Low $\alpha^2$ proton structure function, using gluon and pseudoscalar meson clouds in the constituent quark framework

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1. Introduction

In hard scattering events, mesons and baryons can be viewed as bound states built up from partonic constituents, i.e. quarks and gluons. This picture changes at low energies, where hadronic effects play a more prominent role in the non-perturbative structure of hadrons. Of particular importance are the pion cloud effects which, for example, have a direct impact on the spin structure of the proton [1]. These need to be incorporated in bound-state calculations aiming at a realistic description of mesons and baryons. Pion effects on quark propagation are important for several reasons. They account for (at least part of the) pion cloud effects in baryons and mesons. Furthermore, they allow for the possibility of hadronic intermediate...
states in bound state calculations and therefore generate part of the finite width of meson spectral functions.

Dynamical chiral symmetry breaking (CSB) is one of the most important properties of low-energy QCD. The breaking pattern has a profound impact on phenomenological quantities, for instance, the appearance of pseudoscalar Goldstone bosons in the chiral limit of QCD and the non-degeneracy of chiral partners. Chiral perturbation theory \cite{2, 3} describes these effects very efficiently on the level of hadrons but has nothing to state about the underlying structure of the full theory. The interplay between the fundamental quark and gluon degrees of freedom and the resulting bound states is also particularly interesting. In full QCD there are hadronic contributions to the fully dressed quark–gluon interaction. These effects are generated by the inclusion of dynamical sea quarks in the quark–gluon interaction and are therefore only present in unquenched QCD. The quark–gluon vertex is also an important ingredient in the quark–antiquark interaction that is responsible for the formation and properties of bound states. On a perturbative level, the quark–gluon vertex has been studied in detail in arbitrary gauges and numbers of dimensions in \cite{4}. However, the nonperturbative properties of this vertex are also still under intensive scrutiny.

At low energies, the idea that baryons are made up of three constituent quarks and mesons of a (constituent) quark–antiquark pair, the naive quark model scenario, accounts for a large number of experimental facts. The quest for a relationship between the two regimes, i.e. between the current quarks of the theory and the constituent quarks of the model, has a long history and, in recent years, this search has been the subject of a considerable research effort. The fundamental problem one would like to understand is how confinement, i.e. the apparent absence of color charges and dynamics in hadron physics, is realized. Detailed quark models of the hadron structure based on the constituent quark concept have been defined in order to explain low-energy properties \cite{5}. To proceed from these models to the asymptotic regime, where deep inelastic scattering (DIS) takes place, a hadronic scale is associated with the model calculations. The experimental conditions are achieved by projecting the leading twist component of the observable and evolving according to perturbative QCD. The procedure describes successfully the gross features of the DIS results. It was long ago, at the time that QCD was being proposed, that a procedure, hereafter called ACMP (Altarelli–Cabibo–Miani–Petronzio) \cite{6}, was developed to understand the relation between the constituent quarks and the partons. In this approach, constituent quarks are complex objects, made up of point-like partons (current quarks, antiquarks and gluons), interacting via a residual interaction described by a quark model. The hadron structure functions are obtained as a convolution of the constituent quark wavefunction with the constituent quark structure function. This procedure has been recently reviewed to estimate the structure function of the pion with success. In the ACMP approach, each constituent quark is dressed by a neutral cloud of quark–antiquark pairs and gluons; thus, this scenario supports the confinement mechanism. A few years earlier a second approach had been developed \cite{7}, in which the proton is assumed to be made out of three valence quarks plus a neutral core of quark–antiquark pairs and gluons, very much in the spirit of recent developments of the Manohar–Georgi model \cite{8}. In modern language this duality of approaches has to do with the implementation of CSB. The naive models do not contain spontaneous CSB and this phenomenon has to be implemented if they are to represent QCD at low energies.

The effective chiral quark theory \cite{8} may provide an alternative explanation to that of the traditional meson cloud approach \cite{9}. In this theory, the relevant degrees of freedom are constituent quarks, gluons and Goldstone bosons. The chiral quark model (\(\chi QM\)) includes both gluon and pion exchange between constituent quarks together with corresponding exchange currents. The relevant degrees of freedom and the related question of whether the
pions effectively couple to the nucleon or to the constituent quarks are extensively discussed in [9, 10]. It is necessary to study the consequences of these different scenarios in a broad range of physical processes to assess their validity.

