Gluon parton density in the chiral quark model

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Abstract

Dynamical chiral symmetry breaking is one of the most important properties of low energy QCD. The breaking pattern has a profound impact on phenomenological quantities. Here we extend the idea of the meson cloud approach in the chiral quark model to include a gluon cloud in order to model the parton densities in the nucleon, based on the constituent quark framework. To obtain parton densities, including the gluon density in the constituent quark approach, we use the splitting functions of a quark to quark-meson and quark-gluon at low $Q^2$ values. The parton densities at high energies can be obtained, using the DGLAP evolution equations. A good agreement with that expected is computed for the fraction of the total momentum of the proton which is carried by gluon.

1. Introduction

The chiral quark model ($\chi$QM) is used to study the flavor structure of the constituents quark and the nucleon within the conventional mesonic cloud picture. Using this model, the effects of $SU(3)_f$ symmetry breaking can be discussed. The implications of the Gottfried sum rule (GSR) violation on the $\Delta$-n mass splitting is also considered. At a low-resolution scale energy the constituent quark picture is successfully described hadron structure function. The sea quark and gluon degrees of freedom are assumed to be absorbed into constituent quarks as quasi-particles [1, 2]. A relation between the two regimes of hadron structure function description; i.e. the chiral quark and the constituent quark models, has a considerable significance which has been investigated widely in literature, and attracts much more attention in recent years [3].

Here, there are two main ingredients. In continuation of our previous work [4] we add a gluon cloud to the $\chi$QM whilst we use an effective lagrangian at low $Q^2$ values. We resort to a constituent quark model to extract parton densities inside the proton. Since the gluon densities are also available to us we are able to calculate $F_2$ structure function for the proton at the NLO perturbative QCD level of approximation.

2. Dressed valence quarks

To extract sea quark density in the constituent quarks, we need to valence quark distributions inside the meson. Instead of using available phenomenological model, we prefer to extract directly these distribution functions. The basic procedure, is to use the valon model [5] and also inverses Mellin transformation in a parameterized form [6], to extract the free parameters of the model.

For a brief description of the valon model, suppose that a nucleon is a composite system of three constituent quarks. To determine their momentum distribution, one may perform a deep inelastic scattering experiment but would find that the data can be understood only if there are an infinite number of quarks and anti-quarks. This is not contradictory to the picture of three constituent quarks if by the later we mean three valence quark clusters, each of which contains quarks, anti-quarks and gluons that can be resolved by high-$Q^2$ probes. At low $Q^2$ the resolution is so poor that only three clusters can be discerned in a nucleon, each having no recognizable internal structure. In bound-state problems they are called the constituent quarks. For brevity we shall refer to the valence-quark clusters as valons. The
valons therefore serve as a bridge between hard and soft processes. As basic units in a bound- stated (low-
\( Q^2 \)) problem, they form the basis in terms of which the hadronic wave function can be described. It is here
that the so called un-calculable hadronic complication occurs.

Qualitatively a valon structure has cloud of quarks and gluons that evolves from a single quark in a way
that is calculable in QCD. In that sense, a valon is just a dressed valence quark. There is remaining a question
whether in an ( infinite- momentum) frame where a hadron moves fast, the valons exhausts the momentum
of the hadron, or carry only a fraction \( \eta \) of the hadron momentum with the balance \( 1-\eta \) being carried by
gluons not included in the valons. The issue can not be settled without reliable theory for the confinement.
However we can learn from experiments the relative important of the valons and gluons that bind them.
That is the basis of the valon model. If the assumption is wrong, there should be difficulty in constructing a
consistent description that can accommodate the date. Alternatively, we require that the nucleon be compatible
with the description of the bound-state problem in terms of constituent quarks. In order to emphasize that in the
scattering problem these constrain should be regarded as valence quark clusters rather than point like objects,
we inevitably restore to valon model as described in above. The momentum distribution of the valons in a
nucleon summarizes the bound-state complications due to confinement.

A physical picture of the nucleon in terms of three
valons is then quite analogous to the usual picture of the deuteron in terms of two nucleons. At low
\( Q^2 \) the resolution is low and the description of the deuteron as a bound state of two nucleons is adequate.
The medium-to-long range part of the deuteron wave function summarizes the bound-state problem. In that
description it is not necessary to introduce the pion as third constituent, even though it is the exchange
of pions that effects the binding. The short-range part of the deuteron wave function is complicated
and is intimately related to the nucleon structure. Exact analogy is attained by substituting nucleons for
deuteron and valons for nucleons. In analogy to the deuteron example, we ignore the gluons that bind the
valons. We shall also assume that three valons carry all the momentum of the nucleon. The implication is
that the exchange of very soft gluons is responsible for
the binding, as is reasonable. It is also the assumption that in deep-inelastic scattering at high \( Q^2 \) the valons
are independently probed, since the shortness of the reaction time makes it reasonable to ignore the response of
the spectator valons.

