An Achievable Rate Region for Interfering Multiple Access Channel and Broadcast Channel with a Cognitive Transmitter

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Abstract—We consider the inter-cluster cognitive behavior of two dissimilar interfering clusters, where the first cluster (i.e., primary cluster) consists of a single multiple access channel (MAC) with two senders and one receiver and the second cluster (i.e., cognitive cluster) is a broadcast channel (BC) which has a single sender and two receivers. The BC is assumed to have cognitive transmitter, who knows the messages transmitted by the other non-cognitive multiple access transmitters, in a non-causal manner. A Gel’fand-Pinsker coding-like technique is used to mitigate inter-cluster interference. An achievable rate region for this model is derived based on the Marton region with common message for the general BC and using a result of Slepian-Wolf rate region for the MAC.

Keywords- Achievable rate region; broadcast channel; cognitive radio channel; multiple access channel, Gel’fand-Pinsker coding

I. INTRODUCTION

The cognitive radio technology in the wireless communications has made significant improvements in spectral efficiency. Cognitive radios could sense their wireless environment, obtain the messages of other already transmitting users and adapt themselves to their surroundings. In [1] and [2], Devroye et al. introduced the cognitive radio channel with 2 senders and 2 receivers, in which one user knows the other user’s message non-causally (genie-aided) and simultaneously transmits its own message. The authors also demonstrated an achievable rate region for this channel. Similar works are presented in [3]-[5], which are referred as “interference channel with unidirectional cooperation” or “interference channel with degraded message sets”. In [6], the authors considered an arbitrary wireless network consisting of cognitive and (possibly) non-cognitive radio devices. Moreover, the wireless network is partitioned into clusters with three different types of intra/inter-cluster behaviors: competitive, cooperative, and cognitive. Inter-cluster cognitive behavior refers to when some interfering clusters obtain the messages to be transmitted by other cluster(s) and use this knowledge to improve the overall rate region of wireless network. The inter-cluster cognitive behavior of two clusters that both of them are multiple access channels (MACs) is presented in [7]. In this case, each cluster consists of one receiver and at least two transmitters.

Another related work is studied in [8], where the inter-cluster cognitive behavior of two broadcast channel (BC) clusters is considered. Each cluster assumed to have a single transmitter and two receivers (i.e. two-user BC), while the transmitter of one BC is non-causally aware of the message transmitted by the adjacent BC. An achievable rate region is derived for this model.

In this paper, we consider the inter-cluster cognitive behavior of two dissimilar interfering clusters, where the first cluster consists of a single MAC with two senders and one receiver and the second cluster is a BC which has a single sender and two receivers. By cognitive behavior of MAC and BC, we mean that the transmitter of the BC is aware of the messages transmitted by the adjacent MAC transmitters, in a non-causal manner. We propose an achievable rate region for this model. The region is derived based on the Marton region for the general broadcast channel [9], using a result of Slepian-Wolf rate region for the multiple access channel [10] and a Gel’fand-Pinsker coding-like technique [11] in order to mitigate inter-cluster interference. We denote the proposed channel with $MAC – BC_G$.

The capacity region for MAC with common message is derived as follows:

$$1 \leq I(\frac{1}{1};Y|X_2W)$$
$$2 \leq I(X_2;Y|X_1W)$$
$$1 + \frac{1}{2} \leq I(\frac{1}{1};Y|W)$$
$$0 + \frac{1}{2} \leq I(\frac{1}{1};\frac{1}{2}|Y)$$

where the capacity region is computed over all joint distributions that factor as:

$$p(w,x_1,x_2,y) = p(w)p(x_1|w)p(x_2|w)p(y|x_1,x_2).$$

The largest achievable rate region for a two-user BC with common message is as follows:[9]:

$$0 + \frac{1}{2} \leq I(\frac{1}{1};U_1W)$$
$$0 + \frac{1}{2} \leq I(\frac{1}{2};U_2W)$$
$$0 + \frac{1}{2} \leq \min\{I(Y_1;W),I(\frac{1}{2};W)\}$$
$$-I(U_1;U_2|W) + I(U_1;\frac{1}{2}|W) + I(U_2;\frac{1}{2}|W)$$

$$2 \leq 0 + \frac{1}{2} \leq I(\frac{1}{1};U_1W) + I(\frac{1}{2};U_2W)$$
$$-I(U_1;U_2|W)$$

where the rate region is calculated over all joint distributions that factor as:
The rest of the paper is organized as follows. In Section II, we define channel model. Section III states main result: an achievable rate region for this channel. In Section IV, we prove the achievability of the proposed rate region. A conclusion is prepared in Section V.

