Progressive Collapse Analysis of Steel Frames: Simplified Procedure and Explicit Expression for Dynamic Increase Factor

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Abstract

Progressive collapse refers to a phenomenon in which local damage in a primary structural element leads to total or partial structural system failure. When investigating the progressive collapse of structures, nonlinear dynamic procedures lead to more accurate results than static procedures. However, nonlinear dynamic procedures are very complicated and the evaluation or validation of the results can become very time consuming. Therefore, it is better to use simpler methods. For static analyses, the gravity force applied to the removed column bay should be multiplied by a constant factor of two. However, using a constant dynamic increase factor (DIF) is only appropriate for elastic systems. According to the optimal design of structures, the assumption of elastic behavior after column removal is conservative. Thus, it is necessary to establish an expression for DIF that considers inelastic responses. In this paper, a simplified analysis procedure for the progressive collapse analysis of steel structures is presented using the load displacement and capacity curve of a fixed end steel beam. The results of the proposed method are in good agreement with nonlinear dynamic analysis results. Also, the capacity curve, obtained by dividing the accumulated area under the nonlinear static load displacement curve by the corresponding displacement of the column removed point, is used to predict the progressive collapse resistance of the column removed structure. Finally, an explicit expression for the DIF is established for elastic-perfectly plastic and elastic plastic with catenary action behavior.

Keywords: progressive collapse, steel frames, simplified analysis, dynamic increase factor, capacity curve, catenary action

1. Introduction

Progressive collapse refers to the phenomenon in which the local damage of a primary structural element leads to total or partial structural system failure, without any proportionality between the initial and final damage. Although the probability of structural collapse is low, if it occurs, it can cause significant losses. In the past few decades, many examples of the total or partial collapse of structures due to fire, explosions or impacts have occurred.

The progressive collapse phenomena was first brought to engineers' attention due to the collapse of a 22 story building in Ronan Point, London (UK), as a result of a gas explosion in 1968 (McGuire, 1974; Leyendecker and Ellingwood, 1977). Furthermore, research in this area has been accelerated due to two significant terrorist attacks in the United States that resulted in the structural collapse of the buildings: the Alfred P. Murrah Federal Building bombing in 1995 in Oklahoma City (USA) (Corley et al., 1998) and the destruction of the World Trade Center (WTC), in New York (USA) in 2001 (Bazant and Zhou, 2002; Bazant and Verdure, 2007; Bazant et al., 2008; Seffen, 2008).

In the United States, the Department of Defense (DOD, 2005) and General Service Administration (GSA, 2003) issued design guidelines in an attempt to mitigate the potential for progressive collapse in structures. Both guidelines use the alternate load path (ALP) method for progressive collapse mitigation. The ALP method is an event independent procedure that does not consider the reason for the removal of the column; instead, a single column is typically assumed to be suddenly missing and an analysis is conducted to determine whether or not the structure can bridge support across the missing column. Progressive collapse is naturally dynamic and causes large displacements in the structure. The DOD and GSA allow three analysis methods: linear static (LS), nonlinear static (NLS), and nonlinear dynamic (NLD) methods. In the linear static analysis, the load combination according to Eq. (1) or (2) is applied to all beams of the frame:

\[ \gamma \times (DL + 0.25LL) \] (GSA)
\[
\gamma \times (1.2DL+0.5LL)+0.2WL \text{ (DOD)}
\]

where

\begin{itemize}
  \item \text{DL:} dead load
  \item \text{LL:} live load
  \item \text{WL:} wind load
  \item \text{\(\gamma\):} load factor that considers the dynamic effects in static analysis.
\end{itemize}

\(\gamma\) is 2 for the beams in bays adjacent to the removed column and to all floors above the removed column; it is equal to 1 for all other beams.

The GSA proposed that the demand to capacity ratio (DCR), the ratio of the member force and member strength, be used as the criterion to determine the failure of the main structural members using linear analysis procedures:

\[
DCR = \frac{Q_{UD}}{Q_{CE}}
\]

where

\begin{itemize}
  \item \(Q_{UD}\): acting force determined in the member
  \item \(Q_{CE}\): expected ultimate unfactored capacity of that member.
\end{itemize}

The acceptance criterion is 3 for all members.

In the DOD regulations, the DCR is not evaluated, instead the design strength of member (the nominal strength multiplied the strength reduction factor) is multiplied by the over strength factor (1.1) and is then compared with the member force to determine the failure of members.

In the nonlinear static analysis, the applied load is similar to the linear static analysis; however, in the nonlinear dynamic analysis, \(\gamma\) is 1 for all beams. The failure criteria in nonlinear static and dynamic analyses are based on the beam chord rotation (\(\theta \leq 12^\circ\)) and ductility of the column removed point (\(m \leq 20\)). Beam chord rotation is the ratio of the vertical displacement of the removed column point to the length of the beam. Ductility is the ratio of the vertical displacement of the removed column point to the final elastic displacement of the same point. Therefore, in nonlinear analyses, evaluating the vertical displacement of the removed column point is very important.

