Novel Image Segmentation Based on Machine Learning and Its Application to Plant Analysis

Jinhai Cai, Mahmood R. Golzarian and Stan J. Miklavcic

Abstract—**A novel algorithm is proposed for background estimation using machine learning and statistical pattern recognition. Usually the segmentation of objects in images is achieved by identifying homogeneous regions in individual images or by finding motions of objects in videos. In this paper, we combine the advantages of these approaches for the estimation of background using only two images. The proposed algorithm uses the difference between images to obtain initial estimation of background and then to refine the estimation using machine learning and statistical pattern recognition. Experimental results have shown that the proposed algorithm can achieve promising performance in terms of accuracy and speed.**

Index Terms—**Background Estimation, Gaussian Mixture Models, Object Segmentation, Expectation-Maximization.**

I. Introduction

With the predicted growth of the global population, the demand for better living standards and the accelerating use of grain for biofuel production, the pressures on global grain supplies are increasing. To satisfy the growing, worldwide demand for grain, it is necessary to improve the productivity of existing farmland. This is a challenging task in the face of global environmental change. Plant breeders need to focus on traits with the greatest potential to increase yield and build resilience into production systems. Hence, new technologies must be developed to accelerate breeding through improving genotyping and phenotyping methods and by increasing the available genetic diversity in breeding germplasm [1].

Despite the importance of the plant phenotypic analysis [2,3,4], little has been done on high-throughput plant phenomics. Phenomics and bioinformatics are becoming two indispensable aspects of molecular biology and genetics. Applied to plant biology, automated phenotypic analysis of plant images, captured as a function of growth conditions, can help us to obtain a large amount of information on the function of genes and to study the impacts of abiotic stress on plants. In order to do this, it is essential to automatically separate plants from the background in these images.

Background estimation for image segmentation has been an important research topic in image processing and computer vision for several decades. A wide range of computational vision problems could in principle make good use of segmented images [5], as evidenced by a range of applications from surveillance [6], plant physiology [7], food engineering [8] to medical science [9]. As a result of this extensive research, many different approaches and algorithms for image segmentation have been developed [10]. However, objects and backgrounds in the real world can be very complicated in terms of colors, textures and shapes. Therefore, image segmentation remains one of the most challenging topics in image processing and computer vision.

In general, algorithms for image segmentation can be divided into two categories: segmentation based on feature similarity [11] and segmentation based on motion [10]. Usually, algorithms for segmenting individual images belong to the first category. Most algorithms in this category focus on modeling the consistent and similar features of an object for segmentation [11]. However, an object may have several distinct features within its boundary, which then become segmented into several regions. To solve this problem, some algorithms obtain models of objects by training, which are then applied to segment similar images. As these algorithms can only be applied to images containing objects similar to the training samples, the application of these algorithms is usually limited. Other algorithms require the human knowledge input via interactive schemes [12] for object segmentation in images. As a result, these algorithms can achieve impressive performance, but they cannot be used in a fully automatic manner. Usually, different objects have different features, which are most prominent at the boundaries between different objects. Therefore, it is logical to segment objects using boundary information. Level set methods, which focus on object boundaries, have been successfully applied in image segmentation, where the features of focus can be colors, textures, and shapes [10]. However, level set methods are sensitive to initialization and parameter selection.

In the second category, algorithms are based on motions, where motions can be estimated from multiple images. In this category, some algorithms directly segment objects in the optical flow space [10]. Others focus on the background estimation [13]. Segmentation in the optical flow space needs

Manuscript received April 15, 2001; revised June 12, 2011. This work was supported by the Phenomics and Bioinformatics Research centre, School of Mathematics and Statistics, University of South Australia and the Australian Centre for Plant Functional Genomics.

J. Cai is with the the Phenomics and Bioinformatics Research centre, School of Mathematics and Statistics, University of South Australia, Mawson Lakes, Adelaide SA5095, Australia. Corresponding author: Tel: +61 8 83025533; Fax: +61 8 83025785; Email: Jinhai.Cai@unisa.edu.au or j.cai@ieee.org.

