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Supplier Selection Based on Fuzzy Inertial Capability Index

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Abstract: The process capability index C_{pi}, which considers the process variance and departure of the process mean from the target value, is important in the manufacturing industry to measure process potential and performance. This paper extends its applications to calculate the process capability index $\tilde{C}_{pi}$ of fuzzy numbers by using Buckley's approach. In fact, the α-cuts of fuzzy observations are first derived based on various values of α, and then, for each supplier the membership function of fuzzy process capability index $\tilde{C}_{pi}$ is constructed based on the α-cuts of fuzzy observations. We apply the Dp,q-distance between fuzzy numbers as a criteria to choose the preferable suppliers. An illustrated example of LED components demonstrates that the proposed method is effective and feasible for the evaluation of competing process capability.

1 INTRODUCTION

Process capability indices have been widely used in the manufacturing industry as a process performance measure. Process capability can be broadly defined as the ability of a process to meet customer expectations which traditionally are defined as specification limits (SLs). This comparison is made by forming the ratio of the width between the process specification limits to the width of the natural tolerance limits which is measured by six process standard deviation units [10]. Important traditional PCIs introduce as follows

$$C_p = \frac{USL - LSL}{6\sigma},$$

$$C_{pk} = C_p(1-k), k = \frac{|\mu - M|}{\frac{USL - LSL}{2}},$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - M)^2}},$$

$$C_{pmk} = \frac{USL - LSL - 2|\mu - M|}{6\sqrt{\sigma^2 + (\mu - M)^2}}.$$

The process capability analysis (PCA) produces important results that show us the process ability satisfying specification limits (SLs) and the ratios of conforming (CIs) and nonconforming (NCIs) items which are the probabilities of producing within and out of SLs. However, there are some limitations which prevent a deep and flexible analysis if the process distribution has not normal or when we have a assembly product.

The traditional tolerancing considers the conformity of a batch when the batch satisfies the specifications. The characteristic is considered for itself and not regarding its incidence on the final assembly resultant. The inertia is not tolerated by a tolerance interval but by a scalar representing the maximum inertia that the characteristic should not exceed. It has been showed that inertial tolerancing proposes another tolerancing method to guarantee the final assembly while allowing larger variability in the case of centered production. The inertial process capability index (C_{pi}) is defined based on the inertial tolerancing that it has many properties, particularly in the case of the mixed batches and it is independent from process distribution. Pillet [13] defined the C_{pi} capability index as follows

$$C_{pi} = \frac{I_{max}}{I_{Batch}} = \frac{I_{max}}{\sqrt{\sigma^2 + \delta^2}}, \quad (1)$$

which indicates the capability considering the process off-centering.

The process capability indices (PCI) have been popularly accepted in the manufacturing industry as management tools for evaluating and improving process quality.

PCIs

Yongting [9] defined a fuzzy tolerance interval and introduced a process capability index, C_p, as a real number and it was used by Sadeghpour-Gildeh [4]. Parchami, Mashinchi and Maleki [2] obtained fuzzy confidence interval for fuzzy process capability ($\tilde{C}_{pm}$) when SLs are imprecise. Tsai and Chen [7] proposed an analytic approach to test the index C_p with fuzzy observations. Chen and Lai and Nien [8] used the fuzzy analytic method concerning process capability index C_{pm} and calculated $\tilde{C}_{pm}$ for fuzzy observation. Perakis and Xekalaki [12] constructed confidence interval for the index C_{pm} with crisp data. Liu, Tang and Zhang [17] considered a new chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables. Parchami and Mashinchi [1] introduced an algorithm based on Buckley's estimation approach, and use a family of confidence intervals to estimate process capability indices C_p, C_{pk} and C_{pm}. Bi-Min Hsu and Ming-Hung Shu [3] developed a realistic approach that allows the consideration of imprecise output data resulting from the measurements of the products quality.

In this paper, by using Buckley's approach obtain a fuzzy estimation for $\widetilde{C}_{pi}$ index and apply this estimation for sorting suppliers [11] in order to selecting the preferable suppliers. We use Dp,q distance [4] for ranking membership functions $\widetilde{C}_{pi}$.