On the other hand, the $\chi QM$ can be used to study the flavor structure of the constituent quark model and the nucleon within the conventional mesonic cloud picture. Using this model the effects of $SU(3)_f$ symmetry breaking can be discussed [11]. The implications of the Gottfried sum rule (GSR) violation for the $\Delta$-n mass splitting were also considered in [11]. At a low-energy resolution scale the constituent quark picture successfully describes hadronic structure functions. The sea quark and gluonic degrees of freedom are assumed to be absorbed into constituent quarks to be considered as quasi-particles [12]. A relation between the two regimes of hadron structure function description, i.e. the chiral quark and the constituent quark models, has a considerable significance which has been investigated widely in the literature, and has attracted much attention in recent years [13].

There are two main ingredients of this paper. In continuation of our previous work [14] we add a gluon cloud to the $\chi QM$ whilst we use an effective Lagrangian at low $Q^2$ values. We resort to a constituent quark model to extract parton densities inside the proton. Since the gluon densities are also available to us, we are able to calculate the $F_2$ structure function for the proton at the NLO perturbative QCD level of approximation.

The organization of the paper is as follows: in section 2 we introduce a quark–meson and similarly a quark–gluon vertex function based on nucleonic Sullivan DIS [15]. Section 3 is allocated to a discussion of the $\chi QM$ and the constituent quark distribution is obtained. In addition we consider the gluon cloud in the constituent quark model. Therefore, we can calculate the gluon distribution function inside the proton. This contribution was not considered in [11]. In this section we also discuss about the phenomenological valon model [16] and its use in extracting valence distributions in the meson. These distributions are required in order to obtain mesonic anti-quark contributions in the constituent quark model. In section 4 we present our result for the $F_2$ structure function in NLO approximation, using the parton densities in the proton which are extracted from the $\chi QM$. To confirm the validity of our calculation, we evolve the gluon distribution to high $Q^2$ to get the momentum fraction of the proton which is carried by gluons. Using the antisymmetric property of sea quark densities which results from the $\chi QM$, we also calculate the GSR to test again the validity of our calculations. Our conclusions are given in section 5.

2. Chiral quark model and effective quark–meson and quark–gluon interactions

This model was introduced by Georgi and Manohar [8] in order to incorporate the chiral symmetry of QCD into the successful features of the constituent quark model [17]. The prime assumption of the model is the possible realization of an effective Lagrangian between the scale of CSB $\Lambda_\chi$ and the confinement scale $\Lambda_{QCD}$. The dynamical degrees of freedom here are constituent quarks, pseudoscalar mesons and arguably gluons. The Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = i \bar{Q} \gamma_\mu (\partial^\mu + i g_5 G^\mu) Q - \frac{g_A}{f_\pi} \bar{Q} \partial_\mu U \gamma^\mu \gamma_5 Q - M_Q \bar{Q} Q + \frac{f_\pi}{4} \text{trac}(D_\mu U D^\mu U^\dagger) - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}. \quad (1)$$

The matrix $U$ contains the pseudoscalar mesons and $Q$ stands for the constituent quark. $G^{\mu\nu}$ is the field strength tensor defined as follows:

$$G^{\mu\nu,a} = \partial^\mu G_{\nu,a} - \partial^\nu G^{a,\mu} - i f^{abc} G^b_\mu G^c_\nu. \quad (2)$$
where $G^\mu$ is the gluon field and $f^{abc}$ are the structure constants. The covariant derivative, $D^\mu$, is associated with the chiral symmetry of QCD in flavor space and is defined by

$$D^\mu = \partial^\mu - i r^\mu U + i U l^\mu$$

where $l^\mu$ and $r^\mu$ are the left-handed and right-handed chiral currents coupled to the Goldstone boson [18]. At this work chiral currents are not relevant, so we can replace the covariant derivative, $D^\mu$, with the usual derivative, $\partial^\mu$.

$M_Q$, $f_\pi$ and $g_A$ are the constituent quark mass, the pion decay constant and the axial-vector constant, respectively. The strong running coupling, $g_s$, has to be considered at some energy below $1/\Lambda_{\chi}$ and we then take it as a constant.