M.R. Golzarian is with the Phenomics and Bioinformatics Research Centre, University of South Australia, Mawson Lakes, SA5095, Australia. He is also an academic member of Department of Agricultural Engineering-Agricultural Machinery, Faculty of Agriculture, Ferdowsi University of Mashhad, Iran. Email: Mahmood.Golzarian@unisa.edu.au.

S.J. Miklavcic is with the the Phenomics and Bioinformatics Research centre, School of Mathematics and Statistics, University of South Australia, Mawson Lakes, SA 5095, Australia. He is also a member of Department of Science and Technology, University of Linkoping, Norrkoping, Sweden, and Australian Centre for Plant Functional Genomics Pty Ltd, Adelaide, SA 5064, Australia. Email: Stan. Miklavcic@unisa.edu.au.

only two images, but significant errors may occur at object boundaries. Segmentation based on background subtraction performs well at object boundaries, but it is based on the assumption that all parts of the background will be unveiled at some time. Thus, background estimation usually needs a sequence of images.

In the application to plant phenomics, plants do not move. With their growth, more and more areas of the background will be covered by plants. Therefore, it is pointless to use images of plants in their late growth stages for background estimation. It is desirable for an automated algorithm to use only a few images of plants in their earliest growth stage to estimate the background. In this paper, we propose a method to estimate the background from two images using Gaussian mixture models and pattern recognition.

II. THE DIFFERENCE IMAGE

The Australian Plant Phenomics Facility (APPF) provides state-of-the-art plant growth environments and the latest technology in high throughput plant imaging for the automatic and non-destructive phenotypic experiments of plants. It can take thousands of plant images by different cameras from different angles, two orthogonal side views and a top view, in one day. Given the amount of data collected, it is desirable to be able to process these images automatically.

A. Lens Distortion Correction

In the high throughput plant imaging facility, we have four cameras for fluorescent, near-infrared, infrared and visible spectral images, respectively. The degree of lens distortion of each camera is different. Therefore, it is necessary to correct for lens distortion before integrating these images for plant phenotype analysis. In most cases, lens distortions are close to radially symmetric. The radial distortion can be classified as one of two types: barrel distortion and pincushion distortion. Mathematically, both barrel and pincushion distortions are quadratic and primarily dominated by the first order radial component:

$$\begin{aligned} x_u &= x_d + k(x_d - x_c)r^2, \\ y_u &= y_d + k(y_d - y_c)r^2, \end{aligned} \tag{1}$$

where (x_u , y_u) are coordinates of points in the undistorted image, (x_d , y_d) are coordinates of points in the distorted image, (x_c , y_c) are coordinates of the distortion centre, k is the radial distortion coefficient, and r is

$$r = \sqrt{(x_d - x_c)^2 + (y_d - y_c)^2}.$$

Man-made objects are usually with straight lines. But these lines appear as curves in distorted images. The idea behind the lens distortion correction is to convert curves in distorted images into straight lines in undistorted images. Although, there are several methods developed for lens distortion correction [14], we used an algebraic approach [15] for distortion correction due to its high accuracy and low computational cost. Some results of which are given in Fig.1.

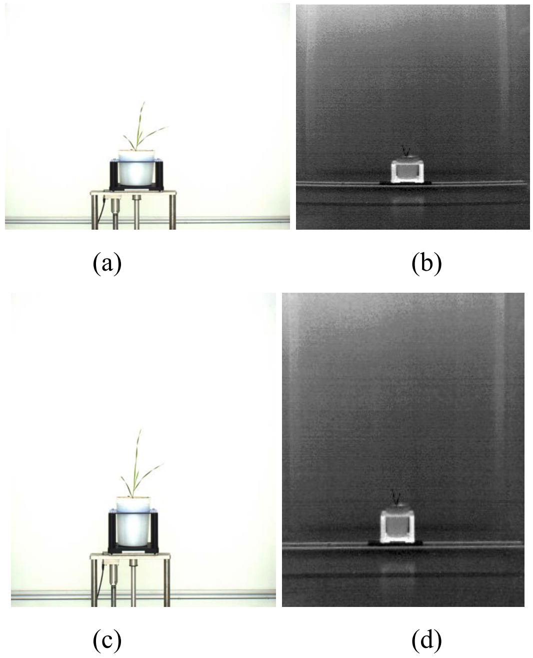
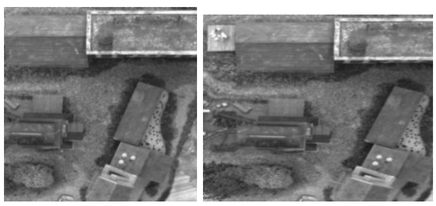