Among the three analysis procedures designated by the DOD and GSA as suitable for structural analysis, the nonlinear dynamic method obtains the most accurate results (Marjanishvili and Agnew, 2006; Fu, 2009; Ruth et al., 2006; Powel, 2005; Tsai and Lin, 2008). As the nonlinear dynamic analysis of 3D frames is very time consuming, most researches are performed on 2D frames (Sucuoğlu et al., 1994, Kim and Kim, 2009; Khandelwal et al., 2009; Kim and Dawoon, 2009). Performing simpler and approximate analyses not only allows the analyst to disregard nonlinear dynamic analyses but provides reliable significant results (Grierson et al., 2005; Izzuddin et al., 2008a; Izzuddin et al., 2008b; Lee et al., 2009).

This paper presents the formulation of a simple model to evaluate the displacement of the removed column point. In this regard, the work done on the structure due to column removal is equal to the energy dissipated by the structure. Although in general this method may be applied to structures with any type of materials, the formulation presented here is only valid for structures with mild steel. To demonstrate the capability and validity of the proposed method, three 2D steel frames with different number of bays and stories were chosen. Nonlinear dynamic analysis of these structures has been performed by other researchers. It is shown that the results of the proposed method is in good agreement with nonlinear dynamic analysis results.

Finally, an explicit expression for the DIF is established for elastic-perfectly plastic and elastic plastic with catenary action behavior.

2. Previous Works

In Marjanishvili and Agnew (2006), an explanation of four methods used to perform progressive collapse analysis (LS, NLS, LD, and NLD) in SAP2000 is presented. Fu (2009) performing nonlinear dynamic analyses of a 20 story 3D structure, found that the columns that are adjacent to the removed column should be designed with an axial force twice that of the static axial force obtained when applying the DL+0.25 LL load combination. Furthermore, Fu (2009) found that column removal in the top stories leads to higher vertical deformations because fewer stories participate in the absorption of the released energy. Mohamed (2009) analyzed 3D concrete structures and investigated the shear stresses that resulted from the torsion in the beams connected to the corner column being removed. The shear stresses in these scenarios lead to brittle failure of the beam, but the 2D analysis models could not trace them.

Ruth et al. (2006) analyzed 2D and 3D steel frames: they illustrated that using load factor of 2 may be conservative, whereas using a load factor of approximately 1.5 captures better dynamic effects when static analyses are performed. However, the authors mentioned that if the behavior of the materials was not elastic-perfectly plastic, if the materials harden after yielding, and if the ductility of structure is high, using load factor of 2 may be more appropriate. As a result, their research suggested that a load factor of 2 should be used for important structures and 1.5 for other structures.

Powel (2005) compared LS, NLS, and ND analyses and found that if a load factor of 2 is used in static analyses, it can display very conservative results. Tsai and Lin (2008) evaluated the progressive collapse resistance of RC buildings and demonstrated that nonlinear static analyses provide more conservative estimations for the collapse resistance than nonlinear dynamic analyses. They also found that the load factor decreases with
increases in the displacement of the removed column point. Sucuoglu et al. (1994) found that the frames that contained a removed column sustained most of the load that was created due to the column removal. Therefore, to evaluate the vertical displacement, the plastic hinge distribution, and rotation in 3D frames, it is sufficient to analyze the 2D frames that contain the removed column.

Kim and Kim (2009) studied the progressive collapse of the steel moment resisting frames. It was observed that the nonlinear static analyses provide smaller structural responses than nonlinear dynamic analyses. However, the linear procedure provides a more conservative decision for the potential of the structure. Khandelwal et al. (2009) analyzed braced frames designed for seismic activity and found that the eccentrically braced frame was less vulnerable to progressive collapse than the concentrically braced frame.

Kim and Dawoon (2009) investigated the effect of the catenary action on the progressive collapse potential of steel moment frame structures. According to the nonlinear static push-down analysis results, the contribution of the catenary action to the progressive collapse demonstrated that the potential of the structures increases as the number of stories and bays increases. Grierson et al. (2005) presented a method for conducting a linear static progressive collapse analysis. They modeled the reduced stiffness during the progressive collapse using an equivalent spring method.

Izzuddin et al. (2008a,b) presented a simplified method for nonlinear static analysis of steel structures. In their research, four stages of simplification were applied to the method. Lee et al. (2009) also developed a simplified trilinear model for the relationship of the vertical resistance and chord rotation of the double span beam. This model depends explicitly on the beam length (l) and beam section depth (d). A response was obtained for three values of l/d: 10, 15, and 20. Lee et al. (2009) state that for other values of l/d, linear interpolation should be used.

3. Initial Formulation

3.1. Middle column removal

Consider a fix-ended mild steel beam with a concentrated load in the mid-span, as presented in Fig. 1. The load deflection curve of this beam has four stages, as shown in Fig. 2.

