Optimization of foam filled spot-welded column for the crashworthiness design

M. Shariati*, H.R. Allahbakhsh**, Jafar Saemi***, M. Sedighi****

*Mechanical Department, Shahrood University of Technology, Shahrood, Iran, E-mail: mshariati44@gmail.com
**Mechanical Department, Shahrood University of Technology, Shahrood, Iran, E-mail: allahbakhshy@gmail.com
***Mechanical Department, Shahrood University of Technology, Shahrood, Iran, E-mail: jafarsaemi@gmail.com
****Mechanical Department, Shahrood University of Technology, Shahrood, Iran, E-mail: nsedighi47@gmail.com

1. Introduction

The welding known as standard method of joining, has received attention in last decades [1]. During recent years, the car body assembly techniques were dominated by spot-welding. Resistance spot-welding is a very quick, cheap and accessible technology to join metal sheets. Also it is controllable and it can be done automatically. Resistance spot-welding does not need special preparation of the parts before joining. On the other hand, weight-saving and impact safety requirements are calling for the application of light-weight materials and structures with high specific energy absorption to energy absorbing structures. Recently, much attention is given to the cellular material filled thin-walled structures. The studies showed that the interaction between metal or polymeric cellular material fillers and the supporting structures produces some desirable crushing behaviours and energy absorption properties. Among many optional cellular fillers, e.g., sawdust, honeycomb, polyurethane foam and metal foams, closed cell aluminium foam is the one which gives some ideal performance.

Peroni [2] compared experimental results on the use of structural adhesives, laser-welding and spot-welding in structures subjected to crash. The obtained results demonstrate that continuously joined structures are at least equivalent to and generally better than spot-welded structures, and have further advantages typical of these joining solutions (higher stiffness and fatigue strength, improved vibration response, especially in the case of adhesive joints). Yujiang Xiang [3] performed crashworthiness optimization of an empty spot-welded thin-walled hat section. Various spot-weld models were first used in a thin-walled hat section to compare with experimental works. An appropriate spot-weld model was then used in the transient nonlinear finite element analysis (FEA), and the number of spot-welds was selected as one of the design variables in optimization. The mass of the thin-walled tubes was optimized subject to constraints on the required mean crushing force and sectional stiffness. Shariati et al [4] performed experimental and numerical quasi-static crushing responses of spot-welded structures they also introduced a pre-crushed trigger to decrease the initial peak force of spot-welded columns under axial loading. The dynamic axial crushing behaviour and the interactive effect due to foam filling of foam-filled hat sections was predicted theoretically by Qing-chun Wang et al [5, 6]. They improved the theoretical model of empty hat sections developed by White and Jones [7], and then based on the experimental results, a theoretical model was created for aluminium foam-filled hat sections. Hong-Wei Song [8] investigated the interaction effect between aluminium foam and the metal spot-welded column. Based on their experimental examination, numerical simulation and analytical models, a systemic approach was developed to partition the energy absorption quantitatively into the foam filler component and the hat section component, and the relative contribution of each component to the overall interaction effect was therefore evaluated. They divided crushed foam filler into two main energy-dissipation regions: densified region and extremely densified region. The volume reduction and volumetric strain of each region were empirically estimated.

To seek for optimal crashworthiness of structure, some alternatives have been exploited for design optimization over the last decade. Response surface method (RSM) is a prevalent technique to model highly-nonlinear systems. The RSM was presented by Myers and Montgomery and advanced by other researchers [9, 10].The idea is to use some simple basis functions such as polynomials [11] to approximate complex crushing response of a structure. This method has been employed to optimize many several other thin-walled structures with crashworthiness criterion [12, 14].

In this paper, the numerical quasi-static crushing responses of foam-filled spot-welded structures are investigated. The numerical crash analyse of foam-filled tubes was performed using the Abaqus finite element software and was validated by comparing against solution published in literature. To seek for the optimal crashworthiness design a set of designs are selected from the design space using the factorial design, which have different thickness column, side length and foam density(continues variables). And also the number of spot-welds (discrete variable) was selected as one of the design variables in optimization. For optimization of discrete variables, discrete optimization methods cannot be directly applied, because a large number of FE simulations would be required. In this paper thickness, side length of the column and foam density (continues variables) and also the number of spot-welds (discrete variable) is optimized by response surface method.