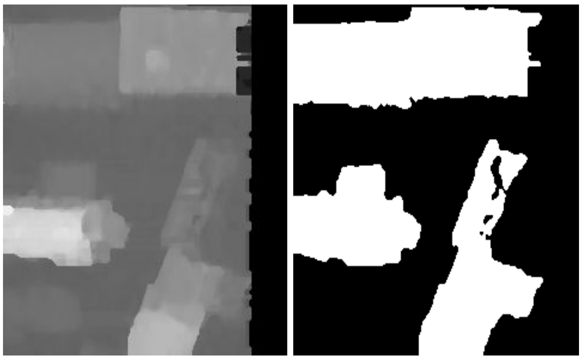
Fig.1 The lens distortion correction. (a) & (b) original images; (c) & (d) images after correcting lens distortion.

B. The Difference Image

It is well known that the performance of segmentation algorithms using individual images is unsatisfactory in many applications. Therefore, it is natural to use multiple images to improve the performance of segmentation. For example, stereo algorithms [16] use two rectified images to calculate the disparity of each pixel, and objects in images can be segmented based on their disparities as shown in Fig.2. The major advantage of motion based segmentation is that it is robust to shades and other interferences. In our application, different plants are imaged against the same background, and the same plant can change its appearance due to its growth. The resolution of plant images is not high enough for texture analysis and the colors of plant leaves are very similar, it is featureless for optical flow to analyze plant leaves at this resolution level. Therefore, it is very difficult to estimate the optical flow, but it is easy to find the difference between two images. Inspired by motion based segmentation, we propose to use the difference image for background estimation and image segmentation. The major advantages of using difference images for segmenting objects from the background are: 1) major parts of the background will be close to zero in the difference images regardless of their colors and textures. This allows us to obtain the initial background estimate; 2) the difference between the background and objects in difference images allows us to obtain the initial estimate of the foreground and the background.



(a) (b)



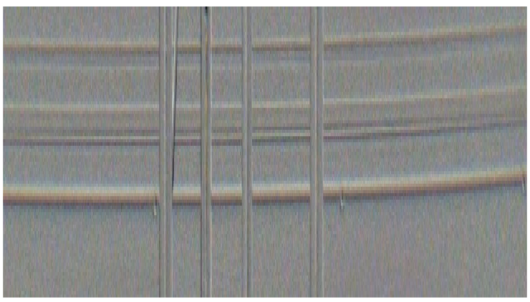
(c) (d)

Fig.2 Segmentation in stereo vision. (a) & (b) original stereo images; (c) the disparity image; (d) the result of building segmentation.

In the available dataset, each plant was imaged every second day by cameras fixed in imaging stations. During the imaging, it is inevitable to have some vibrations. As a consequence, there are some displacements in the images. In order to remove the effects of camera vibrations, the optical flow is used as it is the fastest method to find matched feature point pairs and its performance is very good if the camera's rotation is small. Let $\{x_i, y_i\}$ denote the position of a point in the reference image and it matches a point in another image, where its location is $\{x_i^{'}, y_i^{'}\}$. We can estimate the affine transformation