2.1. Quark–meson effective vertex function

The effective chiral quark model [8] is applied in order to study the pseudoscalar meson clouds in the constituent quarks [11]. In [11] it is found that the pionic clouds can explain the violation of the GSR and introduce an enhancement of the non-perturbative effects on the sea quark pairs. To this end the pion–quark splitting function is introduced in analogy to the nucleonic Sullivan DIS and expressed in [15] as

$$f_{Q \rightarrow M_Q'}(x_M, k_1^2) = \frac{g_{Q \rightarrow M_Q}^2}{16\pi^2} \frac{1}{x_M(1-x_M)} |G_{Q \rightarrow M_Q}(x_M, k_1^2)|^2 \times \frac{((1-x_M)m_Q - m_{Q'})^2 + k_1^2}{(1-x_M)(m_Q^2 - M_{M_Q'}^2)^2},$$

where $x_M$ is the longitudinal (light cone) momentum fraction of the constituent quark for the Goldstone boson and $k_1$ is the transverse momentum of the quark $Q'$. The $g_{Q \rightarrow M_Q}$ is the effective coupling constant of the pion-constituent quark:

$$g_{Q \rightarrow M_Q}^2 = \frac{g_A^2 (m_Q + m_{Q'})^2}{4 f^2},$$

where $g_A$ is the axial-vector coupling constant of the constituent quark which is equal to 1. We take for the light up and down quarks $m_Q = m_{Q'} = \frac{m_N}{2} = 313$ MeV and for the strange quark $m_s = m_{Q'} = 567$ MeV.

$M_{M_Q'}^2$ is the invariant mass squared of the $M_Q'$ system which is defined as

$$M_{M_Q'}^2 = m_Q^2 + k_1^2 + m_{Q'}^2 + k_1^2 \frac{1}{1-x_M}.$$  

The $G_{Q \rightarrow M_Q}$ is a vertex function or a phenomenological form factor for which we adopt the exponential form

$$G_{Q \rightarrow M_Q}(x_M, k_1^2) = \exp \left( \frac{m_Q^2 - M_{M_Q}^2(x_M, k_1^2)}{2\Lambda_{\chi}^2} \right).$$

$\Lambda_{\chi}$ is a cut-off parameter which can be taken equal for all fluctuations involving pseudoscalar or vector mesons. Integrating the splitting function over $k_1$ [$f_{Q \rightarrow M_Q}(x_M) = \int_0^\infty f_{Q \rightarrow M_Q}(x_M, k_1^2) dk_1^2$] and then over $x_M$ and finally summing over the intermediate quarks ($Q'$) yields

$$P_{M/Q} = |a_{M/Q}|^2 = \sum_{Q'} \int_0^1 f_{Q \rightarrow M_Q}(x_M) \, dx_M,$$

which is the probability of finding a Goldstone boson $M$ in the constituent quark $Q$. 

4
2.2. Quark–gluon effective vertex function

Gluon distributions can be obtained by dressing quarks with gluons in the nonperturbative regime with massive effective gluons \( m_{\text{eff}} \) and frozen running \( \alpha_s \). Rather heavy effective gluons \( m_{\text{eff}} > 0.4 \) GeV and small \( \alpha_s < 0.5 \) are required in order to limit the momentum carried by quarks to approximately what is required by the phenomenology [19]. Now in order to include the gluon clouds in the constituent quark picture we need to assume almost the same form of the splitting function for the gluon–quark interaction as is assumed for the quark–meson interaction. There are two main differences. The first one is that the quark–meson coupling constant should be replaced with the strong coupling constant at some low-energy scale. Secondly, we need to know the relevant vertex function for the quark–gluon interaction. The vertex function encodes the extended structure of the gluon and the constituent quarks. The extraction of the vertex function is rather difficult since it incorporates the non-perturbative effects. However, in a series of recent studies [20, 21], the authors have calculated the non-perturbative corrections to the quark–gluon vertex in the framework of the Dyson–Schwinger and Bethe–Salpeter equation. Their predictions for the light meson properties seem satisfactory [20]. We find that our ansatz where the quark–gluon vertex which is assumed similar to the quark–meson vertex has qualitatively the same momentum behavior.