2 MEMBERSHIP FUNCTIONS OF FUZZY ESTIMATIONS FOR $\overline{X}$ AND S_n^2

We refer to Buckley's approach [6] with some modifications to obtain fuzzy numbers for parameter estimation from a set of confidence intervals. Let Y be a random variable with probability density function (pdf) f(y,θ). In practical situations, θ is unknown and it must be estimated from a random sample $Y_1, Y_2, ..., Y_n$. Let $\pi(Y_1, Y_2, ..., Y_n)$ be a statistic used to estimate θ. Given the values of these random variables Y_i, $i = 1,2,..., n$, we obtain a point estimate $\hat{\theta} = \pi(Y_1, Y_2, ..., Y_n)$ for θ. We would never expect this point estimate be exactly equal θ , so a $(1-\alpha)100\%$ confidence interval for θ is also required to compute. In this confidence interval, one usually sets α equal to 0.025, 0.05 or 0.1.

By using Buckley's approach the α -cuts of $\widetilde{\overline{X}}$ and $\widetilde{S}_n^2$ are obtained as follows

$$\widetilde{\overline{X}}[\alpha] = \left[\overline{X} - t_{1-\alpha/2,n-1} \frac{S_n}{\sqrt{n-1}}, \overline{X} + t_{1-\alpha/2,n-1} \frac{S_n}{\sqrt{n-1}} \right]$$

$$\widetilde{S}_n^2[\alpha] = \left[\frac{nS_n^2}{\chi^2_{1-\alpha/2,n-1}}, \frac{nS_n^2}{\chi^2_{\alpha/2,n-1}} \right],$$

where $t_{1-\alpha/2,n-1}$ is the $(1-\alpha/2)$ percentile of the t distribution with n-1 degrees of freedom and $\chi^2_{\alpha/2,n-1}$ is the $(\alpha/2)$ percentile of the ordinary central χ^2 with n-1 degrees of freedom. Now place these α -cut intervals, one on top up of the other, to produce triangular shaped fuzzy numbers $\widetilde{\overline{X}}$ and $\widetilde{S}_n^2$, respectively. In this way we use more information than just a point estimate, or just a single interval estimation.

3 MEMBERSHIP FUNCTION OF FUZZY ESTIMATOR FOR C_{PI}

From equation (1), choosing $g \in \widetilde{\overline{X}}[\alpha]$ and $h \in \widetilde{S}_n^2[\alpha]$, we obtain

$$\widetilde{C}_{pi}(g,h) = \frac{I_{max}}{\sqrt{h + (g-T)^2}}.$$

If $\overline{X} \le T$, $\widetilde{C}_{pi}(g,h)$ is increasing function of g and decreasing function of h. In the case of $\overline{X} \ge T$, $\widetilde{C}_{pi}(g,h)$ is decreasing function of g and increasing function of h. We can formulate the α -cut of the fuzzy estimator $\widetilde{C}_{pi}$, $\widetilde{C}_{pi}[\alpha]$ for all $\beta \in (0,1)$ as follows

$$\overline{X} \le T \quad \begin{cases} l_{\widetilde{C}_{pi}} = \dfrac{I_{max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{\alpha/2,n-1}} + (\overline{X} - t_{1-\alpha/2,n-1}\dfrac{S_n}{\sqrt{n-1}} - T)^2}} \\ \psi_{\widetilde{C}_{pi}} = \dfrac{I_{max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{1-\alpha/2,n-1}} + (\overline{X} + t_{1-\alpha/2,n-1}\dfrac{S_n}{\sqrt{n-1}} - T)^2}} \end{cases}$$

$$\overline{X} \ge T \quad \begin{cases} l_{\widetilde{C}_{pi}} = \dfrac{I_{max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{\alpha/2,n-1}} + (\overline{X} + t_{1-\alpha/2,n-1}\dfrac{S_n}{\sqrt{n-1}} - T)^2}} \\ \psi_{\widetilde{C}_{pi}} = \dfrac{I_{max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{1-\alpha/2,n-1}} + (\overline{X} - t_{1-\alpha/2,n-1}\dfrac{S_n}{\sqrt{n-1}} - T)^2}} \end{cases}$$

