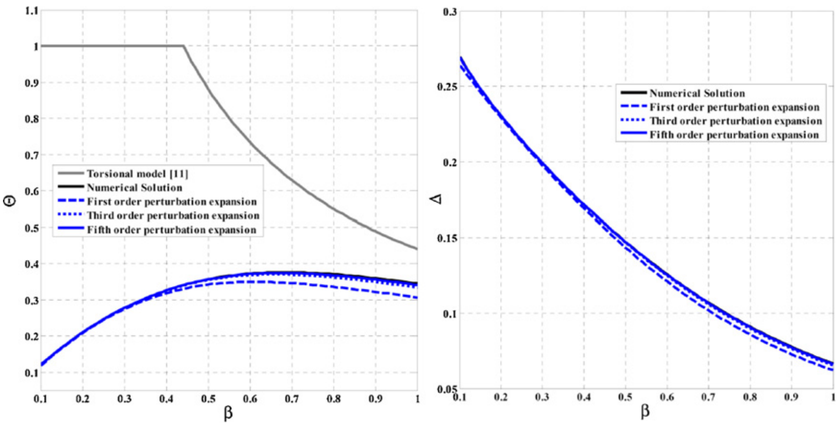

Fig. 2. The values of Θ and Δ at pull-in versus β at $\Gamma = 9$ for micromirrors with $\alpha = 0$.

$$L_1(\Delta_0,\Theta_0) + \left(\Delta_1 \frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Delta_0} + \Theta_1 \frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Theta_0} + N_1(\Delta_0,\Theta_0)\right) P + \left(\frac{1}{2}\Delta_1^2 \frac{\partial^2 L_1(\Delta_0,\Theta_0)}{\partial \Delta_0^2} + \Delta_1\Theta_1 \frac{\partial^2 L_1(\Delta_0,\Theta_0)}{\partial \Delta_0 \partial\Theta_0} + \Delta_2 \frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Delta_0} + \frac{1}{2}\Theta_1^2 \frac{\partial^2 L_1(\Delta_0,\Theta_0)}{\partial \Theta_0^2} + \Theta_2 \frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Theta_0} + \Delta_1 \frac{\partial N_1(\Delta_0,\Theta_0)}{\partial \Delta_0} + \Theta_1 \frac{\partial N_1(\Delta_0,\Theta_0)}{\partial \Theta_0}\right) P^2 + \cdots = 0 \tag{37}$$

$$L_2(\Delta_0,\Theta_0) + \left(\Delta_1 \frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Delta_0} + \Theta_1 \frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Theta_0} + N_2(\Delta_0,\Theta_0)\right) P + \left(\frac{1}{2}\Delta_1^2 \frac{\partial^2 L_2(\Delta_0,\Theta_0)}{\partial \Delta_0^2} + \Delta_1\Theta_1 \frac{\partial^2 L_2(\Delta_0,\Theta_0)}{\partial \Delta_0 \partial\Theta_0} + \Delta_2 \frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Delta_0} + \frac{1}{2}{\Theta_1}^2 \frac{\partial^2 L_2(\Delta_0,\Theta_0)}{\partial \Theta_0^2} + \Theta_2 \frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Theta_0} + \Delta_1 \frac{\partial N_2(\Delta_0,\Theta_0)}{\partial \Delta_0} + \Theta_1 \frac{\partial N_2(\Delta_0,\Theta_0)}{\partial \Theta_0}\right) P^2 + \cdots = 0 \tag{38}$$

equating the coefficients of each power of P with zero in Eqs. (37) and (38), leads to the following sets of equations:

$$\begin{aligned} P^0 &: \quad L_1(\Delta_0,\Theta_0) = 0 \\ P^0 &: \quad L_2(\Delta_0,\Theta_0) = 0 \end{aligned} \tag{39}$$