II. DEFINITIONS AND CHANNEL MODEL

We denote random variables by $X_1, X_2, Y_1, \ldots$ with values $x_1, x_2, y_1, \ldots$ in finite sets $X_1, X_2, Y_1, \ldots$ respectively; $n$-tuple vectors of $X_1, X_2, Y_1, \ldots$ are denoted with $x_1^n, x_2^n, y_1^n, \ldots$

We consider the channel with finite input alphabets $X_1, X_2, \ldots$, finite output alphabets $Y_1, Y_2, \ldots$, transition probability function $p(y_t, x_{t+1}, x_t | x_t, y_t)$ with conditional marginal distributions $p(x_t | x_t, y_t)$ and $p(y_{t+1}, x_{t+1}, x_t | x_t, y_t)$. The channel is memoryless in the sense that for $n$ channel uses, we have

$$p(y_{1:n} | x_{1:n}, x_{1:n}) = \prod_{i=1}^{n} p(y_{1:n}, x_{1:n}, x_{1:n})$$

where $x_{1:n} = (x_{1:t}, \ldots, x_{1:n}) \in X_1^n$, $x_2 = (x_2, \ldots, x_2) \in X_2$, $y_{1:t} = (y_{1:t}, \ldots, y_{1:t}) \in Y_1^n$ and $p(y_{1:n}, x_{1:n}, x_{1:n})$ is $p(y_{1:n})$ for $t = 1, 2$. We depict an example in Fig. 1. Sender 1, $t = 1, 2$ sends a private message $m_1 \in M_1$ together with a common message $m_2 \in M_2 = \{1, \ldots, m_2\}$ to the first receiver over the channel $p(y_{1:n}, x_{1:n}, x_{1:n})$. Sender 2, $t = 1, 2$; together with a common message $m_2 \in M_2 = \{1, \ldots, m_2\}$ to the second and third receivers over the channel $p(y_{1:n}, x_{1:n}, x_{1:n})$.

Channels

A nonnegative rate tuple $(R_{10}, R_{12}, R_{20}, R_{22}, R_{22})$ is achievable for the channel $C$ if for any given $0 < \varepsilon < 1$, and for any sufficiently large $n$, there exists a code $(n, [2^{nR_{10}}], [2^{nR_{11}}], [2^{nR_{20}}], [2^{nR_{21}}], [2^{nR_{22}}], \varepsilon)$. The capacity region for the channel $C$ is defined as the closure of the set of all the achievable rate triples, while an achievable rate region for the channel $C$ is a subset of the capacity region.
III. MAIN RESULT: AN ACHIEVABLE RATE REGION FOR THE DISCRETE MEMORYLESS MAC – BC$_G$

Let us consider auxiliary random variables $W_{10}, W_{20}, U_{23}$, and $U_{22}$ defined on arbitrary finite sets $\mathcal{W}_{10}, \mathcal{W}_{20}, \mathcal{U}_{23}$, and $\mathcal{U}_{22}$ respectively. The auxiliary random variable $W_{10}$ contains common message $m_{10}$ and is decoded by the receiver 1. Two private messages $m_{11}$ and $m_{12}$ which are superimposed over $m_{10}$, are conveyed by $X_{11}$ and $X_{12}$ respectively. The auxiliary random variables $W_{20}, U_{23}$, and $U_{22}$ are used to encode messages $m_{20}, m_{21}$, and $m_{22}$ respectively. $W_{20}$ contains common message $m_{20}$ and is decoded by both receivers 21 and 22. However, $U_{22}$ is just decoded by receiver 2t, $t = 1, 2, \ldots$ . For the MAC – $B_{CG}$ (Fig. 1), let $Z = (W_{10}W_{20}U_{23}X_{11}X_{12}Y_{1}Y_{21}Y_{22})$ and let $\mathcal{P}$ be the set of all distributions of the form (1): (hereafter, for brevity, let $p(w_{10}w_{20}u_{23}u_{22}x_{11}x_{12}y_{1}y_{21}y_{22}) = p(\cdot)$)

$p(\cdot) = p(w_{10})p(x_{11}|w_{10})p(x_{12}|w_{10})p(w_{20}|w_{10}x_{11}x_{12}) 
\cdot p(u_{23}u_{22}|w_{20}w_{10}x_{11}x_{12})p(x_{2}|w_{10}x_{11}x_{12}w_{20}u_{23}u_{22}) 
\cdot p(y_{1}y_{21}y_{22}|x_{11}x_{12}x_{2})$ (1)

The joint distribution (1) results in the following Markov chains.