Stage 1

The beam has an elastic response and the load deflection curve in this stage is linear. The following equations represent the situation:

\[ P(y) = Ky \]  \hspace{1cm} (4)

\[ K = \frac{192EI}{L^3} \]  \hspace{1cm} (5)

where

- \( K \): bending stiffness of the beam
- \( P \): load
- \( y \): mid-span deflection
- \( E \): modulus of elasticity of the beam
- \( I \): moment of inertia of the beam
- \( L \): length of the beam

By increasing \( P \), the bending moment of the beam increases until the beam reaches its maximum bending resistance at load \( P_p \) and plastic hinges form. According to Fig. 3:

\[ \sum M_o = 0 \rightarrow \frac{P_p L}{2} - \frac{L}{2} = 2M_p \rightarrow P_p = \frac{8M_p}{L} \]  \hspace{1cm} (6)

where

- \( M_p \): the plastic moment of the beam
- \( P_p \): concentrated load applied at the middle of the beam which causes the beam to enter plastic region
- \( L \): length of the beam
obtained:

\[ P(y_p) = Ky_p \]  \hspace{1cm} (7)

The displacement at the end of this stage is:

\[ y_p = \frac{M_p L^2}{24EI} \]  \hspace{1cm} (8)

It should be mentioned that the plastic hinges at the ends and mid-span of the fix-ended beam are not formed simultaneously and stage 1 is bilinear, but in this paper, for sake of simplicity, it is assumed that stage 1 is linear. The same assumption has been used in other researches (Fu, 2009; Izzuddin, 2005; Izzuddin et al., 2008a; Izzuddin et al., 2008b; Marjanishvili and Agnew, 2006; McKay, 2008; Ruth et al., 2006).

**Stage 2**

In this stage, the beam begins to yield and enters the plastic region. According to Fig. 2, the load remains constant at \( P_p \) which is easily determined from the plastic bending moment:

\[ P(y) = \frac{8M_p}{L} \]  \hspace{1cm} (9)

This response is continued until the displacement is equal to \( y_c \) as presented in (Izzuddin, 2005):

\[ y_c = \max(y_h, r_p) \]  \hspace{1cm} (10)

where
- \( y_c \): displacement at the beginning of catenary action region
- \( r_p \): plastic interaction radius
  - For typical I sections \( r_p \) is approximately half the cross section depth\(^{1/2} \left( r_p = \frac{d}{2} \right) \)

When displacement reaches \( y_c \), the beam enters Stage 3.

**Stage 3**

In this stage, the catenary action is activated and the load deflection response is (Izzuddin, 2005):

\[ P(y) = \frac{8}{L} M_p + K_e \frac{2(y - y_c)(y + y_c - 2r_p)(y - y_c)}{L} \]  \hspace{1cm} (11)

where \( K_e \) (effective axial stiffness) is determined from

\[
\frac{1}{K_e} = \frac{1}{K_{hr}} + \frac{1}{K_{hl}} + \frac{1}{K_{hr}} \]

\( EA \) is the elastic axial rigidity of the beam, and \( K_{hr} \) and \( K_{hl} \) are the stiffness of right and left frames of the removed column bays, respectively. As an example, Fig. 4 is considered to calculate \( K_{hr} \) and \( K_{hl} \). To determine the stiffness of the left bay, the stiffness of the frame in Fig. 5 should be considered.

If the number of stories is high, the frame in Fig. 5 can be simplified into frame in Fig. 6.

\[ K_{hr,i} = \frac{24nEI}{L^3} \]  \hspace{1cm} (12)

The restraint condition for each column is according to Fig. 7.

The stiffness of the column in Fig. 7 is \( \frac{12EI}{L^3} \). For general cases in which there are \( n \) columns in each side, stiffness is calculated using:

\[ K_{hr,i} = \frac{24nEI}{L^3} \]  \hspace{1cm} (12)
This response is valuable until \( y_{cr} \) reaches the value in Eq. (13) (Izzuddin, 2005):

\[
y_{cr} = y_c + \frac{F_p L}{2 K_c} \left( y_c - r_p \right) \left( y_c - r_p \right)
\]

where

\( F_p \): plastic tension force

Using the linear interaction between the moment and axial force, \( F_p \frac{M_p}{r_p} \) is obtained.

\section*{Stage 4}

Beyond \( y_{cr} \), the beam reaches its full plastic axial capacity with a zero bending moment. Therefore, the load deflection response in the final catenary stage may be expressed as (Izzuddin, 2005):

\[
P(y) = \frac{8F_p y}{L}
\]

Using the response of a single beam, the response of a beam in a frame must be formulated prior to the analysis. Consider the beam in Fig. 8.

If the deflections of the adjacent beams and columns are neglected, according to Fig. 1, the response of the beam may be calculated using equations above.