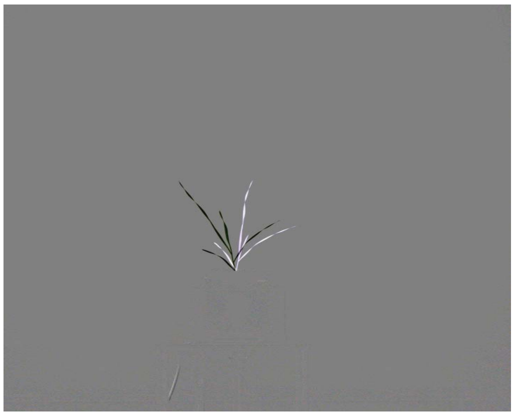
$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i^{'} \\ y_i^{'} \\ 1 \end{bmatrix} \tag{2}$$

where m_{11} , m_{12} , …, m_{23} are elements of the transformation matrix. In this way, we align all images to the reference image. Fig.3(a) shows the difference image without lens distortion correction or alignment. Clearly, differences in some regions, particularly at object boundaries, are salient. In contrast, differences in calibrated images as shown in Fig.3(b) are much small at object boundaries. This is mainly due to the fact that calibration removes the effects of camera vibrations. Fig.3(c) shows the whole difference image.



(a)

(b)



(c)

Fig.3 Difference images: (a) a section of the difference image without calibration; (b) a section of the difference image with calibration; (c) the whole difference image.

III. Learning the Background and Foreground

From Fig.3(c) we find that the difference image can be divided into three distinct parts. In the first part, the differences are close to zero. Since we know that the backgrounds of the two images are almost identical, the difference between them is very small after the calibration. Consequently, we can conclude that the region with very small image difference is likely to be the background. This part appears as the medium-gray region in the difference image. In the second part, the differences are negative. The plant appearing in the reference image will likely appear in the negative part of the difference image as intensities of plant leaves are usually lower than the background. In the third part, the differences are positive. The plant appearing in the second image will likely appear in the positive part of the difference image. The negative and positive parts are represented by the dark gray regions and light gray regions, respectively, in the difference image. The properties of three parts in the difference image can easily be quantified by histograms as shown in Fig.4, where the histogram of the difference image in red color channel has three blocks and each block corresponds to one part in the difference image. The other color channels have the similar property as the red color channel.

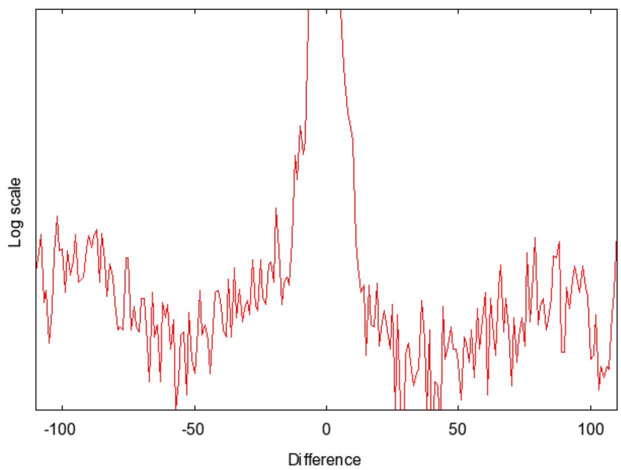


Figure 4. The histogram of the difference image.

A. Gaussian Mixture Models for the Difference Image

The most direct way for segmentation is to apply thresholds on histograms of the difference image for each color channel. The key issue of thresholding methods is the choice of the threshold value. As the histograms of the difference image are not smooth, it is difficult to automatically select threshold values. One method is to smooth the histograms using low-pass filters. We can repeatedly apply the low-pass filters on each histogram until there are three peaks in the histogram. Then, we can automatically select the valleys as threshold values. However, there are two major problems for low-pass filtering. The first problem is that the low-pass filtering process changes the positions of valleys, thus the estimated threshold values are not optimal. Another problem is that the segmentation results based on individual histograms are not always consistent with each other.

Statistics based machine learning methods can always perform better than methods based on filtering. The k-means clustering algorithm [17] in one dimensional case has been proven to converge at a local optical solution. However, the k-means clustering algorithm needs initial threshold values. This means that different initial threshold values may give different final threshold values. The solution to this problem is to integrate the filtering based method and the k-means clustering algorithm. We can use the filtering based method to estimate initial threshold values and use the k-means clustering algorithm to refine the final results. However, this does not solve the inconsistence of segmentation results from different color histograms of the difference image.