The derived parton distributions and nucleon structure functions, with virtually no arbitrary assumptions
or parameters except in the specification of the number of relevant flavors, also agreed with experimental
results. The pion and meson structure functions in the valon model framework have been analyzed. Recently
the valon model has been used to extract polarized parton distributions and proton structure functions in the
Leading and Next-to-Leading order approximations. As an implication of the polarized valon model, the broken
light sea quark distribution has also been analyzed and considered in \[7\].

3. Meson clouds in \( \chi \)QM

As is well known, chiral fluctuations of valence quarks inside hadrons, play a very important dynamical
role in the nucleon structure function, as they have been studied in the context of cloudy chiral models.
In the \( \chi \)QM, the effective degrees of freedom are dressed or dynamical quarks along with the Goldstone
bosons (constituent quarks). The effective interaction Lagrangian between the goldstone boson fields (\( \Pi \)) is
\[ 1 \]

\[
\mathcal{L}_{\text{int}} = \frac{g_A}{f} \bar{q} \partial \mu \Pi q \gamma^\mu q,
\]

where \( q \) is the constituent quark field, \( g_A \), being the axial-vector constant and \( f = 93 \text{ MeV} \) is the pion decay constant.

The constituent quark Fock-state \( |\Omega\rangle \), can be expressed in terms of a series of light-cone Fock-states:
\[ 2 \]

\[
|\Omega\rangle = \sqrt{Z}|q\rangle + \sum_{q'} \alpha_{B|q'}|q', B\rangle,
\]

where \(|q\rangle\) is the “bare” but massive state, \( \sqrt{Z} \) denotes the renormalization factor for a “bare” constituent quark and \( |\alpha_{B|q}\rangle^2 \) are probabilities to find Goldstone bosons and gluon distribution in the constituent quark states.

The bare quark distribution in the constituent quarks has the form:
\[ 3 \]

\[
u^{(0)}_u(x) = d^{(0)}_D(x) = \left( 1 - \sum_B P_{B|q} \right) \delta(x - 1),
\]
where these distributions play the role of the valance quark distributions inside the constituent quarks. In Eq. (3), $P_{B/Q}$ refers to the probability of finding a Goldstone boson and gluon in the constituent quark $Q$. So referring back to Eq.(2), we have $P_{B/Q} = |a_{B/Q}|^2$.

Mesonic anti-quark contributions in the constituent quarks are given by the equations:

$$
\bar{u}^{(s)}_U(x) = \frac{1}{6} I_\pi(x), \quad \bar{u}^{(s)}_D(x) = \frac{5}{6} I_\pi(x),
$$

$$
\bar{d}^{(K)}_U(x) = I_K(x),
$$

where

$$
I_M(x) = \int_x^1 f_{M/Q}(y) q_M \left( \frac{x}{y} \right) \frac{dy}{y}. \quad (5)
$$

Here $q_M(x)$ denotes the valence quark distribution of the meson and $f_{M/Q}$ has been defined in [8]. These valence distributions can be obtained, using the phenomenological valon model as described in previous section. By using Eq. (3), sea quark distributions inside the constituent quark can be determined. But as we told before, we need first to obtain the valence distribution in a meson.

4. Effective quark-gluon interaction

Gluon distributions can be obtained by dressing quarks with gluons in the nonperturbative regime with massive effective gluons ($m_{g^{eff}}^2$) and frozen running $\alpha_s$. Rather heavy effective gluons $m_{g^{eff}}^2 > 0.4$ GeV and small $\alpha_s < 0.5$ are required in order to limit the momentum carried by quarks to approximately what is required by the phenomenology [9]. Now in order to include the gluon clouds in the constituent quark picture we need to assume almost the same form of splitting function is assumed for the gluon-quark interaction as for the quark-meson interaction. There are two main differences. The first one is that the quark-meson coupling constant should be replaced with the strong coupling constant at some low energy scale. Secondly, we need to know the relevant vertex function for the quark-gluon interaction. The vertex function encodes the extended structure of the gluon and the constituent quarks. The extraction of the vertex function is rather difficult since it incorporates the non-perturbative effects. However, in a series of recent studies [10, 11], the authors have calculated the non-perturbative corrections to the quark-gluon vertex in the framework of the Dyson-Schwinger and Bethe-Salpeter equation. Their predictions for the light meson properties seems satisfactory [10]. We find that our ansatz where the quark-gluon vertex which is assumed similar to the quark-meson vertex has qualitatively the same momentum behavior.