Consequently, we have quark–gluon fluctuations which lead to the following splitting function:

\[
\begin{align*}
 f_{Q \rightarrow Q'}(x_g, k^2_\perp) &= C_f \frac{\alpha_s(Q^2)}{4\pi} \frac{1}{x_g(1-x_g)} |G_{Q \rightarrow Q'}(x_g, k^2_\perp)|^2 \\
 &\times \frac{((1-x_g)m_Q - m_{Q'})^2 + k^2_\perp}{(1-x_g)(m_Q^2 - M_{Q'}^2)^2},
\end{align*}
\]

(9)

where \( C_f \) is the color factor, defined by \( \frac{N^2}{N-1} \) for the SU(\( N \)) symmetry group which is equal to \( \frac{4}{3} \) with \( N = 3 \), \( x_g \) is the longitudinal (light cone) momentum fraction of the constituent quark and \( k_\perp \) is the transverse momentum of the quark \( Q' \). Here \( G_{Q \rightarrow Q'}(x_g, k^2_\perp) \) is defined like equation (7) while instead of \( M_{Q'}^2 \), a new invariant mass squared is defined by

\[
M_{Q'}^2 = \frac{m_g^2 + k^2_\perp}{x_g} + \frac{m_Q^2 + k^2_\perp}{1-x_g},
\]

(10)

where \( m_g \) implicates the effective gluon mass. Our result for both meson and gluon clouds may enhance equally in the infrared region at small \( k_\perp \). It is of course obvious that our results depend on the cut-off chosen. We choose a numerical value for the cut-off which firstly is compatible with the quoted values in [11, 12] and secondly yields the best calculated results.

The integration of the quark–gluon splitting function over \( k_\perp \) and then over \( x_g \) and finally summing over the intermediate quarks \( (Q') \) yields

\[
P_{g/Q} = |a_{g/Q}|^2 = \sum_{Q'} \int_0^1 f_{Q \rightarrow g'Q}(x_g) \, dx_g,
\]

(11)

3. Constituent quark distribution function in the chiral quark model

The constituent quark Fock state \( |Q\rangle \) can be expressed in terms of a series of light cone Fock states [11, 12]:

\[
|Q\rangle = \sqrt{Z}|q\rangle + \sum_{B} a_{B/Q}|q', B\rangle,
\]

(12)
where $|q \rangle$ is the ‘bare’ but massive state, $\sqrt{Z}$ denotes the renormalization factor for a ‘bare’ constituent quark and $|a_{B/Q}^2|$ are the probabilities of finding Goldstone bosons and gluon distribution in the constituent quark states. Then the dressed $u$- and $d$-quark Fock states are

$$|U\rangle = \sqrt{Z} |u\rangle + \frac{\sqrt{2}}{3} a_{\pi^+/U} |u, \pi^+\rangle + \frac{\sqrt{2}}{3} a_{K^+/U} |s, K^+\rangle + a_{g/U} |u, g\rangle + \cdots,$$

(13)

$$|D\rangle = \sqrt{Z} |d\rangle + \frac{\sqrt{2}}{3} a_{\pi^-/D} |d, \pi^-\rangle + \frac{\sqrt{2}}{3} a_{K^-/D} |s, K^-\rangle + a_{g/D} |d, g\rangle + \cdots.$$  

(14)

The above expressions can be depicted as in the following graphs:

\begin{align*}
U & \rightarrow \ 
\begin{array}{cccccccc}
\pi^0 & \pi^+ & K^+ & g \\
u & u & u & d & s & u & u & u \\
\end{array} + \\
D & \rightarrow \\
\begin{array}{cccccccc}
\pi^0 & \pi^+ & K^0 & g \\
d & d & d & u & d & d & d & d \\
\end{array} + ... \\
\end{align*}

In these graphs the thick lines indicate the quark propagators, the dashed lines meson fields and the wiggly curves stand for the gluons, respectively.

We consider the nucleon to be a bound state of three constituent quarks ($U$ and $D$). The quark distributions in the constituent quark, at some QCD initial scale, can be written as

$$u_U(x) = u_U^{(0)}(x) + u_U^{(i)}(x) + u_U^{(\pi)}(x) + u_U^{(g)}(x),$$  

(15)

$$u_D(x) = u_D^{(i)}(x) + u_D^{(\pi)}(x),$$  

(16)

$$d_D(x) = d_D^{(0)}(x) + d_D^{(i)}(x) + d_D^{(\pi)}(x) + d_D^{(g)}(x),$$  

(17)

$$d_U(x) = d_U^{(i)}(x) + d_U^{(\pi)}(x),$$  

(18)

$$s_U(D)(x) = s_U^{(i)}(D)(x) + s_U^{(K)}(D)(x),$$  

(19)

$$g_U(x) = u_U^{(g)}(1-x),$$  

(20)

$$g_D(x) = d_U^{(g)}(1-x).$$  

(21)