2. Numerical analysis using the finite element method

The numerical simulations were carried out using the finite element software Abaqus/Explicit. In this simulation, a self-contact algorithm was used to prevent interpenetration during the folding of the columns and the spot-welds are modelled by surface-based tie constraints option
in the Abaqus/Explicit. The plastic behavior of foam is taken into account using the CRUSHABLE FOAM and the CRUSHABLE FOAM HARDENING options in the Abaqus/Explicit software package [14].

2.1. Geometry and mechanical properties of the foam filled columns

The structures considered in this study are spot-welded thin-walled hat section, as illustrated in Fig. 1.

![Fig. 1 Schematic drawings of the spot-welded column used in the current study](image)

Design variables \(a, t, \rho\) are chosen as the design variables, and the constraints of these three design parameters are given as \(50 \leq a \leq 70\), \(1 \leq t \leq 1.5\) mm and \(250 \leq \rho_f \leq 400\) kg/m\(^3\). The effects of these parameters on the following response of the filled spot-welded column evaluate for quasi-static loading. In this work, the lengths \(L\) of the spot-welded structures are a constant of 200 mm.

The spot-welded columns used for this study were made of mild steel FEE355 with mechanical properties of stress at 0.2% strain \(\sigma_{0.2} = 380\) MPa and ultimate stress \(\sigma_u = 430\) MPa. Furthermore, the value of Poisson ratio was assumed to be \(\nu = 0.3\) [8].

Four 250, 300, 340 and 400 kg/m\(^3\) (\(\rho_1, \rho_2, \rho_3, \rho_4\), respectively) average densities of aluminium foam are used in this paper. The uniaxial and hydrostatic stress-strain curves and other information for the aluminum foams have been reported [15]. The aluminum foam was modeled with the foam model of Dehspande and Fleck [16] in Abaqus software. In this model, the foam is considered as an isotropic material. The following yield criterion is assumed for this model

\[
\Phi = \sigma - Y \leq 0
\]

where

\[
\sigma^2 = \frac{1}{[1/(\alpha / 3)]^2} [\sigma_v^2 + \alpha \sigma_m^2]
\]

\[
\alpha_2 = \frac{2(1-2\nu_p)}{9(1+\nu_p)}
\]

The following hardening rule, which includes the variation of the foam density, is implemented in this model

\[
Y = \sigma_p + \gamma \frac{\hat{\varepsilon}}{e_D} + \alpha \ln \left( \frac{1}{1 - \left( \frac{\hat{\varepsilon}}{e_D} \right)^\beta} \right)
\]

where \(\sigma_p, \alpha, \gamma, \beta\) and \(\rho_f\) are material parameters, and \(\hat{\varepsilon}\) the equivalent strain. If the strain hardening rule is calibrated to a uniaxial compression test, the compaction strain, \(e_D\) can be expressed as

\[
e_D = \frac{9 + \alpha^2}{3\alpha^2} \ln \left( \frac{\rho_f}{\rho_{f0}} \right)
\]

where \(\rho_f\) is the foam density and \(\rho_{f0}\) the density of the base material.[17-19].

2.2. Boundary conditions and element formulation

For applying boundary conditions on the edges of the spot-welded columns, two rigid plates were used that were placed to the ends of the columns. All degrees of freedom in the lower plate and all degrees of freedom in the upper plate, except in the direction of longitudinal axis, were constrained.

For this analysis, the linear element S4R, which is a four-node element, is suitable for analysis of thin shells, and element C3D8R, which is an 8-node linear brick element, is suitable for analysis of foam.