4 DP,Q-DISTANCE

The Dp,q-distance, indexed by parameter 1< p <∞, 0 ≤q ≤ ∞, between two fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ is a non-negative function on F(R) × F(R) given as follows

$$D_{p,q}(\widetilde{A},\widetilde{B}) = \begin{cases} \left[(1-q) \int_0^1 \left|A_\alpha^- - B_\alpha^-\right|^p d\alpha + q \int_0^1 \left|A_\alpha^+ - B_\alpha^+\right|^p d\alpha \right]^{\frac{1}{p}} & if\ p < \infty \\ (1-q) \sup\limits_{0<\alpha\le 1}\left(\left|A_\alpha^- - B_\alpha^-\right|\right) + q \inf\limits_{0<\alpha\le 1}\left(\left|A_\alpha^+ - B_\alpha^+\right|\right) & if\ p = \infty. \end{cases}$$

The analytical properties of Dp,q depend on the first parameter p, while the second parameter q of Dp,q characterizes the subjective weight attributed to the sides of the fuzzy numbers. If there are no reason to distinguish any side of fuzzy numbers, $D_{p,1/2}$ is recommended.

Definition 4.1:

Let R be the set of real numbers. Set: $F(R) = \{\widetilde{A} \mid \widetilde{A} : R \to [0,1], \widetilde{A}\ \text{is a continuous function}\}$ and $F_T(R) = \{tri(a,b,c) \mid a,b,c \in R, a \le b \le c\}$, where

$$tri(a,b,c) = \begin{cases} (x-a)/(b-a) & if\ a \le x \le b, \\ (c-x)/(c-b) & if\ b \le x \le c, \\ 0 & elsewhere. \end{cases} \qquad (2.1)$$

Any $\widetilde{A} \in F(R)$ is called a fuzzy set on R and any $tri(a,b,c) \in F_T(R)$ is called a triangular fuzzy number.

Proposition:

Assume that $\widetilde{A} = tri(a_1, a_2, a_3)$ and $\widetilde{B} = tri(b_1, b_2, b_3)$, the α -cuts of $\widetilde{A}$ and $\widetilde{B}$ are as follows

$$A_\alpha = [(1-\alpha)a_1 + a_2\alpha, a_2\alpha + (1-\alpha)a_3],$$

$$B_\alpha = [(1-\alpha)b_1 + b_2\alpha, b_2\alpha + (1-\alpha)b_3].$$

It can establish that

$$\left[D_{2,\frac{1}{2}}(\widetilde{A},\widetilde{B})\right]^2 = \frac{1}{6}\left[(b_1-a_1)^2+2(b_2-a_2)^2+(b_3-a_3)^2+(b_1-a_1)(b_2-a_2)+(b_3-a_3)(b_2-a_2)\right].$$

Definition 4.2:
Fuzzy number A is positive (negative) if $\forall x \le 0 \;\; \widetilde{A}(x)=0$ ($\forall x \ge 0 \;\; \widetilde{A}(x)=0$).

Definition 4.3:
The central $D_{2,q}$-mean square dispersion of $\widetilde{X}$ about $\widetilde{E}(\widetilde{X})$ (or $\widetilde{\mu}_{\widetilde{X}}$) is called $DVAR(\widetilde{X})$ given by the value (if it exists)

$$D\,\text{var}(\widetilde{X}) = E([D_{2,q}(\widetilde{X},\widetilde{\mu}_{\widetilde{X}})]^2) = \int_{\Omega}\left[(1-q)\int_0^1\left(X_\alpha^-(w)-(\mu_{\widetilde{X}})_\alpha^-\right)^2 d\alpha + q\int_0^1\left(X_\alpha^+(w)-(\mu_{\widetilde{X}})_\alpha^+\right)^2 d\alpha\right]dp(w).$$

We estimate $D\,\text{var}(\widetilde{X})$ by $\hat{D}\,\text{var}(\widetilde{X})$ given by

$$\hat{D}\,\text{var}(\widetilde{X}) = \frac{1}{n}\sum_{i=1}^{n}\left[D_{2,q}\left(\widetilde{X},\widetilde{\widetilde{X}}\right)\right]^2.$$

We propose that use Dp,q-distance for comparison two positive fuzzy numbers. With calculate the Dp,q-distance between positive fuzzy numbers and zero $\left(\widetilde{0}=tri(0\ 0\ 0)\right)$, and we obtain a criterion for sorting them.