The anti-quark distributions become

$$\bar{u}_U(x) = \bar{u}_U^{(\pi,K)} = \bar{d}_D(x) = \bar{d}_D^{(\pi,K)},$$  

(22)

$$\bar{u}_D(x) = \bar{u}_D^{(\pi)} = \bar{d}_U(x) = \bar{d}_U^{(\pi)};$$  

(23)

$$\bar{s}_U(x) = \bar{s}_U^{(K)} = \bar{s}_D(x) = \bar{s}_D^{(K)}.$$  

(24)

It should be noted that for sea quark densities we have $\bar{q}_Q^{(M)} = q_Q^{(M)}$. The superscripts denoted with (0) correspond to the ‘bare’ quark distributions, those denoted with (i and g) to the intermediate quark distributions associated with mesons and gluons respectively and those denoted with (\pi) originate from mesons (pions).

The bare quark distribution in the constituent quarks has the form

$$u_U^{(0)}(x) = d_D^{(0)}(x) = \left(1 - \sum_B P_B/Q\right)\delta(x - 1),$$  

(25)
where these distributions play the role of the valence quark distributions inside the constituent quarks. In equation (25), $P_B/Q$ refers to the probability of finding a Goldstone boson and gluon in the constituent quark $Q$. So referring back to equation (12), we have $P_B/Q = |a_B/Q|^2$.

A convinced explanation for the existence of the Dirac delta function in equation (25) is as follows. The total momentum which is carried by a constituent quark, $U$ or $D$, is assumed to be 1. At low $Q^2$ value we just have valence quarks. The bare quarks, which are assumed here as valence quarks in the constituent quark, are carrying all the momentum of the constituent quark. So the momentum which is carried by the valence quark should be a fraction of the momentum which is assigned to the constituent quark. In a naive quark model this fraction is $\frac{1}{3}$ and the related distribution as a function of $x$ is represented by this fraction multiplies the Dirac delta function $\delta(x - 1)$. In the meson and gluon cloud model which we use here, this fraction is obtained by a subtraction of the contribution of all meson and gluon clouds from the total momentum of the constituent quark which is equal to 1, as in equation (25). To improve the recent work, we can represent the bare (valence) quark by a Gaussian function which can be considered as a new research proposal in future.

The intermediate quark distribution function in the constituent quark is calculated from the meson splitting function [11]:

\begin{align*}
    u_U^{(i)}(x) &= d_U^{(i)}(x) = \frac{1}{3} f_{\pi/Q}(1 - x), \\
    u_D^{(i)}(x) &= d_D^{(i)}(x) = \frac{5}{3} f_{\pi/Q}(1 - x), \\
    s_U^{(i)}(x) &= s_D^{(i)}(x) = f_{K/Q}(1 - x),
\end{align*}

where $f_{\pi}^{(i)}$ is the total splitting function of the constituent quark and is defined as

\begin{equation}
    f_{\pi}^{(i)} \equiv \sum_Q f_Q \rightarrow M^{(i)}.
\end{equation}

Mesonic anti-quark contributions in the constituent quarks which finally give us the sea quark distributions inside the constituent quarks are given by the equations

\begin{align*}
    \bar{u}_U^{(i)}(x) &= \frac{1}{3} I_{\pi}(x), \\
    \bar{u}_D^{(i)}(x) &= \frac{5}{3} I_{\pi}(x), \\
    \bar{s}_U^{(i)}(x) &= I_{K}(x),
\end{align*}

where

\begin{equation}
    I_M(x) = \int_{x}^{1} f_{M/Q}(y) q_M \left( \frac{x}{y} \right) \frac{dy}{y}.
\end{equation}

Here $q_M(x)$ denotes the valence quark distribution of the meson. These valence distributions are required to extract sea quark densities in the constituent quarks of the proton, using equation (30). The valence quark distribution of the meson is obtained in the next section, using the phenomenological valon model. By accessing the sea quark distribution in the constituent quark, equation (42) yields the sea quark distribution in the proton.

Using the constituent quark model for the proton which is different with respect to the valon model for meson, the other parton densities in the proton at low $Q^2$ value, for instance, 0.5 GeV$^2$ can be obtained. More details regarding the constituent model employed are explained in section 4.