3. Response surface method (RSM)

Response Surface Methodology (RSM) is a method for understanding the correlation between multiple input variables and one output variable. In this approach, an approximation \(\hat{y}(x)\) to the response of the spot-welded columns is assumed a series of the basic functions in a form of

\[
\hat{y}(x) = \sum_{j=1}^{N} a_j \varphi_j(x)
\]

where \(N\) represents number of basis function \(\varphi_j(x), x \in R^n\). A typical class of basis functions is the polynomials, for instances, whose full quadratic form is given as

\[
\varphi_j(x) = x_j^2
\]
\[\hat{y} = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + \cdots \quad \text{Linear terms}\]
\[a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 + \cdots \quad \text{Interaction terms}\]
\[a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + \cdots \quad \text{Quadratic terms}\]

To determine the regression coefficient \(a = (a_1, a_2, \ldots, a_n)\) in Eq. (6), a large number of FE analyses \(y^{(i)} (i = 1, 2, \ldots, M)\) are needed \((M>N)\). The method of least-square can be used to determine the regression coefficient vector \(a\) by minimizing the errors between the FE analysis \(y\) and the response function \(\hat{y}\). The least square function can be expressed as

\[
E(a) = \sum_{i=1}^{M} e_i^2 = \sum_{i=1}^{M} [y^{(i)} - \sum_{j=1}^{N} a_j \varphi_{ij}(x^{(i)})]^2
\]

The regression coefficient vector \(a = (a_1, a_2, \ldots, a_n)\) can be evaluate by \(\frac{\partial E(a)}{\partial x}\), which is

\[
a = (\Phi^T \Phi)^{-1} (\Phi^T y)
\]

Where matrix \(\varphi\) denotes the values of basis functions evaluated at these \(M\) sampling points, which is

\[
\varphi = \begin{bmatrix}
\varphi_1(x^{(1)}) & \cdots & \varphi_N(x^{(1)}) \\
\vdots & \ddots & \vdots \\
\varphi_1(x^{(M)}) & \cdots & \varphi_N(x^{(M)})
\end{bmatrix}
\]

By substituting Eq. (9) into Eq. (4), the RS model can be fully defined \([10, 11]\).

4. Problem description

The crashworthiness of the spot-welded columns is expressed in terms of specific energy absorption SEA. The SEA is defined as

\[
SEA = \frac{\text{Total energy absorption } E_{\text{total}}}{\text{Total structural weight}}
\]

The area under the load–displacement curve gives the total energy absorption. High values for SEA indicate a lightweight absorber.

Fig. 2 Flowcharts of the two optimization processes: a - first algorithm, b - second algorithm
The optimization problem is
\[
\begin{align*}
\text{Minimize} & : y = P(x) \\
\text{St} & : \text{SEA}_i \geq \text{SEA}_{\text{const.}}, \\
& : x^L \leq x \leq x^U
\end{align*}
\]
where \( \text{SEA}_{\text{const.}} = 9 \text{ kJ/kg.} \), \( x^L = (x_1^L, x_2^L, \ldots, x_i^L) \) and \( x^U = (x_1^U, x_2^U, \ldots, x_i^U) \) are respectively the lower and upper bounds of the design variables.

The numbers of spot-weld in foam-filled spot-welded columns are optimized by two methods. First method is to first optimize the geometry based on a section with a large number of spot-welds or a complete weld. Then the minimum number of spot-welds is determined by satisfying the optimization problem without changing the geometry obtained in the first step. The assumption here is that the crash behaviour does not change much when the number of spot-welds is relatively large and the peak crashing force decreases with decreasing the number of spot-weld. This assumption is shown to be valid by the numerical results in Section 4.3. In second method, a large number of attempts would be to create RSM models corresponding to different numbers of spot-welds, and optimization results from all the RSM models are compared to determine the best solution. This method could result in significant computational cost if the numbers of spot-welds to be considered is large.

4.1. FE models and crashworthiness analysis

FE models are created for spot-welded columns and they are used for the crashworthiness analyses. For the three continuous variables, \((a,t,\rho)\) the factorial design method was adopted in design of experiments (DOE). \( \text{SEA} \) and the maximum crushing force \( P_m \) acquire from the analyses and will later be used for constructing corresponding RS models.