$$\begin{aligned} P^1 &: \quad \left(\frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Delta_0}\right)\Delta_1 + \left(\frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Theta_0}\right)\Theta_1 = -N_1(\Delta_0,\Theta_0) \\ P^1 &: \quad \left(\frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Delta_0}\right)\Delta_1 + \left(\frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Theta_0}\right)\Theta_1 = -N_2(\Delta_0,\Theta_0) \end{aligned} \tag{40}$$

$$\begin{aligned} P^2 &: \quad \Delta_2\left(\frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Delta_0}\right) + \Theta_2\left(\frac{\partial L_1(\Delta_0,\Theta_0)}{\partial \Theta_0}\right) = -\frac{1}{2}\Delta_1^2\frac{\partial^2 L_1(\Delta_0,\Theta_0)}{\partial \Delta_0^2} - \Delta_1\Theta_1\frac{\partial^2 L_1(\Delta_0,\Theta_0)}{\partial \Delta_0\partial\Theta_0} - \frac{1}{2}\Theta_1^2\frac{\partial^2 L_1(\Delta_0,\Theta_0)}{\partial \Theta_0^2} - \Delta_1\frac{\partial N_1(\Delta_0,\Theta_0)}{\partial \Delta_0} - \Theta_1\frac{\partial N_1(\Delta_0,\Theta_0)}{\partial \Theta_0} \\ P^2 &: \quad \Delta_2\left(\frac{\partial L_2(\Delta_0,\Theta_0)}{\partial \Delta_0}\right) + \Theta_2\left(\frac{\partial L_2\left(\Delta_0,\Theta_0\right)}{\partial \Theta_0}\right) = -\frac{1}{2}\Delta_1^2\frac{\partial^2 L_2(\Delta_0,\Theta_0)}{\partial \Delta_0^2} - \Delta_1\Theta_1\frac{\partial^2 L_2(\Delta_0,\Theta_0)}{\partial \Delta_0\partial\Theta_0} - \frac{1}{2}\Theta_1^2\frac{\partial^2 L_2(\Delta_0,\Theta_0)}{\partial \Theta_0^2} - \Delta_1\frac{\partial N_2(\Delta_0,\Theta_0)}{\partial \Delta_0} - \Theta_1\frac{\partial N_2(\Delta_0,\Theta_0)}{\partial \Theta_0} \end{aligned} \tag{41}$$

Eqs. (39)–(41) can be solved iteratively for finding (Δ_i, Θ_i), $0 \le i \le 2$. Then Δ and Θ are obtained by substituting $P = 1$ and (Δ_i, Θ_i), $0 \le i \le 2$ in Eqs. (35) and (36), respectively. The accuracy of the obtained results can be further improved by using higher perturbation expansions in Eqs. (37) and (38).

In Figs. 2 and 3 the pull-in angle and pull-in displacement of the micromirror is plotted against β for $\alpha = 0$. It is observed that a fifth order perturbation expansion gives sufficiently precise results. In addition, one can conclude that ignoring the bending effect of torsion beams can lead to significantly large errors up to several hundred percent in the pull-in angle especially at low values of β. Comparing Figs. 2 and 3, shows that at small values of Γ which is the sign of low bending strength of the torsion beams, the dominant instability mode of the micromirror is the bending mode, while at large values of Γ, the dominant instability mode, is the torsion mode. It is also observed that at low values of β, the bending instability mode is strengthened.

General case of $\alpha \ge 0$

Since the value of α is usually small, it can be used as perturbation parameter. Doing so, Δ and Θ are expanded as:

$$\Delta = \Delta'_0 + \alpha\Delta'_1 + \alpha^2\Delta'_2 + O(\alpha^3) \tag{42}$$

$$\Theta = \Theta'_0 + \alpha\Theta'_1 + \alpha^2\Theta'_2 + O(\alpha^3) \tag{43}$$