3.1. Valon model

According to the valon model [16], a valon is a dressed valence quark so that there is a one-to-one identification of a valon with the associated valence quark as probed at high $Q^2$. In this
model a meson, for instance, is a bound state of two valons. They contribute independently in an inclusive hard collision with a $Q^2$-dependence that can be calculated in QCD at high $Q^2$. The valon picture suggests that the structure function of a meson involves a convolution of two distributions: the valon distribution in the meson and the structure function for each valon, so that one has
\[ F_2^M(x, Q^2) = \sum_v \int_x^1 \mathrm{d}y \, G_{v/M}(y) \, F_2^v \left( \frac{z}{y}, Q^2 \right), \] (32)
where the summation is over the two valons. Here $F_2^M(z, Q^2)$ is the meson structure function, $F_2^v$ is the corresponding structure function of a $v$ valon and $G_{v/M}(y)$ indicates the probability for the $v$ valon to have a momentum fraction $y$ in the meson. We shall assume that the two valons carry all the momentum of the meson.

We assume the following simple form for the exclusive valon distribution inside the mesons which facilitates the phenomenological analysis
\[ G_v(y_1, y_2) = g \left( \frac{y_1}{y_2} \right)^p \left( 1 - \frac{y_1 + y_2}{y_2} \right)^q, \] (33)
where $p$ and $q$ are the two free parameters and $y_i$ is the momentum fraction of the $i$th valon. The $U$- and $D$-type inclusive valon distributions can be obtained by integration over the specified variable:
\[ G_{v_1}(y) = \int_0^1 \mathrm{d}y_2 \, G_v(y, y_2) = g y^p \left( 1 - y \right)^q, \] (34)
\[ G_{v_2}(y) = \int_0^1 \mathrm{d}y_1 \, G_v(y_1, y) = g y^q \left( 1 - y \right)^p. \] (35)

The normalization parameter $g$ has been fixed by requiring
\[ \int_0^1 G_{v_1}(y) \, \mathrm{d}y = \int_0^1 G_{v_2}(y) \, \mathrm{d}y = 1, \] (36)
and is given by $g = \frac{1}{B(p+1, q+1)}$, where $B(m, n)$ is the Euler beta-function.

Consequently, we will get the following inclusive valon distributions for mesons:
\[ G_{v_1}(y) = \frac{1}{B(p+1, q+1)} y^p \left( 1 - y \right)^q, \quad G_{v_2}(y) = \frac{1}{B(q+1, p+1)} y^q \left( 1 - y \right)^p. \] (37)

The dirac delta function, $\delta(y_1 + y_2 - 1)$, automatically ensures the momentum sum rule:
\[ \int_0^1 y G_{v_1} \, \mathrm{d}y + \int_0^1 y G_{v_2} \, \mathrm{d}y = 1. \] (38)

The Mellin transformation from equation (37) will yield the following moment distributions for valons:
\[ V_1(n) = \frac{B(p+n, q+1)}{B(p+1, q+1)}, \quad V_2(n) = \frac{B(q+n, p+1)}{B(q+1, p+1)}. \] (39)
Analyzing the experimental data [22, 23] and according to the theoretical model [24, 25], the distribution function of each valence quark inside the pion is equal. Consequently, for the pion we have $p = q$. There is no such constraint for the kaon. The moments of quark and gluon distributions inside the meson are defined as follows [16]:
\[ M_{val, 1, 2}(n, Q^2) = V_{1, 2}(n) M_{NS}(n, Q^2), \]
\[ M_{sea}(n, Q^2) = \frac{1}{2 f}(V_1(n) + V_2(n))(M_q(n, Q^2) - M_{NS}(n, Q^2)), \]
\[ M_g(n, Q^2) = (V_1(n) + V_2(n))M_{gg}. \] (40)
In the above equations, there are four free parameters: $\Lambda_{\pi\gamma y}$, $Q_0$, $q$, $p$. To obtain these free parameters, we take an inverse Mellin transformation of equation (40):

$$x q(x, Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{dn}{x^{n-1}} M(n, Q^2).$$  

(41)

Then we fit the distribution function obtained above to the available experimental data to extract the free parameters. By employing the obtained free parameters and using equation (40) the valence quark densities inside the mesons will be obtained [26]. The meson structure functions in the valon model framework have also been analyzed in [27, 28] where our calculations have been done independently from them.