For validation of FEA, deformation mode and mean crushing force are of interest. Fig. 3 shows the comparison of results for energy absorbed from the present simulations with experimental and theoretical results [6] for the foam-filled spot-welded column, and indicates good agreement between simulations with experimental and theoretical results. According to the response surface method, 36 design points are chosen in the design space to establish the following \( \text{SEA} \) response function in terms of design variables. Based on the FEM, the RSM model for \( \text{SEA} \) and peak crushing force was constructed using a quadratic polynomial given as follows.

\[
P = -16.01818 - 0.00206a + 5.32228t + 0.12976 \rho - 0.00089a^2 - 0.02116t - 0.00007a \rho - 0.67777t^2 - 0.00145t \rho - 0.00017 \rho^2
\]

\[
\text{SEA} = 139.1767 + 0.2493a - 204.4902t - 0.1472 \rho - 0.01177a^2 + 1.09855at + 0.0011a \rho + 88.06932t^2 + 0.04073t \rho + 0.0002 \rho^2
\]

Figs. 4 and 5 show the RS of absorbed energy and peak force as a function of \( a, t \) and \( \rho \) at a constant deformation of 120 mm. It can be seen that in Fig. 4 with increasing \( t \) and decreasing \( a \), the \( \text{SEA} \) increases and with increasing \( t \) and \( a \) the peak force increases.

From Fig. 4, it can clearly deduced that the foam filling solution does not always result to the efficient and light energy absorbers. It can be seen that while the foam density increases the energy absorption does not increase monotonically. In other words, there is one optimal foam density where the maximum \( \text{SEA} \) occurs. When foam with lower or higher density than this optimum density is selected, the \( \text{SEA} \) decreases.

The optimal results acquired using the nonlinear programming (fmincon), which is provided by MATLAB. “fmincon” attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate [20]. Table 1 shows the optimized tube geometry and foam density and its energy absorption.
Fig. 4 Response surface of SEA for the foam filled spot welded column

Fig. 5 Response surface of peak force for the foam filled spot welded column

Table 1

<table>
<thead>
<tr>
<th>Optimum foam-filled square column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (mm)</td>
</tr>
<tr>
<td>Optimum value</td>
</tr>
</tbody>
</table>

4.2. Step 2: number of spot-weld optimization

In this step, the optimized variables in Table 1 were selected and the algorithm in Fig. 2, a optimized the number of spot-welds. The SEA for various $n$ ($n = 3, 4, ...$) were obtained from FEA, and the smallest $n$ with a specific energy absorption greater than 9 kJ was selected as the final solution. Figs. 6-8 shows the trend of SEA and peak crushing force with changes of $n$. The results show that the SEA for ($n < 10$) does not meet the constraint of ($SEA \geq 9$); therefore, the minimum number of spot-welds is 10. The peak crushing force corresponding to the optimum solution is 97 kN, which 5.5% reduction over the section with a complete weld which this reduction of peak crushing force is desirable. Fig. 7 also shows that the trend of the SEA tends to be flat from $n = 10$ to a complete weld. This verifies the assumption of the first optimization algorithm that the number of spot-welds can be decoupled from the rest of design variables for the crushing force when the number of spot-welds is large enough. The fluctuation of the values of SEA for ($n > 10$) is due to the highly nonlinear behaviors of crushing processes, because varying the number of spot-welds may result in different deformation modes and shapes [3]. The deformed sections with $n = 5, 10, 15$, and a complete weld are shown in Fig. 8.
4.3. Second optimization algorithm

To verify the optimal result obtained by the first algorithm, the second algorithm was also used to compute the optimum design variable. This is done by enumerating the number of spot-welds \( n \) and optimizing the design variables for each \( n \). The values of \( SEA \) based on the RSM are given in Table 2.