Manufacturers increasingly purchase components from suppliers or hire contract manufacturers to produce necessary parts, and they assemble these parts to deliver finished products to customers. The quality of parts obtained from suppliers determines the quality of the finished products produced by manufacturers as well as the customers' satisfaction and loyalty. Therefore, the evaluation of supplier performance and supplier selection is becoming major challenges faced by the manufacturing and purchasing managers. Now we study a numerical example.

5 NUMERICAL EXAMPLE

We study the example given in [11].
In this example due to high demands on the LED-LFs, the company does not have enough production capacity to supply one type of LED components used in the LED-LFs. Therefore, the decision-makers decide to purchase the LED components from some possible suppliers. The luminous intensity of LED sources is a critical characteristic for this type of LEDs. The decision-makers need to choose preferable suppliers based on the fuzzy sample data of the luminous intensity which have been collected from each supplier with size 20, as listed in Table 1, where the data $\widetilde{x}_{in}=T(x_{in1},x_{in2},x_{in3})$ with i=1,2,3,4 and n =1,2,3,…,20 are assumed as triangular fuzzy numbers.

Maximum inertial is 8.33 and T=66.2. The mean for each supplier is a triangular fuzzy number that given in Table 2 and Figure 1 indicate $\widetilde{\widetilde{X}}$'s.

Table 2: for each supplier

Supplier	$\widetilde{\widetilde{X}}$
S1	[66.8840 68.2495 69.6950]
S2	[65.1645 66.1930 67.5620]
S3	[63.9710 66.2055 68.1680]
S4	[60.3990 64.8200 70.3995]

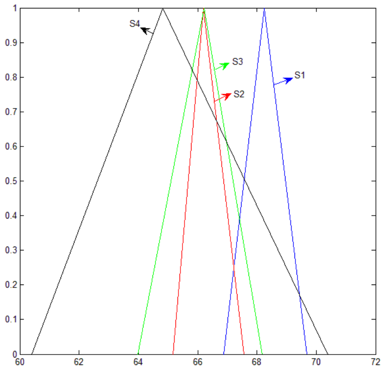

Figure 1: $\widetilde{\widetilde{X}}$ for each Supplier

In order to obtain membership function for $\widetilde{\widetilde{X}}$ and $\widetilde{S}_n^2$ by using Buckley's approach, we estimate $\widetilde{\widetilde{X}}$ with root of Dp,q-distance of $\widetilde{\widetilde{X}}$ than zero $\left(\widetilde{0}=tri[0\ 0\ 0]\right)$ and $\widetilde{S}_n^2$'s with $\hat{D}\,\text{var}$ such as Table 3.

Table 3

Supplier	$\sqrt{D_{2,1/2}(\widetilde{\widetilde{X}},\widetilde{0})}$	$\hat{D}\,\text{var}$
S1	68.2743	20.3124
S2	66.2818	25.4101
S3	66.1486	18.8456
S4	65.1738	20.7705

Thus membership function for $\widetilde{\widetilde{X}}$, $\widetilde{S}_n^2$ and $\widetilde{C}_{pi}$'s with Buckley's approach indicate in Figure 2, Figure 3 and Figure 4 respectively.

In this paper we apply $\widetilde{C}_{pi}$ as criterion for selection supplier's. In rder to sorting $\widetilde{C}_{pi}$'s, we calculate the Dp,q distance between $\widetilde{C}_{pi}$ and $\widetilde{0}$. Thus with considering Table 4 sort of selection suppliers is $\{S3, S4, S1, S2\}$.