It should be noted that simplifying the frame, as in Fig. 1, presents the beam behavior as stiffer than the actual situation. As the beam in a frame is not completely restrained at the ends, the effects of rotations of the joints should be considered in the calculations. Assume that the inflection point of the columns is in the middle of their height (only the first and last story columns do not have this limitation): the load deflection response of the beam is corrected as follows (Dussenberry and Hamburger, 2006):

\[
P(y) = \frac{192EI}{L^3(1+3R_b)} y
\]

\[
y_p = \frac{M_p L^2}{24EI} \frac{(1+3R_b)}{L^2(1+3R_b)}
\]

where \( R_b \) is:

\[
R_b = \frac{L}{I+4I_{ab}+6I \frac{L}{H}}
\]

where

\( I_{ab} \): moment of inertia of the adjacent beam

\( I \): moment of inertia of adjacent columns

\( H \): length of adjacent columns

As in many practical cases, the connections do not fail until the end of catenary action behavior region (Lee et al., 2009), in this study the maximum expected displacement of the column removed point is assumed to be \( y_{cr} \) and it is assumed that connection failure will not occur.

\section*{3.2. Corner column removal}

Consider a beam with the load and restrain conditions as presented in Fig. 9.

The difference between the load deflection response of this beam and that in the beam in Fig. 2 is that, this beam has moment resistance only and does not have catenary action. By increasing \( P \), plastic hinges are formed at both ends of the beam. According to Fig. 10, the following is obtained:

\[
\sum M_0 = 0 \rightarrow P = 2M_p \rightarrow P = \frac{2M_p}{L}
\]

where

\( M_p \): plastic moment of the beam

According to the beam stiffness in the elastic stage \( (K = \frac{12EI}{L^3}) \), the final displacement in this stage is:

\[
y_p = \frac{M_p L^2}{6EI}
\]

At the end of the elastic stage, the beam enters a plastic stage and results in the following:
\[ P(y) = \frac{2M_p}{L} \]  \hspace{1cm} (20)

Displacement continues until \( y_c \) reaches the value in Eq. (10) and because the beam does not enter the catenary action, \( y_c \) is its final value.

According to the restraint rotations, the load deflection response in the elastic stage may be corrected as follows:

\[ P(y) = \frac{24EI}{L^2(2+3R_b)} y \]  \hspace{1cm} (21)

\[ y_p = \frac{M_p L^2 (1+3R_b)}{12EI} \]  \hspace{1cm} (22)

where \( R_b \) is:

\[ R_b = \frac{I}{I + 4I_{ab} + 12I \frac{L}{L_{cH}}} \]  \hspace{1cm} (23)

4. Progressive Collapse Formulation

Consider the frame in Fig. 11 under distributed load \( W \).

The gravity axial force in column \( C \) is \( R \); therefore, the behavior of this frame is similar to the frame depicted in Fig. 12.

The removal of column \( C \) is almost equivalent to replacing it by load \( R \), as in Fig. 13.

After removal of the column according to Fig. 14, vertical displacements occur in the middle of removed column bay. If it is assumed that these displacements are equal in all stories, the behavior of the beam in the removed column bay can be considered equivalent to that of the beam in Fig. 8. Thus, the equations developed in the previous section may be used.

In a similar manner, corner column removal may be modeled according to Fig. 15.

If the displacement of column removed point is \( y \), the external work done by load \( R \) after column removal scenario is:

\[ W_e = R \cdot y \]  \hspace{1cm} (24)

The internal work of the beams for each story is equal to the area under the load deflection response (Fig. 2) according to displacement \( y \). Thus, the following can be calculated:
Figure 15. Model of corner column removal by applying a transverse load.

\[ w_y = \int_0^y P \, dy \]  

Because the structural system of Fig. 14 is a parallel system, the total internal work is the sum of the internal work of each beam:

\[ W_i = \sum_{N=0}^y P \, dy \]  

where \( N \) is the number of stories above the removed column point.

Then, equating Eqs. (24) and (26) leads to:

\[ y = \frac{1}{R} \sum_{N=0}^y P \, dy \]  

It can be seen that instead of performing a dynamic nonlinear analysis using Eq. (27), the displacement of the removed column point is easily calculated.

5. Structural Safety Control

By equating Eqs. (24) and (26), the following is obtained:

\[ R = \frac{1}{\sum_{N=0}^y P \, dy} \]  

where \( R(y) \) is the capacity curve.

According to Eq. (28), the capacity curve can be constructed by dividing the accumulated stored energy by its corresponding displacement. It can be shown that this curve is identical to the nonlinear dynamic response curve of structures (Tsai and Lin, 2008; Izzuddin et al., 2008a,b). The maximum point of this curve represents the ultimate resistance of the structure against progressive collapse. Therefore, if this curve intersects with the applied load line, it indicates that the structure is safe against progressive collapse; if not, the structure risks severe damage (Abruzzo et al., 2006).