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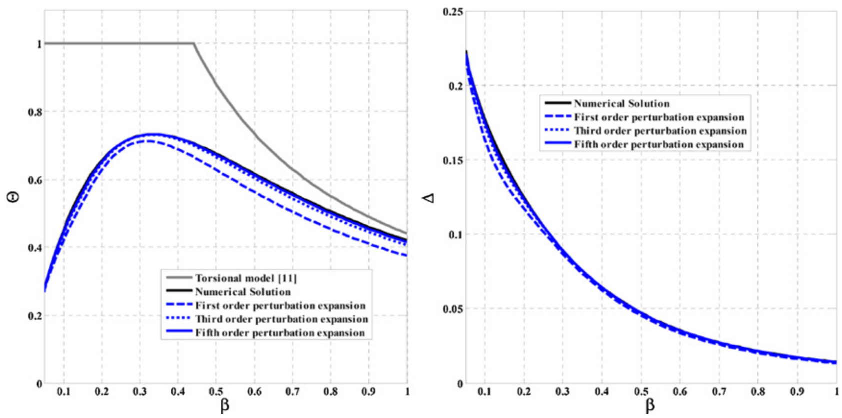

Fig. 3. The values of Θ and Δ at pull-in versus β at $\Gamma = 9$ for micromirrors with $\alpha = 0$.

these equations are then substituted in Eqs. (24) and (25) and the Taylor series expansion of the resulting equations with respect to α are then obtained which results in Eqs. (44) and (45), respectively.

$$\begin{aligned} & f_1(\Delta'_0, \Theta'_0, 0) + \left(\Delta'_1 \frac{\partial f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'_0} + \Theta'_1 \frac{\partial f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Theta'_0} + \left. \frac{\partial f_1\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \alpha} \right]_{\alpha=0} \right) \alpha \\ & + \left(\Delta'_2 \frac{\partial f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'_0} + \Theta'_2 \frac{\partial f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Theta'_0} + \Delta'_1 \Theta'_1 \frac{\partial^2 f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'_0 \partial \Theta'_0} + \left. \frac{1}{2} \frac{\partial^2 f_1\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \alpha^2} \right]_{\alpha=0} \right. \\ & \left. + \frac{1}{2} \Delta'^2_1 \frac{\partial^2 f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'^2_0} + \Delta'_1 \frac{\partial^2 f_1\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \Delta'_0 \partial \alpha} + \frac{1}{2} \Theta'^2_1 \frac{\partial^2 f_1\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Theta'^2_0} + \left. \Theta'_1 \frac{\partial^2 f_1\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \Theta'_0 \partial \alpha} \right]_{\alpha=0} \right) \alpha^2 + \ldots = 0 \end{aligned} \tag{44}$$

$$\begin{aligned} & f_2(\Delta'_0, \Theta'_0, 0) + \left(\Delta'_1 \frac{\partial f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0} + \Theta'_1 \frac{\partial f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'_0} + \left. \frac{\partial f_2(\Delta'_0, \Theta'_0, \alpha)}{\partial \alpha} \right]_{\alpha=0} \right) \alpha \\ & + \left(\Delta'_2 \frac{\partial f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0} + \Theta'_2 \frac{\partial f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'_0} + \Delta'_1 \Theta'_1 \frac{\partial^2 f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0 \partial \Theta'_0} + \left. \frac{1}{2} \frac{\partial^2 f_2(\Delta'_0, \Theta'_0, \alpha)}{\partial \alpha^2} \right]_{\alpha=0} \right. \\ & \left. + \frac{1}{2} \Delta'^2_1 \frac{\partial^2 f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'^2_0} + \Delta'_1 \frac{\partial^2 f_2(\Delta'_0, \Theta'_0, \alpha)}{\partial \Delta'_0 \partial \alpha} + \frac{1}{2} \Theta'^2_1 \frac{\partial^2 f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'^2_0} + \left. \Theta'_1 \frac{\partial^2 f_2(\Delta'_0, \Theta'_0, \alpha)}{\partial \Theta'_0 \partial \alpha} \right]_{\alpha=0} \right) \alpha^2 + \ldots = 0 \end{aligned} \tag{45}$$