Gaussian mixture models (GMMs) have been applied successfully to approximate density functions in many applications. We will use GMMs for the separation of the foreground from the background. In the difference image, the feature vector of a pixel is $\chi = \{\chi_j\}$, where χ_j is the feature vector for the jth channel where j = 1, 2 or 3. So, we have a density function $p(\chi \mid \vartheta)$ that is governed by the set of parameters ϑ. We assume that each channel is independent, so we have

$$p(\chi \mid \vartheta) = \prod_{j=1}^{N} p(\chi_j \mid \vartheta_j), \tag{3}$$

where ϑ_j is the set of parameters for channel j and N is the number of channels. For each channel, there are three distinct parts in the difference image as shown in Fig.3(c) and Fig.4. Accordingly, we assume that the density function for each channel is in the form of the Gaussian mixture model and the number of mixtures is 3. We have

$$p(\chi_j \mid \vartheta_j) = \sum_{i=1}^{M} \alpha_i p_i(\chi_j \mid \vartheta_{ji}) \tag{4}$$

where M=3, $\sum_{i=1}^{M} \alpha_i = 1$, p_i is a Gaussian function and ϑ_{ji} includes the mean and the variance. The parameters, α_i and ϑ_{ji} can be estimated by using the Expectation-Maximization (EM) algorithm [18].

B. Learn the background and foreground

As each Gaussian function corresponds to one part in the difference image, we can use Bayes's rule to separate each part in the difference image:

$$\Gamma(\chi \mid C_i) = \prod_{j=1}^{N} \alpha_i p_i(\chi_j \mid \vartheta_{ji}), \tag{5}$$

where $\Gamma(\chi \mid C_i)$ is the likelihood of the feature χ belonging to class C_i , and C_i is a class label of the background part, plant part in the reference image and plant part in another image. The classification can be performed by

$$C = \arg\max_i \Gamma(\chi \mid C_i).$$

In this way, we can obtain the initial segmented plants and the background.

There is some chance that plant regions in one image overlap with these in another image. Since plants share similar colors, the differences of two images in these overlap regions are small. Consequently, the overlapped plant regions might be classified as the background if we used the difference image alone. To avoid this problem, we utilize the two original images as well as the difference image for the learning of background and foreground. As the total area of overlap is very small, its impact on the background estimation is negligible. Therefore, we need only the initially segmented regions in the original images to learn the background using the Gaussian mixture model. Similarly, we can use the initially segmented plant regions in the original images to learn the foreground.

For the estimation of background from two images, there is no information on the background in regions blocked by plants in both images. These blocked regions cause the same problem in the estimation of digital terrain models (DTMs) [19], where the invisible parts are approximated by neighboring visible regions. We use the similar approach to approximate the background in regions of plant overlap by replacing the occluded regions with their neighboring regions. In this way, we can estimate the whole background as shown in Fig.6(c).

IV. EXPERIMENTAL RESULTS

Background estimation and plant segmentation are the first step in an automated phenotyping process. They are essential components in high-throughput image analyses. To evaluate the performance of the proposed algorithm, images from the APPF Plant Accelerator©, which is equipped with LemnaTec's© cutting edge imaging and conveyor belt technologies, were used in our experiments.

In our experiments, we estimated the background from two images as shown in Fig.6(a) and (b). The estimated background is given in Fig.6(c). The overall performance of the algorithm is excellent with the exception of a few minor errors at edges of leaves as shown in Fig.6(d). It is inevitable that there will be some errors in the estimation as there is no clear-cut boundary between the background and plants in the histogram as shown in Fig.4. Fortunately, most of these minor errors occur in strips of one pixel width, thus they can be removed easily by a median filter.