Consequently, we have quark-gluon fluctuations which lead to the splitting function:

$$
f_{Q\to Q'}(x_\pi, k^2_\perp) = \frac{\alpha_s(Q^2)}{4\pi} \frac{1}{x_\pi(1-x_\pi)} \left| G_{Q\to Q'}(x_\pi, k^2_\perp) \right|^2 
$$

\begin{equation}
\times \frac{((1-x_\pi)m_Q - m_{Q'})^2 + k^2_\perp}{(1-x_\pi)(m_Q^2 - M_{g^{eff}}^2)^2}, \quad (6)
\end{equation}

where $x_\pi$ is the longitudinal (light cone) momentum fraction of the constituent quark for the gluon and $k_\perp$ is the transverse momentum of the quark $Q'$.

The integration of the quark-gluon splitting function over $k_\perp$ and then over $x_\pi$ and finally summing over the intermediate quarks ($Q'$) yields:

$$
P_{B/Q} = |a_{B/Q}|^2 = \sum_{Q'} \int_0^1 f_{Q\to Q'}(x_\pi) dx_\pi, \quad (7)
$$

We consider the nucleon to be a bound state of three constituent quarks ($U$ and $D$). The gluon distributions in the constituent quark, at some QCD initial scale, can be written as [8]:

$$
g_U(x) = u^{(q)}_U(1-x) \quad \text{and} \quad g_D(x) = d^{(q)}_D(1-x). \quad (8)
$$

Since we have access to the gluon distribution, to confirm the validity of the calculation at low $Q^2$ values, we can evolve it to high $Q^2$ and calculate the fraction of the momentum of the proton which is carried by gluons. We obtained 41.2% which is what we expect.

5. Conclusion

In order to follow the calculations and can use from experimental data to extract unknown parameters which exist in computations, we need to quark distribution in a nucleon, $g_N(x)$, which can be obtained, using the convolution of the corresponding quark distributions in the constituent quark ($g_{q_U,D}(x/y)$) with the light-cone momentum distribution of the constituent quark in the nucleon ($U(y), D(y)$), so as:

$$
q_N(x) = \int_x^1 \left[ 2U(y)q_U \left( \frac{x}{y} \right) \right] dy
$$
\[ + D(y) q_\mu \left( \frac{x}{y} \right) \frac{dy}{y}. \tag{10} \]

The Eq. (10) is the base of the constituent quark model in which we can obtain the quark densities in a nucleon.

Taking the Goldstone bosons as one of the degrees of freedom in the $\chi QM$ leads to a contradiction when considering the $Q^2$ behavior of the sea quark distribution inside the constituent quark. All theoretical calculations and phenomenological models, and also experimental data, confirm that the valence quark distributions inside the mesons (and all hadrons) will decrease with increasing resolution energy scale $Q^2$. Consequently, the sea quark distributions inside the constituent quark will decrease with increasing the $Q^2$. However, we know sea quark distributions, oppositely to valence distributions, should increase with increasing $Q^2$. We suggest a way to resolve this difficulty.

If the Goldstone boson-quark splitting function increases with increasing $Q^2$, the $f_{M Q}^\mu(y)$ in Eq. (5) will also increase. On the other hand, the valence quark distribution of meson, decreases with increasing $Q^2$. Now, if increasing the total splitting function $f_M^\mu(y)$, can compensate and indeed exceed the decrease of the valence quark distribution inside the meson ($q_M^\mu(\frac{x}{y})$) in Eq. (5), the resulting convolution integral, $I_m(x)$, increases with increasing $Q^2$. Then we can claim that the difficulty of the unsatisfactory $Q^2$-dependence of sea quark distributions inside the constituent quarks has been resolved. This strategy has been employed in [6]. However in [8] new approach has been used.

The first physics issue of this letter at low $Q^2$ is the direct extraction of gluon distribution at this energy scale, based on the $\chi QM$. This has not been done before. As a second physics issue we are able to extract the other parton densities inside the proton-like sea and valence densities at low $Q^2$ without resorting any global fit to extract patron densities at an initial $Q^2$ value as many phenomenological models

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