$X_{11} \leftrightarrow W_{10} \leftrightarrow X_{12}$ (1-1)
$(W_{10}, W_{20}, U_{23}, U_{22}) \leftrightarrow (X_{11}, X_{12}, X_{2}) \leftrightarrow (Y_{1}, Y_{21}, Y_{22})$ (1-2)

Theorem 1: For the MAC – $B_{CG}$ (Fig. 1), let $Z = (W_{10}W_{20}U_{23}U_{22}X_{11}X_{12}X_{2}Y_{1}Y_{21}Y_{22})$ and let $\mathcal{P}$ be the set of all distributions of the form (1). For any $Z \epsilon \mathcal{P}$ let $S(Z)$ be the set of all rate tuples ($R_{10}, R_{11}, R_{12}, R_{20}, R_{21}, R_{22}$) of non-negative real numbers such that

$R_{11} \leq I(Y_{1}; X_{11}|W_{10}X_{12})$ (2-1)
$R_{12} \leq I(Y_{1}; X_{12}|W_{10}X_{11})$ (2-2)
$R_{11} + R_{12} \leq I(Y_{1}; X_{11}X_{12}|W_{10})$ (2-3)
$R_{10} + R_{11} + R_{12} \leq I(Y_{1}; X_{11}X_{12})$ (2-4)
$R_{20} + R_{21} \leq I(Y_{21}; U_{21}W_{20}) - I(W_{20}U_{21}; W_{10}X_{11}X_{12})$ (2-5)
$R_{20} + R_{22} \leq I(Y_{22}; U_{22}W_{20}) - I(W_{20}U_{22}; W_{10}X_{11}X_{12})$ (2-6)
$R_{20} + R_{21} + R_{22} \leq (I(Y_{21}; U_{21}W_{20}) + I(Y_{22}; U_{22}W_{20}) 
- I(U_{21}; W_{10}X_{11}X_{12}|W_{20}) 
- I(U_{22}; W_{10}X_{11}X_{12}|W_{20}) 
+ \min(I(Y_{21}; W_{20}), I(Y_{22}; W_{20})) 
- I(W_{20}; W_{10}X_{11}X_{12}) 
- I(U_{22}; U_{21}W_{20}X_{11}X_{12}) 
+ I(U_{22}; U_{21}W_{20}X_{11}X_{12}) 
- I(W_{20}U_{21}; W_{10}X_{11}X_{12}) 
- I(U_{22}; U_{21}W_{20}X_{11}X_{12}))$ (2-7)
$2R_{20} + R_{21} + R_{22} \leq I(Y_{21}; U_{21}W_{20}) + I(Y_{22}; U_{22}W_{20}) 
- I(W_{20}U_{21}; W_{10}X_{11}X_{12}) 
- I(U_{22}U_{21}W_{20}; W_{10}X_{11}X_{12})$ (2-8)

then any element of the closure of $\cup_{Z \in \mathcal{P}} S(Z)$ is achievable.

Proof: Refer to Section IV.

Remark 1: Note that the rate region $S$ is convex, and therefore no convex hull operation or time sharing is necessary.

Remark 2: From inequalities (2-5)-(2-8), note that if $I(U_{2t}; Y_{2t}|W_{20}) < I(U_{2t}; W_{10}X_{11}X_{12}|W_{20})$

by setting $U_{2t} = \emptyset$ (i.e. $U_{2t}$ be a null random variable), and if

$max\{I(Y_{21}; W_{20}), I(Y_{22}; W_{20})\} < I(W_{20}; W_{10}X_{11}X_{12})$

by letting $W_{20} = \emptyset$, we can improve the achievable rate region of the theorem.

Remark 3: By isolating the second cluster (BC) from the first one (MAC) and eliminating the cognitive behavior in inequalities (2-5)-(2-8), we have $I(U_{22}, U_{21}; W_{20}; W_{10}; X_{11}; X_{12}) = 0$ . Therefore, after removing redundant inequality (i.e. (2-8)) from the resulting inequalities, we can readily obtain the Marton achievable rate region with common message [9].