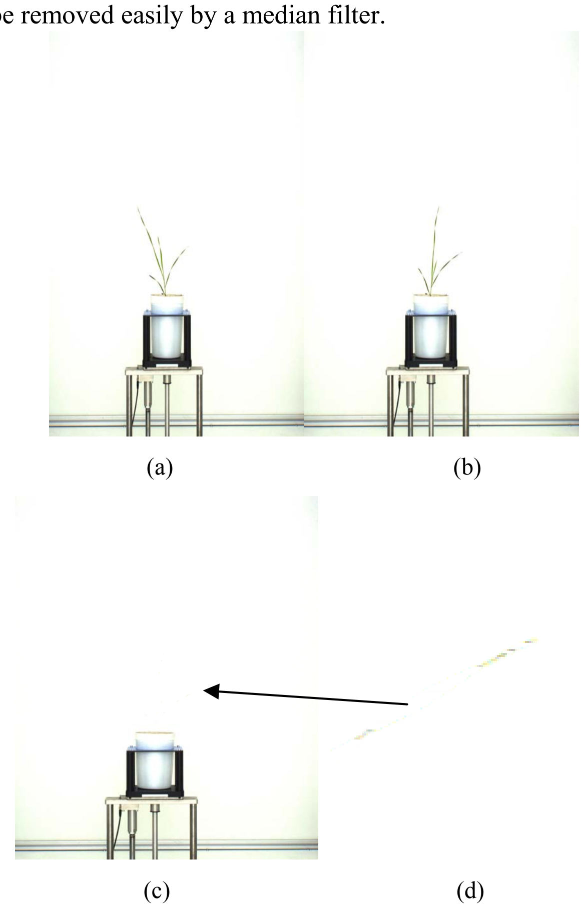


Fig.6 The background estimation: (a) and (b) original images after the correcting for lens distortion; (c) the estimated background; and (d) the magnified area in the background image (c), which shows some minor errors in the background estimation.

Once we have the estimated background, we can perform the segmentation of plants easily. Fig.7(a) and (b) show the segmented plants from Fig.6(a) and (b), respectively. The algorithm performs very well in most cases as shown in Fig.7(c) and (d). However, there are some minor errors in plant segmentation as shown in Fig.7(e), where a small area of the pot rim covered by dust has been classified as part of the plant. However, in the post-processing stage, leaf detection and tip detection should be able to remove these errors.

It is interesting to compare the performance of plant segmentation using the proposed algorithm to that of level-set methods. We first manually cropped a rectangular region around a plant out its whole image as shown in Fig.8(a). We evaluated several level-set methods using this image. We found that [20] and [21] can achieve good results as shown in Fig.8(b) and (c), respectively. The result of [21] on this plant is very similar to the result shown in Fig.7(d). It is also interesting to apply different approaches to whole images for evaluation. However, [20] and [21] do not perform well when whole images are used for segmentation [22,23]. Although the initial contours cover the plant only, the evolved contours extend to whole images as shown in Fig.8(d) and (e).

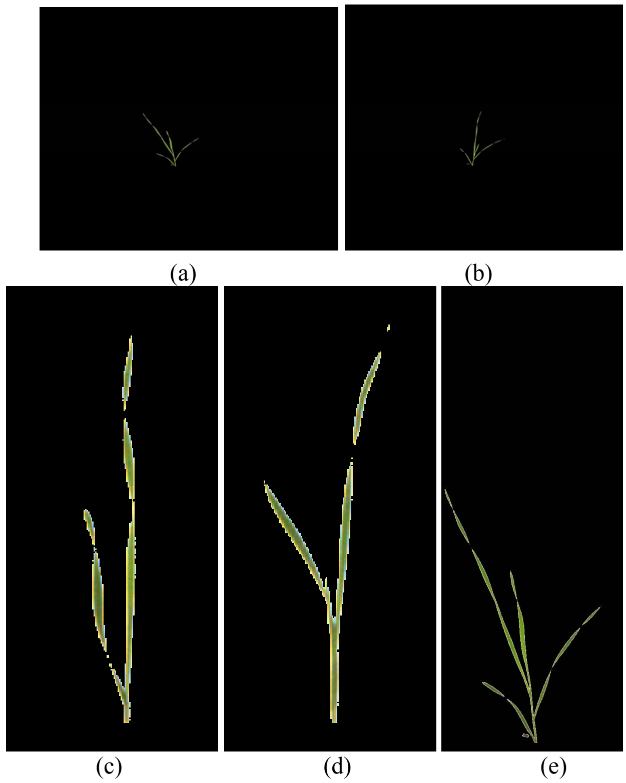


Fig.7 The segmentation of plants: (a) and (b) are plants segmented from Fig.6(a) and (b), respectively; (c) and (d) are two other plants segmented by the proposed method. (e) is the same plant in (a) but magnified to show the error at the bottom of the image.