6. Proposed Method Verification

6.1. Example 1

The steel frame with \( f_y = 325 \) (MPa) presented in Fig. 16 is under a distributed load of 70 kN/m; the beams dimensions are: \( L = 6.1 \text{ m} \), \( A = 7.742 \times 10^{-3} \text{ m}^2 \), \( I = 8.325 \times 10^{-4} \text{ m}^4 \), \( F_{p} = 2001690 \text{ N} \), and \( M_{p} = 734.4 \text{ kN} \text{m} \). The column dimensions are as follows: \( L = 3.66 \text{ m} \) and \( I = 4.995 \times 10^{-4} \text{ cm}^4 \). Under an applied distributed load, the axial forces in the right column and middle column are \( R = 427 \text{ kN} \) and \( R = 854 \text{ kN} \), respectively.

Using the proposed formulation, the \( y \) values for the removal of the right and middle columns are 0.13 \( \text{m} \) and 0.15 \( \text{m} \), respectively. The responses obtained from the nonlinear dynamic analysis for the right and middle column removals were 0.14 \( \text{m} \) (Kaewkulchai and Williamson, 2004) and 0.17 \( \text{m} \) (Kaewkulchai, 2003), respectively.

In Fig. 17, the capacity curve of this structure for middle column removal and the line of axial force of the removed column are plotted. As shown, these curves intersect each other, so this structure is safe against progressive collapse.

6.2. Example 2

Consider the steel frame presented in Fig. 18. The loading, beam, and column identifications are similar to Example 1. Under an applied distributed load, the axial force in the removed column is \( R = 1281 \text{ kN} \). Using the proposed formulation, \( y \) is 0.143 \( \text{m} \). The response obtained from the nonlinear dynamic analysis is 0.16 \( \text{m} \) (Kaewkulchai, 2003).

The capacity curve of this structure for middle column removal and the line of axial force of the removed column are plotted in Fig. 19. As shown, these curves...
intersect each other, so the structure is safe against progressive collapse.

6.3. Example 3
Consider the nine story steel frame with $f_c = 345$ (Mpa) depicted in Fig. 20. Distributed loads of $32.37$ kN/m and $12.67$ kN/m are applied to the floor and roof beams, respectively. The beam lengths and column lengths are $8.25$ and $4.3$ m, respectively. All beams are W21×57. Under the applied load, the axial force in the removed column is $R = 2240,947$ kN. Using the proposed formulation, $y$ is $0.14$ m. The response obtained from the nonlinear dynamic analysis was $0.132$ m (Lee et al., 2009). A more detailed calculation of this example is presented in Appendix 1.

The capacity curve of the structure in Fig. 20 for middle column removal and the line of axial force of the removed column are plotted in Fig. 21. As shown, these curves intersect each other, so the structure is considered safe against progressive collapse.

7. Dynamic Increase Factor
The load factor, $x$, in Eqs. (1) and (2), which considers the dynamic effects in static analyses, is called the dynamic magnification factor (Biggs, 1964; Clough and Penzin, 1993) or the displacement response factor (Chopra, 2007) in structural dynamic books. Chopra (2007) expresses the displacement response factor for a single degree of freedom system with an elastic behavior under a step force according to Fig. 22 as follows:

$$DIF = 1 + \frac{\sin(\pi t_r/T_n)}{\pi t_r/T_n}$$

(29)

where

$T_n$: natural period of the structure
$t_r$: rising time of the step force

The load $R$ in Fig. 13 may be assumed as a step force with very finite $t_r$. In this case, the value of $t_r/T_n$ tends towards zero and the value of the DIF tends towards $2$. The constant value used in Eqs. (1) and (2) is obtained by assuming elastic behavior. However, if the material has plastic behavior, the value of the DIF is less than 2 (Ruth et al., 2006). According to the optimal design of structures, the assumption of elastic behavior after column removal scenario is conservative.

After analyzing 2D and 3D frames, McKay (2008)
suggested the following equations:

\[ DIF = 1.04 + \frac{0.45}{m - 0.52} \quad \text{(concrete structures)} \] (30)

\[ DIF = 1.08 + \frac{0.76}{m - 0.17} \quad \text{(steel structures)} \] (31)

where \( m \) is the displacement ductility of the structure. Stevens et al. (2008) expresses the following equation:

\[ DIF = 1.44(m-1)^{0.12} \] (32)

The DIF for a SDOF system is defined as:

\[ DIF = \Delta_{dy}/\Delta_{dy} \] (33)

where \( \Delta_{dy} \) and \( \Delta_{dy} \) are the responses obtained from dynamic and static analysis, respectively, under the same load. The DIF can be defined, according to Fig. 23, as:

\[ DIF = \frac{P_{dy}}{P_{dy}} \] (34)

where \( P_{dy} \) and \( P_{dy} \) are static and dynamic forces for a same displacement, respectively.