Remark 4: By setting $S = (W_{10}, X_{11}, X_{12})$ from inequalities (2-5)-(2-8) we obtain the achievable rate region for the two-user BC with common message and side information known non-causally at the transmitter (the Marton region with common message for the BC with non-causal side information) which is the set of all rate triples ($R_{20}, R_{21}, R_{22}$) satisfying

$R_{20} + R_{21} \leq I(W_{20}; U_{21}; Y_{21}) 
- I(W_{20}; U_{21}; S)$ (3-1)
$R_{20} + R_{22} \leq I(W_{20}; U_{22}; Y_{22}) 
- I(W_{20}; U_{22}; S)$ (3-2)
$R_{20} + R_{21} + R_{22} \leq \min\{I(W_{20}; Y_{21}), I(W_{20}; Y_{22})\} 
+ I(U_{21}; Y_{21}|W_{20}) 
+ I(U_{22}; Y_{22}|W_{20}) 
- I(U_{22}; U_{21}W_{20}) 
- I(U_{22}; U_{21}W_{20}) 
- I(U_{22}; U_{21}W_{20}; S)$ (3-3)
$2R_{20} + R_{21} + R_{22} \leq I(W_{20}; U_{21}; Y_{21}) 
+ I(W_{20}; U_{22}; Y_{22}) 
- I(W_{20}; U_{22}; W_{20}; S)$ (3-4)

for some joint distribution of the form $P(s)p(y_{21}|s)p(y_{22}|s)p(x_{12}|w_{20}, u_{21}, u_{22})p(w_{20}|s)$

$P(x_{2}|w_{20}, u_{21}, u_{22}, s)p(y_{21}, y_{22}|x_{2}, s)$.

IV. PROOF OF THEOREM I

It is sufficient to show that any element of $S(Z)$ for each $Z \epsilon \mathcal{P}$ is achievable. So, fix $Z = (W_{10}W_{20}U_{23}U_{22}X_{11}X_{12}X_{2}Y_{1}Y_{21}Y_{22})$ and take any $(R_{10}, R_{11}, R_{12}, R_{20}, R_{21}, R_{22})$ satisfying the constraints of the theorem.

Codebook generation: Consider $n > 0$ , some distribution of the form (1). We construct the codebook as follows:

1. generate $[2^{nR_{10}}]$ independent codewords $w_{10}(j), j \in \{1, \ldots, [2^{nR_{10}}]\}$ according to $\prod_{i=1}^{n} p(w_{10i})$.
2. For each codeword $w_{10}(j)$:
   2.1 at encoder 1, generate $[2^{nR_{11}}]$ independent codewords $x_{11}(j,k), k \in \{1, \ldots, [2^{nR_{11}}]\}$, according to $\prod_{i=1}^{n} p(x_{11i}(w_{10i}))$.
   2.2 at encoder 2, generate $[2^{nR_{12}}]$ independent codewords $x_{12}(j,l), l \in \{1, \ldots, [2^{nR_{12}}]\}$.
three components channel throw them randomly into generate. The cognitive sender as follows.

4.1 generate \([2^{n(r_2 + 2r_3)}]\] independent codewords \(u_{21}(m,n), n \in \{1, \cdots, [2^{n(r_2 + 2r_3)}]\}\) according to \(\prod_{i=1}^n p(u_{21}(W_{20i}))\) and throw them randomly into \([2^{nR_2}]\) bins such that the sequence \(u_{21}(m,n)\) in bin \(b_1\) is denoted as \(u_{21}(m,b_2,n), b_2 \in \{1, \cdots, [2^{nR_2}]\}\)

4.2 generate \([2^{n(r_2 + 2r_3)}]\] independent codewords \(u_{22}(m,o), o \in \{1, \cdots, [2^{n(r_2 + 2r_3)}]\}\) according to \(\prod_{i=1}^n p(u_{22}(W_{20i}))\) and throw them randomly into \([2^{nR_2}]\) bins such that the sequence \(u_{22}(m,o)\) in bin \(b_3\) is denoted as \(u_{22}(m,b_2,o), b_3 \in \{1, \cdots, [2^{nR_2}]\}\)

**Encoding & transmission:** The aim is to send a six dimensional message \((j,k,l,b_1,b_2,b_3)\) whose first three components \(j,k,l\) and \(m\) are message indices and whose last three components \(b_1,b_2,b_3\) are bin indices. The message actually sent over the genie-aided cognitive radio channel \(x_{11}, x_{12}\), and \(x_2\). The message and bin indices are mapped into \(x_{11}, x_{12}, x_2\), as follows.