4. Results and discussions

Using the $\chiQM$ we are able to extract the valence, sea and gluon densities inside the constituent quarks. To obtain the parton densities inside the proton, we employ a constituent model which is different with respect to the valon model in subsection 3.1. We need the quark distribution in a proton, $q_N(x)$, which can be obtained using the convolution of the corresponding quark distributions in the constituent quark ($q_{U,D}(x/y)$) with the light cone momentum distribution of the constituent quark in the proton ($U(y), D(y)$), so that

$$q_N(x) = \int_1^x \left[ 2U(y)q_U \left( \frac{x}{y} \right) + D(y)q_D \left( \frac{x}{y} \right) \right] \frac{dy}{y}.$$  

(42)

The required quark distributions in the constituent quark have been obtained in previous sections. The calculation of sea quark distributions in the constituent quark does specifically depend on the valon model as described in subsection 3.1.

Equation (42) is the basis for the constituent quark model which we can use to obtain the quark densities in a proton. In our calculations, $U(y)$ and $D(y)$ are parameterized as

$$U(y) = B(\alpha_Q + 1, \beta_Q + 1) y^{\alpha_Q} (1 - y)^{\beta_Q},$$

$$D(y) = B(\gamma_Q + 1, \eta_Q + 1) y^{\gamma_Q} (1 - y)^{\eta_Q},$$  

(43)

where $B(\alpha_Q + 1, \beta_Q + 1)$ and $B(\gamma_Q + 1, \eta_Q + 1)$ are the Euler beta-functions. The unknown parameters which exist in equation (43) have been fixed by requiring the number sum rule for valance quark densities inside the proton and also the momentum sum rule for the parton densities inside the proton. Consequently in this stage of our calculations, existing a renormalized constituent distribution is not essential for us. Sea quark densities in the constituent are obtained, using equations (30) and (31) based on the $\chiQM$. Gluon densities in the constituent are obtained using equations (20) and (21) where $g_U(x)$ and $g_D(x)$ are given by integrating the new splitting function for quark to gluon–quark in equation (9) over $k_{\perp}$.

The distributions in the proton are obtained finally, using equation (42).

Requiring sum rules, we will get the following numerical values for the parameters associated with the constituent quark distributions:

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<th>$\alpha_Q$</th>
<th>$\beta_Q$</th>
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<th>$D(y)$</th>
<th>$\gamma_Q$</th>
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The obtained densities are at low $Q^2 = 0.5 \text{GeV}^2$ which corresponds to the chosen value for $\Lambda_y = 1.26 \text{GeV}$ in our calculations according to the model A of [11]. The results for sea and gluon densities in the proton are depicted in figures 1 and 2 respectively. The asymmetry
of sea quark densities is obvious as we expected from the $\chi QM$. In figure 3 we plot the result for valence $u$ and $d$ quark densities in the proton and compare them with the available experimental data [29, 30]. Since the data are at $Q^2 = 15$ GeV$^2$, we evolve the valence densities to this energy scale, using the DGLAP evolution equation.
Figure 3. Valence quark distributions in the proton at $Q^2 = 15 \text{ GeV}^2$. A comparison with available experimental data [29, 30] has also been done.

By using the nucleon parton densities, the $F_2$ structure function at the NLO level of perturbative QCD approximation which is defined by

$$ F_{2}^{ep}(x, Q^2) = x \sum_{q} e_{q}^{2} q(x, Q^2) + \bar{q}(x, Q^2) + \frac{\alpha_{s}(Q^2)}{2\pi} [C_{q,2} \otimes (q + \bar{q}) + 2C_{g,2} \otimes g] $$

(44)

can be calculated. The convolutions are defined as usual:

$$ C \otimes q = \int_{x}^{1} \frac{dy}{y} C \left( \frac{x}{y} \right) q(y, Q^2). $$

(45)

In equation (44) $q$, $\bar{q}$ and $g$ refer respectively to quark, anti-quark and gluon distributions inside the proton and $C$ terms are the Wilson coefficients which are defined in [19]. Parton densities in the constituent quarks, including gluon, valence and sea quark distributions, can be obtained, using equations (20), (21), (25) and equation (30). The parton densities in the proton will be obtained, using equation (42), based on the constituent quark model which we used in our calculations. The output of equation (42) for quark and gluon densities in the proton should be replaced in equation (44) to yield the proton structure function.