For any value of displacement in a load displacement curve, the value of the static to dynamic force may result in the DIF. As a result, with the load displacement curve under nonlinear static and nonlinear dynamic analyses and using Eq. (34), the DIF is obtained according to the post yield behavior of the structure. Thus, if the load displacement response of the structure in Fig. 14 is presented, the dynamic load displacement response is obtained using Eq. (28) (Tsai and Lin, 2008); according to Eq. (34), the DIF is the ratio of these two responses.

In the previous section, using the load displacement of a fixed end steel beam, the behavior of the frame after the removal of the column is simulated. As the load displacement response of the beam in the catenary action region is nonlinear, and because many parameters contribute to its response, it is not possible to obtain an explicit expression for DIF. Therefore, a simpler tri-linear load displacement curve is used. Finally, using Eq. (34), an explicit expression for DIF is expressed. To compare these results with previous researches (Eqs. (30)-(32)), an expression is derived for the elastic-perfectly plastic behavior.

8. Tri-linear Load Displacement Response

While plotting the load displacement response for twelve compact steel beams with span \( (l) \) to depth \( (d) \) ratios equal to 10, 15 and 20, Lee et al. (2009) found that these curves are similar for identical \( l/d \) ratios \( (l=\frac{l}{2}) \) according to Fig. 1). Therefore, they present a tri-linear curve, according to Fig. 24, for the load displacement response, as the areas below the presented curves and real curves are similar. It should be noted that the limiting value for displacement is at the end of the catenary action region.

The parameters are defined as follows (Lee et al., 2009):

\[ \alpha = 0.033, \quad \beta = 0.085, \quad y_{y/l} = 0.110, \quad y_{y/l} = 0.172, \]

for \( l/d = 10 \): (35)

\[ \alpha = 0.071, \quad \beta = 0.179, \quad y_{y/l} = 0.073, \quad y_{y/l} = 0.132, \]

for \( l/d = 15 \): (36)

\[ \alpha = 0.135, \quad \beta = 0.335, \quad y_{y/l} = 0.060, \quad y_{y/l} = 0.116, \]

for \( l/d = 20 \): (37)

For other values, \( 10 \leq l/d \leq 15 \), or \( 15 \leq l/d \leq 20 \) linear interpolation is required. Therefore, the expressions below are derived:

\[ \alpha = \begin{cases} 1/131.58 (l/d - 5.66) & (10 \leq l/d < 15) \\ 1/78.125 (l/d - 9.453) & (15 \leq l/d \leq 20) \end{cases} \] (38)

\[ \beta = \begin{cases} 1/53.19 (l/d - 5.479) & (10 \leq l/d < 15) \\ 1/32.05 (l/d - 9.263) & (15 \leq l/d \leq 20) \end{cases} \] (39)

\[ y_{y/l} = \begin{cases} -1/135135 (l/d - 24865) & (10 \leq l/d < 15) \\ -1/384165 (l/d - 43077) & (15 \leq l/d \leq 20) \end{cases} \] (40)

\[ y_{y/l} = \begin{cases} -1/125 (l/d - 31.5) & (10 \leq l/d < 15) \\ -1/312.5 (l/d - 56.25) & (15 \leq l/d \leq 20) \end{cases} \] (41)
where
\( y_1 \): displacement at the beginning of catenary action behavior in the tri-linear load-displacement curve
\( y_2 \): displacement at the end of catenary action behavior in the tri-linear load-displacement curve
\( \alpha \): slope of plastic region in the tri-linear load-displacement curve
\( \beta \): slope of catenary action region in the tri-linear load-displacement curve

According to the above expressions, in addition to the linear relations between the load and displacement, the response only depends on \( l/d \) which simplifies the derivation of an explicit expression for the DIF.

9. DIF Formulation

An explicit expression for the DIF is derived using the tri-linear load displacement curve that was introduced in the previous section. According to the calculations (detailed calculations are presented in Appendixes 2 and 3), the DIF for elastoplastic behavior with catenary action is:

\[
DIF = \frac{m\{(\lambda(m-a)+\alpha(a-1)+1\)}{-0.5+0.5\alpha(a-1)^2+0.5\beta(m-a)^2+m+(a-1)(m-a)\alpha}
\]

(42)

where \( m \) is the displacement ductility and \( \alpha = y_2/y_1 \): is:

\[
\alpha = m'\lambda
\]

(43)

for \( 10 \leq l/d \leq 15 \):

\[
\lambda = 1.0811 + \frac{0.0531}{0.184-7.4 \times 10^{-3} \times l/d}
\]

(44)

for \( 15 \leq l/d \leq 20 \):

\[
\lambda = 1.2308 + \frac{0.0421}{0.112-2.6 \times 10^{-3} \times l/d}
\]

(45)

where \( \lambda = y_2/y_1 \) is the ratio of the displacement at the end of the catenary action to the displacement at the beginning of the catenary action.