Table 4: Dp,q distance C_{pi}'s of $\widetilde{0}$

Supplier	$D_{2,1/2}(\widetilde{C}_{pi},\widetilde{0})$
S1	2.9142
S2	2.3024

S3	3.1091
S4	2.9569

Table 1. Triangular fuzzy data collected from suppliers

S_1	S_2	S_3	S_4
(68.26,70.46,72.18)	(72.69,73.96,75.20)	(54.24,64.27,70.69)	(58.53,60.58,66.15)
(70.22,72.88,73.10)	(57.90,58.69,60.26)	(66.67,72.00,73.61)	(54.79,61.16,69.86)
(62.26,63.52,66.82)	(76.09,77.08,79.03)	(66.31,70.38,74.15)	(58.37,58.92,72.53)
(66.64,68.10,70.09)	(64.40,66.80,68.54)	(59.89,66.92,71.78)	(55.20,56.13,66.92)
(67.80,69.17,70.22)	(67.28,68.07,69.70)	(69.28,70.23,73.45)	(59.92,65.08,65.57)
(65.33,66.79,68.20)	(64.46,65.61,67.98)	(65.90,66.92,67.90)	(65.16,70.91,73.60)
(59.90,61.71,62.90)	(65.90,67.48,68.67)	(62.73,63.03,64.02)	(54.03,58.64,61.83)
(68.54,69.38,70.32)	(67.67,68.12,69.29)	(59.88,60.12,62.54)	(66.50,66.63,72.08)
(72.10,73.28,74.90)	(67.25,68.62,68.98)	(69.25,70.03,71.32)	(62.17,65.77,66.84)
(72.19,74.26,75.32)	(60.80,61.01,62.20)	(70.00,71.39,72.12)	(65.24,73.35,79.83)
(72.69,73.96,75.20)	(65.90,66.92,67.90)	(63.89,64.74,66.23)	(65.65,70.76,72.17)
(57.90,58.69,60.26)	(62.73,63.03,64.02)	(65.80,66.85,67.92)	(68.26,72.26,75.81)
(76.09,77.08,79.03)	(59.88,60.12,62.54)	(54.42,55.75,57.34)	(63.88,64.98,71.10)
(64.40,66.80,68.54)	(69.25,70.03,71.32)	(67.56,69.47,70.22)	(61.98,62.58,74.81)
(67.28,68.07,69.70)	(70.00,71.39,72.12)	(59.42,60.12,61.78)	(58.42,59.82,63.24)
(64.46,65.61,67.98)	(63.89,64.74,66.23)	(64.90,68.48,69.67)	(58.59,68.87,70.05)
(65.90,67.48,68.67)	(65.80,66.85,67.92)	(62.69,63.22,65.79)	(55.25,66.51,68.72)
(67.67,68.12,69.29)	(54.42,55.75,57.34)	(68.35,69.72,70.98)	(54.72,66.88,74.94)
(67.25,68.62,68.98)	(67.56,69.47,70.22)	(62.94,63.51,64.20)	(63.69,65.03,75.51)
(60.80,61.01,62.20)	(59.42,60.12,61.78)	(65.30,66.96,67.65)	(57.63,61.54,66.43)