In figure 4 the $F_2$ structure function for the proton at low $Q^2 = 0.5 \text{ GeV}^2$ is shown and compared with the GRSV model [19]. A comparison with the available experimental data [31] has also been done. We achieve good agreement between our result and the GRSV model and also the available experimental data at $Q^2 = 0.5 \text{ GeV}^2$ as can be seen in figure 4. Up to equation (44) and even at equation (44) all calculations have been done at $Q^2 = 0.5 \text{ GeV}^2$ since we use the $\chi QM$ which is based on an effective Lagrangian at low fixed value as our initial inputs. The result of equation (44) for the proton structure function, which is plotted in figure 4 at low $Q^2 = 0.5 \text{ GeV}^2$, confirms the validity of our calculation. The agreement between our result with experimental data and the GRSV model is the sign of our reliable calculations. To obtain the results at high $Q^2$ we should use DGLAP equations to evolve the parton densities to a high-energy scale.
Since we have access to the gluon distribution, to confirm the validity of the calculation at low $Q^2$ values, we can evolve it to high $Q^2$ and calculate the fraction of the momentum of the proton which is carried by gluons. We obtained for this fraction 41.2% at $Q^2 = 15 \text{ GeV}^2$ which is what we expect. The authors in [19] claimed that their extracted gluon distribution carries about 50% of total momentum of the proton. This is more than we got. From the inspection of figure 2 which indicates the gluon distribution in the proton at low $Q^2$ it is understandable that we obtain a lower momentum fraction at the evolved $Q^2$ value in our model. A possible reason for this difference is that the results in [19] are based on a global fit whilst we employed the $\chiQM$ to obtain the gluon contribution in our calculation.

We use the following relation [11]:

$$S_G = \int_0^1 \left[ F_2^p (x) - F_2^n (x) \right] \frac{dx}{x} = \frac{1}{3} + \frac{2}{3} \int_0^1 (\bar{u}(x) - \bar{d}(x)) \, dx = \frac{1}{3} - \frac{4}{9} P_\pi (46)$$

to obtain the GSR. The numerical value which is obtained by our model is 0.2339 which is very close to the experimental value $0.235 \pm 0.026$ quoted by the NMC group [32, 33]. Once again the validity of the calculation using the $\chiQM$ at low $Q^2$ values is confirmed.

5. Conclusion

The flavor structure of the nucleon in the effective chiral quark model ($\chiQM$) has been studied. In this model the Goldstone bosons couple directly to the constituent quarks. This idea has been extended to include a gluon cloud in the $\chiQM$ at low $Q^2$ values. Consequently we could obtain an improved result for the sea quark density and also calculated the gluon distribution directly in the $\chiQM$ which had not been done in previous works [11, 14]. To obtain the sea quark densities inside the constituent quark we needed the valence quark distributions.
of the meson. We got these valence distributions using the phenomenological valon model [16]. Furthermore, the sea quark densities in the proton have been obtained by convoluting the required distributions in the constituent quark model. This yielded a result in which the sea quark densities in the proton are asymmetric. By including the other partons in the proton, we could calculate the $F_2$ structure function at $Q^2 = 0.5 \text{ GeV}^2$ which confirmed the anticipated result of the model. To further test the validity of the model the fraction of the momentum of the proton which is carried by gluons at $Q^2 = 15 \text{ GeV}^2$ has been calculated. The numerical result which was obtained for this fraction at $Q^2 = 15 \text{ GeV}^2$ and the numerical value for GSR are very close to what are expected. As a brief description, the first physics issue of this paper at low $Q^2$ is the direct extraction of gluon distribution at this energy scale, based on the $\chiQM$. This has not been done before. As a second physics issue we are able to extract the other parton densities inside the proton-like sea and valence densities at low $Q^2$ without resorting any global fit to extract parton densities at an initial $Q^2$ value as many phenomenological models such as MRST, GRSV, CTEQ, etc do. Accessing to parton densities at a high-energy scale will be done, using the DGLAP evolution equations. One of the other features of the $\chiQM$ is that to lead us to a symmetry breaking of sea quark densities without using the Pauli-blocking ansatz as was done in articles like [34] and references therein. Whilst the results obtained in this paper are satisfactory, one can use a different vertex function, quoted in [35]. By comparing the results, one can choose the best candidate to model meson and gluon clouds at low values of $Q^2$. We intend to carry out this exercise in future research. A further suggestion is to change the valence quark density inside the constituent quark to a Gaussian form rather than the Dirac $\delta$-function assumed in equation (25). In this case we would expect to satisfy the number sum rules for the both constituent and valence quarks in a more straightforward way. This will also be a task for future research.

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