By setting the coefficients of each power of α with zero the following sets of equations are obtained:

$$\begin{aligned} \alpha^0: \quad & f_1(\Delta'_0, \Theta'_0, 0) = 0 \\ \alpha^0: \quad & f_2(\Delta'_0, \Theta'_0, 0) = 0 \end{aligned} \tag{46}$$

$$\begin{aligned} \alpha^1: \quad & \Delta'_1 \frac{\partial f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0} + \Theta'_1 \frac{\partial f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'_0} = - \left. \frac{\partial f_1(\Delta'_0, \Theta'_0, \alpha)}{\partial \alpha} \right]_{\alpha=0} \\ \alpha^1: \quad & \Delta'_1 \frac{\partial f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0} + \Theta'_1 \frac{\partial f_2(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'_0} = - \left. \frac{\partial f_2(\Delta'_0, \Theta'_0, \alpha)}{\partial \alpha} \right]_{\alpha=0} \end{aligned} \tag{47}$$

$$\begin{aligned} \alpha^2: \quad & \Delta'_2 \frac{\partial f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0} + \Theta'_2 \frac{\partial f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'_0} = -\Delta'_1 \Theta'_1 \frac{\partial^2 f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'_0 \partial \Theta'_0} - \left. \frac{1}{2} \frac{\partial^2 f_1(\Delta'_0, \Theta'_0, \alpha)}{\partial \alpha^2} \right]_{\alpha=0} \\ & - \frac{1}{2} \Delta'^2_1 \frac{\partial^2 f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Delta'^2_0} - \Delta'_1 \frac{\partial^2 f_1(\Delta'_0, \Theta'_0, \alpha)}{\partial \Delta'_0 \partial \alpha} - \frac{1}{2} \Theta'^2_1 \frac{\partial^2 f_1(\Delta'_0, \Theta'_0, 0)}{\partial \Theta'^2_0} - \left. \Theta'_1 \frac{\partial^2 f_1(\Delta'_0, \Theta'_0, \alpha)}{\partial \Theta'_0 \partial \alpha} \right]_{\alpha=0} \\ \alpha^2: \quad & \Delta'_2 \frac{\partial f_2\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'_0} + \Theta'_2 \frac{\partial f_2\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Theta'_0} = -\Delta'_1 \Theta'_1 \frac{\partial^2 f_2\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'_0 \partial \Theta'_0} - \left. \frac{1}{2} \frac{\partial^2 f_2\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \alpha^2} \right]_{\alpha=0} \\ & - \frac{1}{2} \Delta'^2_1 \frac{\partial^2 f_2\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Delta'^2_0} - \Delta'_1 \frac{\partial^2 f_2\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \Delta'_0 \partial \alpha} - \frac{1}{2} \Theta'^2_1 \frac{\partial^2 f_2\left(\Delta'_0, \Theta'_0, 0\right)}{\partial \Theta'^2_0} - \left. \Theta'_1 \frac{\partial^2 f_2\left(\Delta'_0, \Theta'_0, \alpha\right)}{\partial \Theta'_0 \partial \alpha} \right]_{\alpha=0} \end{aligned} \tag{48}$$

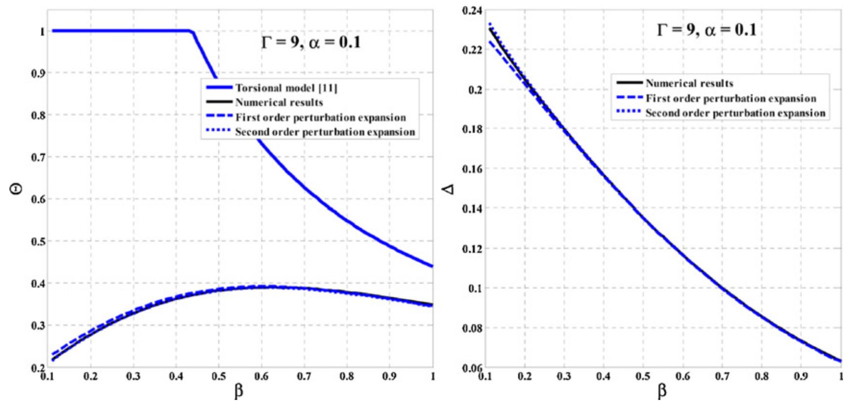

Fig. 4. The values of Θ and Δ at pull-in versus β at $\Gamma = 9$ for micromirrors with $\alpha = 0.1$.