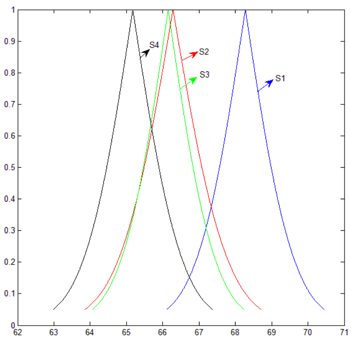

Figure 2: $\tilde{\tilde{X}}$ for each supplier with Buckley's approach

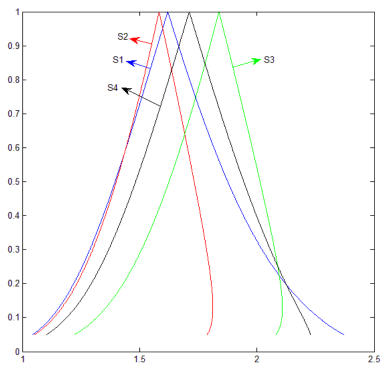

Figure 4: $\tilde{C}_{pi}$ for each suppliers with Buckley's approach

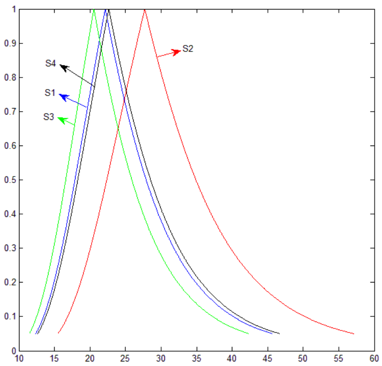

Figure 3: $\tilde{S}_n^2$ for each supplier with Buckley's approach

6 CONCLUSION

A constructive methodology for obtaining the fuzzy estimate of inertial process capability index C_{pi} with the help of "Confidence Interval" and "Dp,q-distance" is proposed in this paper. The main advantage of this methodology is that the fuzzy data are now able to be handled. By applying the Dp,q distance between two fuzzy numbers, we estimated C_{pi} and proposed a method to compare different Cpi's. Our results are illustrated by a numerical example.

REFERENCE

[1] A. Parchami and M. Mashinchi: Fuzzy estimation for process capability indices, Information Sciences, Vol. pp. 177, 1452–1462, 2007.

[2] A. Parchami, M. Mashinchi and H.R. Maleki: Fuzzy confidence interval for fuzzy process capability index, Journal of Intelligent & Fuzzy Systems, Vol. 17, pp. 287-295, 2006.

[3] B. M. Hsu and M. H. Shu: Fuzzy inference to assess manufacturing process capability with imprecise data,

European Journal of Operational Research, Vol. 186, pp. 652–670, 2008.

[4] B. Sadeghpour Gildeh and D. Gien: Dp,q-distance and the correlation coeficient between two fuzzy random variables, Rencontres Franceophones sur la logique floue et ses applications, Mons, Belgique, pp. 97-101, 2001.

[5] B. Sadeghpour-Gildeh: Comparison of Cp, Cpk and $\widetilde{C}_p$ Process Capability Indices in the case of Measurement Error Occurrence, IFSA World Congress, Istanbul, Turkey, pp. 563–567, 2003.

[6] J. J. Buckley and E. Eslami: Uncertain probabilities II: The continuous case. Soft Computing , Vol. 8, pp. 193–199, 2004.

[7] C. C. Tsai and C. C. Chen: Making decision to evaluate process capability index Cp with fuzzy numbers, Int J Adv Manuf Technol Vol. 30, pp. 334–339, 2006.

[8] C. C. Chen, C. M. Lai and H. Y. Nien: Measuring process capability index Cpm with fuzzy data, Qual Quant , Vol. 44, pp. 529–535, 2010.

[9] C. Yongting, Fuzzy quality and analysis on fuzzy probability: Fuzzy Sets and Systems, Vol. 83, pp. 283–290,1996.

[10] D. C. Montgomery: Introduction to statistical quality control (5th ed.), John Wiley & Sons, Inc, 2005.

[11] M. H. Shu and H. C. Wu: Quality-based supplier selection and evaluation using fuzzy data, Computers & Industrial Engineering , Vol. 57 , pp. 1072–1079, 2009.

[12] M. Perakis and E. Xekalaki: A new method for constructing confidence intervals for the index Cpm. Quality and reability engineering international, Vol. 20, pp. 651-665, 2004.

[13] M. Pillet: Inertial tolerancing, The TQM magazine, Vol. 16, No. 3, pp. 202-209, 2004.

[14] P.A. Adragna, M. Pillet, F. Formosa and S.Samper, Inertial tolerancing and capability indices in an assembly production, Revuev Internationale d'Ingenierie Numerique , Vol. 2, No. 1-2 , pp. 71-88, 2006.

[15] R. A. Boyles, The Taguchi capability index, Journal of Quality Technology , Vol. 23, pp. 17–26, 1991.

[16] W. L. Pearn and M. H. Shu: Manufacturing capability control for multiple power distribution switch processes based on modified MPPAC, Microelectronics & Reliability, Vol. 43, No. 6, pp. 963–975, 2003.

[17] H. Liu, Y. Tang and H. H. Zhangc: A new chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables, Computational Statistics and Data Analysis, Vol. 53, pp. 853-856, 2009.