Table 2
Comparison of the two optimization algorithms

<table>
<thead>
<tr>
<th>Number of spot-weld</th>
<th>( t ), mm</th>
<th>( a ), mm</th>
<th>( \rho_f ), kg/m(^3)</th>
<th>( P_{max} ), kN</th>
<th>( SEA ), kJ/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>1.47</td>
<td>50</td>
<td>351.1</td>
<td>103.8</td>
<td>9.0</td>
</tr>
<tr>
<td>12</td>
<td>1.46</td>
<td>50</td>
<td>353</td>
<td>104.4</td>
<td>9.0</td>
</tr>
<tr>
<td>14</td>
<td>1.47</td>
<td>50</td>
<td>352</td>
<td>105.3</td>
<td>9.0</td>
</tr>
</tbody>
</table>

It can be seen from Table 2 that there are no meaningful changes for design variables \((a, t, \rho)\), constraints \( SEA \) and Peak crushing force for \( n \geq 10 \) and also it can be seen for \( n \leq 10 \) the \( SEA \) is lesser than 9 kJ/kg and do not satisfy the optimization problem. Table 2 also verifies that the number of spot-welds can be separated from the other design variables for the optimization. The final optimal results of the first and second algorithms are very close, as given in Table 3. However, the first algorithm is better than the second algorithm, because that requires less FE simulations. For example, if the number of spot-welds \( n \) varies from 8 to 13, the required number of FEA is 42 in the second algorithm (36 for factorial design method plus 6 for enumerations); while it is 144 in the second algorithm (4 enumerations each with 36 for factorial design method).

Table 3
Optimum foam-filled square tube

<table>
<thead>
<tr>
<th>( n )</th>
<th>( t ), mm</th>
<th>( a ), mm</th>
<th>( \rho_f ), kg/m(^3)</th>
<th>( P_{max} ), kN</th>
<th>( SEA ), kJ/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>First algorithm</td>
<td>10</td>
<td>1.45</td>
<td>50</td>
<td>350</td>
<td>102.3</td>
</tr>
<tr>
<td>Second algorithm</td>
<td>10</td>
<td>1.47</td>
<td>50</td>
<td>351.1</td>
<td>103.8</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper, crashworthiness optimization of spot-welded columns was studied when no spot-weld failure occurred. The number of spot-welds and cross-sectional of spot-welded columns are optimized. The effect of foam density, wall thicknesses and side length on SEAs and peak crushing forces are also presented through the plots of the response surfaces. In this study two different design algorithm were used for optimization of the number of spot-weld. The illustrative example indicated that first algorithm is more efficiency than second.

References

M. Shariati, H.R. Allahbakhsh, J. Saemi, M. Sedighi

OPTIMIZATION OF FOAM FILLED SPOT-WELDED COLUMN FOR THE CRASHWORTHINESS

SUMMARY

To improve crashworthiness efficiency, aluminum foam has been adopted as one of new filler materials in engineering. Introduction of the foam material changes the crash behavior of structural component and make necessary exploration of more sophisticated design optimization methodology. This paper presents a crashworthiness design of foam-filled columns made from mild steels and joined by spot-weld. During the design optimizations, maximum crushing force (Pm) is set as the design objective, wall thickness, foam density and side length (continuous variables) and also the number of spot-weld (discrete variable) are selected as design variables, and specific energy absorption (SEA) is set as the design constraint. To formulate the complex crashworthiness design problem, the response surface method (RSM), is used.

M. Шариати, Н.Р. Аллахбакхш, Я. Саеми, М. Седигхи

ОПТИМИЗАЦИЯ ПРОЕКТИРОВАНИЯ ТОЧЕЧНОЙ СВАРКОЙ СВАРЕННЫХ И ПЕНОЙ ЗАПОЛНЕННЫХ БЕЗОПАСНЫХ КОЛОНОК

Резюме

Для повышения безопасности использовалась алюминиевая пена как новый заполнитель колонок. Использование пены изменило вид разрушения компонентов колонок и потребовало более сложной методики оптимизации проектирования. В этой статье предложено проектирования безопасных, пеної заполненных, изготовленных из мягкой стали точечной сваркой сваренных колонок. При оптимизации максимальная разрушающая сила (Pm) использовалась как цель проекта; толщина стенки, плотность пены, длина стороны (непрерывные переменные) и так же число точек сварки (дискретная переменная) подобраны как переменные, а специфическая аборбированная энергия использована как целевая функция. При формулировке комплексной проблемы проектирования безопасной конструкции использован метод реакции поверхности.

Accepted January 28, 2010
Received June 03, 2010