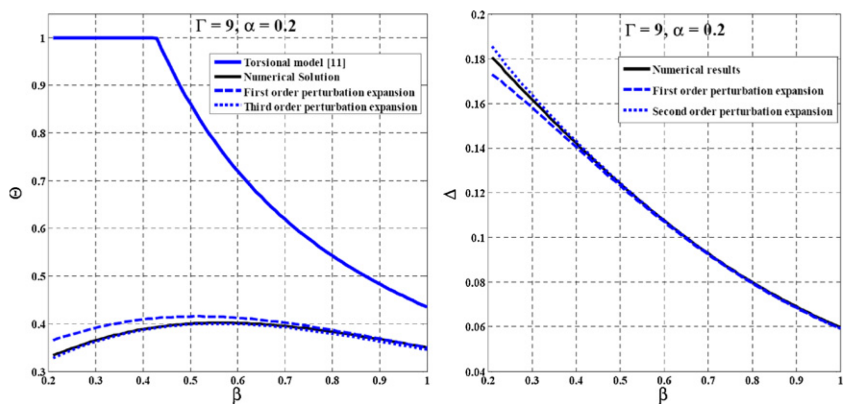

Fig. 5. The values of Θ and Δ at pull-in versus β at $\Gamma = 9$ for micromirrors with $\alpha = 0.2$.

the solution of Eq. (46) is the solution of Eqs. (27) and (28) for the special case of $\alpha = 0$ whose solution was obtained in the in the previous step. So Δ'_0 and Θ'_0 are already known. Then Eqs. (47) and (48) can be solved iteratively for finding sets of (Δ_i, Θ_i), $1 \le i \le 2$. The accuracy of the obtained results can be further improved by using higher order perturbation expansions.

In Figs. 4–6, the pull-in angle and pull-in displacement of micromirrors with different nonzero values of α is presented. It can be seen that the convergence of the perturbation method is so fast that even a second order perturbation expansion is sufficient for obtaining a highly precise response. Obviously, the lower the value of α, the faster the perturbed response converge. Again, just like the special case of $\alpha = 0$, neglecting the bending effect can cause several hundred percent error in the pull-in angle and pull-in displacement of the micromirror.

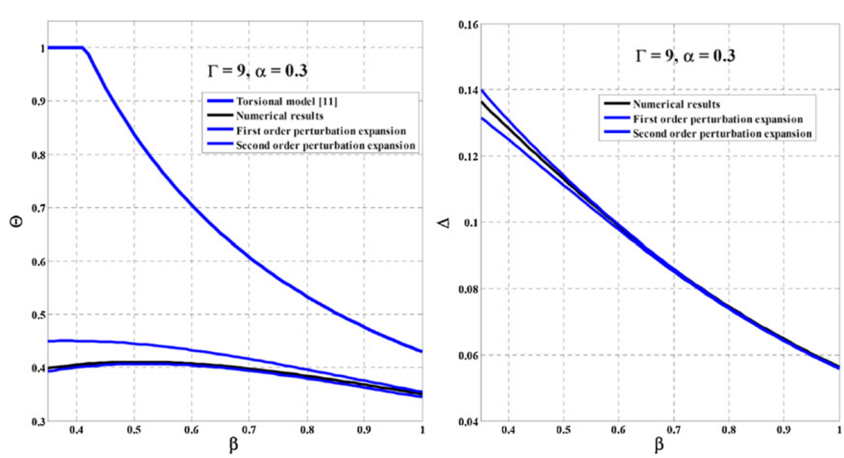

Fig. 6. The values of Θ and Δ at pull-in versus β at $\Gamma = 9$ for micromirrors with $\alpha = 0.3$.