To compare the results of the proposed method with previous researches, the DIF for elastic-perfectly plastic behavior (Fig. 25) is derived as:

\[
DIF = \frac{2m}{2m-1}
\]

(46)

It should be noted that for \( m = 1 \), which represents the elastic behavior, the DIF value is equal to 2. Figure 26 plots the DIF against \( m \) for the values of \( m \) between 1.2 and 12 for Eqs. (30), (31), (32), and (46).

Figure 27 plots the DIF against \( m \) for the values of \( m \) between 1.2 and 12 for Eq. (42) for six values of \( l/d \). As can be seen, an increasing \( l/d \) causes the DIF to increase. Also, until a ductility value around 4, the increasing ductility causes the DIF to decrease; however, after 4, the DIF value increases, which is comparable with the results presented in Ruth et al. (2006).

10. Conclusion

In this paper, the formulation of a simplified method for the progressive collapse analysis of steel moment resisting frames is proposed using the load deflection response of a fix-ended beam. This formulation is obtained by equating the external work of an applied load with the internal work of the beams in the removed column bay. According
to the examples calculated using the proposed method and the results of nonlinear dynamic analyses from other researches, the proposed method is sufficiently accurate. Furthermore, an explicit expression for dynamic increase factor (DIF) was presented. Using a constant DIF equal to two for considering dynamic effects in static analysis is only appropriate for elastic systems while according to the optimal design of structures, assuming elastic behavior after column removal is conservative. The DIF is defined as the ratio of the static force to dynamic force for the same displacement. As a result, having a load displacement curve under nonlinear static and nonlinear dynamic analyses, the DIF is obtained according to the post yield behavior of the structure. For nonlinear static analyses, a tri-linear load displacement curve of a fix-ended steel beam is used. The capacity curve is used for the nonlinear dynamic responses of the structure. The DIF is plotted against the ductility curve for elastic-perfectly plastic and elastoplastic with catenary action behaviors. If the catenary action is not considered, the DIF may increase as the ductility increases. However, when the ductility reaches 12, the DIF approaches 1. If the catenary action is considered, for a ductility value of up to 4, an increasing ductility causes the DIF to decrease; however, after that point, the DIF increases. For a ductility value equal to 12, the value of the DIF is approximately 2. As a result, if the catenary action is considered and the ductility of structure is high, using load factor of 2 is not conservative.

Appendix 1:
Detailed calculation of example 3 is presented here. (for first 5 story)
\[
R_b = \frac{4.87 \times 10^{-4}}{4.87 \times 10^{-4} + 4 \times 4.87 \times 10^{-4} + 6 \times 7.908 \times 10^{-4}} = 0.0236
\]

(for last 4 story)
\[
R_b = \frac{4.87 \times 10^{-4}}{4.87 \times 10^{-4} + 4 \times 4.87 \times 10^{-4} + 6 \times 4.158 \times 10^{-4}} = 0.0406
\]

(\text{for first 5 story})
\[
P_i(y) = \frac{192 \times 2 \times 10^{11} \times 4.87 \times 10^{-4}}{16.5^3 (1 + 3 \times 0.0236)} = 3887764y
\]

(\text{for last 4 story})
\[
P_i(y) = \frac{192 \times 2 \times 10^{11} \times 4.87 \times 10^{-4}}{16.5^3 (1 + 3 \times 0.0406)} = 3711016y
\]

\[
M_p =ZF_y = 2.114 \times 10^{-3} \times 384.7 \times 10^9 = 813255.8
\]

\[
y_p = \frac{M_p L^2 (1 + 3 R_b)}{24EI} = \frac{813255 \times 16.5^2 \times (1 + 3 \times 0.0406)}{24 \times 2 \times 10^{11} \times 4.84 \times 10^{-4}} = 0.107 (m)
\]

Assuming elastic response and using Eq. (27):
\[
y = \frac{1}{R} \sum \int Pdy = \frac{1}{2240947} (5 \times 3887764 \times 0.5 \times y^2 + 4 \times 0.5 \times 0.0236) \Rightarrow y = 0.13 (m)
\]

As \( y > y_n \) the above assumption is not true and the response may be plastic.

(\text{for all stories})
\[
P(y) = \frac{8 M_p}{L} = \frac{8 \times 813255}{16.5} = 394305.94
\]

\[
y_c = \max(y_n, r_p) = \max(0.107, \frac{0.5359}{2}) = 0.268
\]

Assuming plastic response and using Eq. (27):
\[
y = \frac{1}{2240947} (5 \times 3887764 \times 0.5 \times 0.107^2 + 4 \times 3711016 \times 0.5 \times 0.107^2 + 9 \times 394305.94 \times (y - 0.107)) \Rightarrow y = 0.14 (m)
\]

As \( y < y_c \) the response may be plastic and the answer is \( y = 0.14 (m) \).