The sender \(T_{X_{11}}\) to send \(j\) and \(k\) looks for \(w_{10}(j), x_{11}(j,k),\) and sends \(x_{11}(j,k)\). The sender \(T_{X_{12}}\) to send \(j\) and \(l\) looks for \(w_{10}(j), x_{12}(j,l)\), and sends \(x_{12}(j,l)\) . The cognitive sender \(T_{X_3}\) knowing \(w_{10}(j), x_{11}(j,k), x_{12}(j,l)\) non-causally, to send \((b_1,b_2,b_3)\) finds a sequence \(w_{20}(m)\) in bin \(b_1\) such that \((w_{10}(j), x_{11}(j,k), x_{12}(j,l), w_{20}(b_0,m)) \in A'_2\); and then finds sequences \(w_{21}(m,n)\) and \(u_{22}(m,o)\) in bin \(b_2\) and bin \(b_3\) respectively, such that \((w_{10}(j), x_{11}(j,k), x_{12}(j,l), w_{20}(b_0,m), u_{21}(m,b_2,n), u_{22}(m,b_3,o)) \in A'_2\); finally \(T_{X_3}\) generates \(x_{2}\) i.i.d. according to \(\prod_{i=1}^n p(x_{2i})\prod_{i=1}^n p(w_{20i}x_{11i}x_{12i}w_{21i}u_{22i}),\) and sends it.

**Error probability analysis:** The messages are decoded based on strong joint typicality as in [12]. Assuming all messages to be equiprobable, we consider the situation where \((j = 1,k = 1,l = 1,b_1 = 1,b_2 = 1,b_3 = 1)\) was sent. First, we consider encoding errors. The probability of encoding error will be negligible if the below binning conditions are held:

\[
\begin{align*}
    r_2 - R_2 &\leq I(W_{20}; W_{10}X_{11}X_{12}) && (A1-1) \\
    r_{21} - R_{21} &\leq I(U_{21}; W_{10}X_{11}X_{12}W_{20}) && (A1-2) \\
    r_{22} - R_{22} &\leq I(U_{22}; W_{10}X_{11}X_{12}W_{20}) && (A1-3) \\
    r_{21} + r_{22} &\leq I(U_{21}; U_{22}W_{10}X_{11}X_{12}W_{20}) && (A1-4)
\end{align*}
\]

Finally, considering decoding errors at the decoders. If we assume no encoding errors, with standard techniques of information theory it can be shown that the decoding errors can be avoided if

\[
\begin{align*}
    R_{11} &\leq I(Y_1; X_{11}W_{10}X_{12}) && (A2-1) \\
    R_{12} &\leq I(Y_1; X_{12}W_{10}X_{11}) && (A2-2) \\
    R_{11} + R_{12} &\leq I(Y_1; X_{11}X_{12}W_{10}) && (A2-3) \\
    R_{10} + R_{11} + R_{12} &\leq I(Y_1; X_{11}X_{12}) && (A2-4) \\
    r_{20} + r_{21} &\leq I(Y_{21}; U_{21}W_{20}) && (A2-5) \\
    r_{20} + r_{21} + r_{22} &\leq I(Y_{21}; U_{22}W_{20}) && (A2-6) \\
    2r_{20} + r_{21} + 2r_{22} &\leq I(Y_{21}; U_{21}W_{20}) + I(Y_{22}; U_{22}W_{20}) && (A2-7) \\
    2r_{20} + r_{21} + 2r_{22} &\leq I(Y_{21}; U_{21}W_{20}) + I(Y_{22}; U_{22}W_{20}) && (A2-8)
\end{align*}
\]

Therefore, by considering the binning conditions (A1-1)-(A1-4) and the inequalities (A2-1)-(A2-8) we reach to the constraints (2-1)-(2-8).

V. CONCLUSION

A wireless network model including interfering multiple access channel and broadcast channel with a cognitive transmitter is considered. The BC is supposed to have cognitive transmitter, who knows the messages transmitted by multiple access transmitters, in a non-causal manner. Using a result of Slepian-Wolf rate region for the multiple access channel and the Marton region with common message for the general broadcast channel along with random binning technique, an achievable rate region for this model is derived.

REFERENCES