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Table 1
Parameters of the micromirror under investigation.

Property	Symbol	Value
Shear modulus of elasticity	G	66 Gpa
Poison ratio of the torsional beams material (polysilicon)	υ	0.29
Micromirror width	a	100 μm
Micromirror length	L	100 μm
Torsional beam length	l	65 μm
Torsional beam width	w	1.55 μm
Torsional beam thickness	t	1.5 μm
Electrode parameter	a_1	6 μm
Electrode parameter	a_2	84 μm
Initial distance between the micromirror and the underneath electrodes	h	2.75 μm

4. Voltage dependent behavior of micromirror

In this section, HPM is utilized for finding analytic solutions of voltage dependent behavior of micromirror. For this purpose, the Taylor expansion is utilized to find the linear part of Eqs. (17) and (18).

$$L_1'(\Delta,\Theta) = -V^2(\beta-\alpha) + (K_b - 2V^2(\beta-\alpha))\Delta - V^2(\beta^2-\alpha^2)\Theta \tag{49}$$

$$L_2'(\Delta,\Theta) = -V^2\frac{(\beta^2-\alpha^2)}{2} - V^2(\beta^2-\alpha^2)\Delta + \left(K_t - \frac{2}{3}(\beta^3-\alpha^3)V^2\right)\Theta \tag{50}$$

obviously the nonlinear parts of Eqs. (49) and (50) are obtained by subtracting $L_1'(\Delta,\Theta)$ from $\Xi_1(\Delta,\Theta)$ and $L_2'(\Delta,\Theta)$ from $\Xi_2(\Delta,\Theta)$ respectively.

$$N_1'(\Delta,\Theta) = \Xi_1(\Delta,\Theta) - L_1'(\Delta,\Theta) \tag{51}$$

$$N_2'(\Delta,\Theta) = \Xi_2(\Delta,\Theta) - L_2'(\Delta,\Theta) \tag{52}$$

now the homotopy form is constructed using the embedded parameter P:

$$L_1'(\Delta,\Theta) + P\cdot N_1'(\Delta,\Theta) = 0 \tag{53}$$

$$L_2'(\Delta,\Theta) + P\cdot N_2'(\Delta,\Theta) = 0 \tag{54}$$

Furthermore, the independent variables (i.e. Δ and Θ) are expanded as:

$$\Delta = \Delta_0 + P\Delta_1 + \ldots \tag{55}$$

$$\Theta = \Theta_0 + P\Theta_1 + \ldots \tag{56}$$

by substituting Eqs. (55) and (56) into Eqs. (53) and (54) and finding the Taylor series expansions of the resulting expressions with respect to P, the following equations are obtained.

$$L_1'(\Delta_0,\Theta_0) + \left(\Delta_1\frac{\partial L_1'(\Delta_0,\Theta_0)}{\partial\Delta_0} + \Theta_1\frac{\partial L_1'(\Delta_0,\Theta_0)}{\partial\Theta_0} + N_1'(\Delta_0,\Theta_0)\right)P + O(P^2) = 0 \tag{57}$$

$$L_2'(\Delta_0,\Theta_0) + \left(\Delta_1\frac{\partial L_2'(\Delta_0,\Theta_0)}{\partial\Delta_0} + \Theta_1\frac{\partial L_2'(\Delta_0,\Theta_0)}{\partial\Theta_0} + N_2'(\Delta_0,\Theta_0)\right)P + O(P^2) = 0 \tag{58}$$

equating the coefficient of each power of P is Eqs. (57) and (58) with zero leads to:

$$\begin{aligned} P^0:&\quad L_1'(\Delta_0,\Theta_0) = 0 \\ P^0:&\quad L_2'(\Delta_0,\Theta_0) = 0 \end{aligned} \tag{59}$$

$$\begin{aligned} P^1:&\quad \Delta_1\frac{\partial L_1'(\Delta_0,\Theta_0)}{\partial\Delta_0} + \Theta_1\frac{\partial L_1'(\Delta_0,\Theta_0)}{\partial\Theta_0} = -N_1'(\Delta_0,\Theta_0) \\ P^1:&\quad \Delta_1\frac{\partial L_2'(\Delta_0,\Theta_0)}{\partial\Delta_0} + \Theta_1\frac{\partial L_2'(\Delta_0,\Theta_0)}{\partial\Theta_0} = -N_2'(\Delta_0,\Theta_0) \end{aligned} \tag{60}$$

Equations (59) and (60) can be solved iteratively for finding Δ_i and Θ_i, $0 \le i \le 1$. By substituting $P=1$ and Δ_i and Θ_i, $0 \le i \le 1$ in Eqs. (55) and (56), Δ and Θ are obtained. The accuracy of the given solution can be further improved by using higher order perturbation expansions in Eqs. (55) and (56). For verification purpose, a micromirror with characteristics given in Table 1 is considered.

In Fig. 7 the results of the presented model has been compared with numerical results and with experimental findings of Huang et al. [7]. It is observed that even a first order perturbation approximation precisely follow the numerical results especially at voltages less than 16V and the analytical results obtained from a fifth order perturbation expansion has almost no error. In addition, it can be seen that the analytical and numerical results well follow the experimental results. Fig. 7 also shows that disregarding the effect of bending in modeling would induce significant errors in the results.

In Fig. 8, the analytical results of dimensionless deflection Δ is compared with those of numerical findings. Again, it is seen that analytical results closely follow numerical ones.

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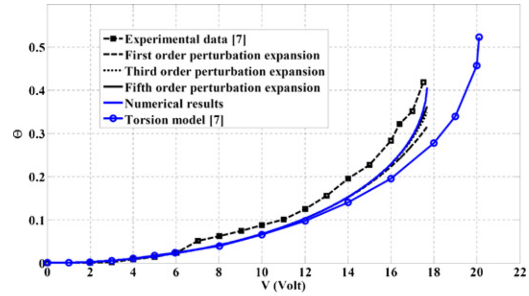

Fig. 7. Voltage-angle behavior of torsional micromirrors, comparison of experimental, numerical and analytical results.

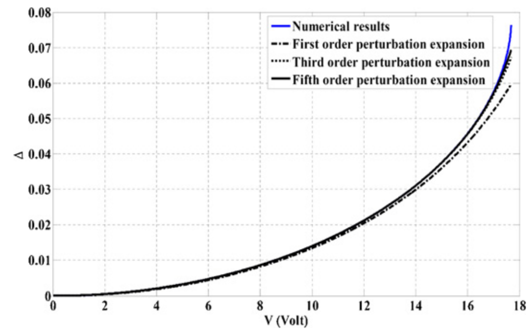

Fig. 8. Voltage-displacement behavior of the torsional micromirror, comparison of numerical and analytical results.

5. Conclusion

In the current paper, analytical solutions were presented for modeling the bending of the torsional beams in electrostatically actuated micromirrors. Energy method was employed to obtain equilibrium equations. Furthermore, the instability mode of the micromirror was modeled using the implicit function theorem. The model was then solved using the combination of HPM and straight forward perturbation expansion. The presented results were in excellent agreement with numerical simulations. Results revealed that neglecting the effect of bending in electrostatic torsion micro actuators can cause several hundred percent overestimation of the stability limits of the device. HPM was used to study the voltage-angle and voltage-displacement behavior of the micromirror. Presented results were in good agreement with numerical simulations and experimental findings. The design tool presented in this paper can be used for efficient, accurate and time consuming modeling of micromirrors and their design optimization.

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