Appendix 2:
(a) Dynamic Increase Factor expression by considering catenary action
\[
P = \beta (y - a) + 1 + \alpha (a - 1)
\]
\[
R = \frac{1}{y} \left[ \int_0^a y dy + \int_a^{y_1} (\beta (y - a) + 1 + \alpha (a - 1)) dy \right]
\]
\[
= \frac{1}{y} \left[ -0.5 + 0.5 \beta (a - 1)^2 + 0.5 \beta (y - a)^2 + y + (a - 1) (y - a) \alpha \right]
\]
\[
DIF = \frac{P}{R}_{y=m}
\]
\[
DIF = \frac{m \left( \beta (m-a) + \alpha (a-1) \right)}{-0.5 + 0.5 \beta (a-1)^2 + 0.5 \beta (m-a)^2 + m + (a-1) (m-a) \alpha}
\]

(b) Dynamic Increase Factor expression for elastoplastic behavior
\[
R = \frac{1}{y} \left[ \int_0^a y dy + \int_a^{y_1} y dy \right] = \frac{1}{y} (y - 0.5)
\]
\[
DIF = \frac{P}{R}_{y=m} = \frac{2m}{2m-1}
\]
Appendix 3:
Demonstration of Eqs. (43) and (44):

For $10 \leq d \leq 15$, according to Eqs. (40) and (41):

$$\lambda = \frac{24.865 - 135.135 \times y_f/d}{31.5 - 125 \times y_f/d}$$

$$y_f/l = 0.925 \times y_f/l - 0.0491$$

$$l = \frac{0.925 y_f}{y_f/l} \times 0.0491$$

According to Eq. (40):

$$y_f/l = 0.184 \times 7.4 \times 10^{-3} \times l/d$$

Therefore we have:

$$\lambda = y_2/y_1 = 1.0811 + \frac{0.0531}{0.184 - 7.4 \times 10^{-3} \times l/d}$$

As $\lambda = y_2/y_1$ and $m = y_2/y_f$:

$$\alpha = m/\lambda$$

For $15 \leq d \leq 20$, Eq. (45) may be proved in a similar way.

Notation:
- $\gamma$: Load factor.
- $DL$: Dead load.
- $LL$: Live load.
- $WL$: Wind load.
- $DCR$: The ratio of member force and member strength.
- $Q_{ud}$: Acting force determined in the member.
- $Q_{ce}$: Expected ultimate unfactored capacity of that member.
- $l$: Span length.
- $d$: Beam section depth.
- $K$: Bending stiffness of the beam.
- $E$: Modulus of elasticity of the beam.
- $I$: Moment of inertia of the beam.
- $L$: Length of beam.
- $y$: Mid-span displacement of the beam.
- $P$: Concentrated load applied at the middle of the fixed-ended beam.
- $P_f$: Concentrated load applied at the middle of the beam which causes the beam to enter plastic behaviour.
- $M_p$: Plastic moment of the beam.
- $y_p$: Displacement at the beginning of plastic behaviour.
- $r_p$: Plastic interaction radius.
- $K_a$: Effective axial stiffness.
- $y_c$: Displacement at the beginning of catenary action behaviour.
- $EA$: Elastic axial rigidity of the beam.
- $K_{br}$: Stiffness of right frame of the removed column bay.
- $K_{bh}$: Stiffness of left frame of the removed column bay.
- $n$: Number of columns in each side of the column removed bay.
- $y_c$: Displacement at the end of catenary action behaviour.
- $F_p$: Plastic tension force.
- $K_w$: Factor to consider the rotations of beams and columns which are adjacent to removed column.
- $I_{bc}$: Moment of inertia of beams adjacent to removed column.
- $I_c$: Moment of inertia of columns adjacent to removed column.
- $H$: Length of columns adjacent to removed columns.
- $R$: Axial force in the removed column.
- $W_{E}$: the external work done after column removal scenario.
- $w_c$: The internal work done by each beam.
- $W_i$: Total internal work done after column removal scenario.
- $N$: Number of stories above the column removed point.
- $f_y$: Yield tension of steel.
- $A$: Area of beam section.
- $t_r$: Rising time of the step force.
- $T_p$: Natural period of the structure.
- $m$: Displacement ductility.
- $\delta$: Response obtained from dynamic analysis.
- $\delta_s$: Response obtained from static analysis.
- $P_{u}$: Static force that cause a known displacement.
- $P_{d}$: Dynamic force that cause a known displacement.
- $y_{i}$: Displacement at the beginning of catenary action behavior in the tri-linear load-displacement curve.
- $y_{f}$: Displacement at the end of catenary action behavior in the tri-linear load-displacement curve.
- $\alpha$: The slope of plastic region in the tri-linear load-displacement curve.
- $\beta$: The slope of catenary action region in the tri-linear load-displacement curve.
- $\gamma$: The ratio of the displacement at the end of the catenary action to the displacement at the beginning of the catenary action.
- $\alpha$: The ratio of the displacement at the end of the plastic behavior to the displacement at the beginning of the plastic behavior.
- $Z$: Section modulus.

References


Progressive Collapse Analysis of Steel Frames: Simplified Procedure and Explicit Expression for Dynamic Increase Factor

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