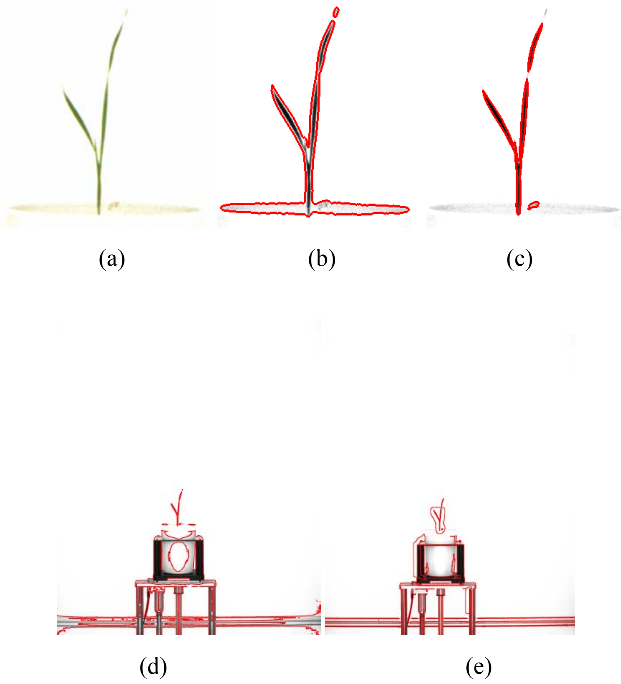


Fig.8 Level-set segmentation. (a) the plant in the original image; (b) the result by [20]; (c) the result by [21]; (d) the result of the whole image by [20]; (e) the result of the whole image by [21].

V. Conclusions and Discussions

We present a novel method for background estimation and image segmentation using two images only. The proposed method uses the difference image for the initial segmentation and uses GMMs and machine learning algorithms for further refinement. The method has been evaluated on a plant image dataset and promising performance has been achieved.

We have conducted a simple performance comparison between the proposed method and some level-set methods. In the near future, we will investigate the possibility of integrating our region-based method with edge-based methods, such as the level-set method, for image segmentation.

In the near future, we will conduct researches on applying the automatic segmentation method to the automated high-throughput phenotypic analysis for cereal plants and other applications.

ACKNOWLEDGMENT

The authors would like to thank the APPF Plant Accelerator for providing the image data for this research.

REFERENCES

[1] M. Tester and P. Langridge, "Breeding Technologies to Increase Crop Production in a Changing World," *Science*, vol.327, 2010, pp.818-822.

[2] B.N. Harris, V.O. Sadras and M. Tester, "A water-centred framework to assess the effects of salinity on the growth and yield of wheat and barley," *Plant Soil*, vol.336, 2010, pp.377-389.

[3] R. T. Furbank, "Plant phenomics: from gene to form and function," *Functional Plant Biology*, vol.36, 2009, pp.5-6.

[4] K.J. Niklas and B.J. Enquist, "On the Vegetative Biomass Partitioning of Seed Plant Leaves, Stems, and Roots," *The American Naturalist*, vol.159, pp.482-497, 2002.

[5] Pedro F. Felzenszwalb, and Daniel P. Huttenlocher, "Efficient Graph-Based Image Segmentation," *International Journal of Computer Vision*, vol.59, no.2, 2004, pp.167-181.

[6] J.M. Ferryman, S.J. Maybank, and A.D. Worrall, "Visual Surveillance for Moving Vehicles, " *International Journal of Computer Vision*, vol.37, no.2, 2000, pp.187-197.

[7] B. Berger, B. Parent, and M Tester, "High-throughput shoot imaging to study drought responses," *Journal of Experimental Botany*, vol.61, no.13, 2010, pp.3519-3528.

[8] D. Mery and F. Pedreschib, "Segmentation of colour food images using a robust algorithm," *Journal of Food Engineering*, vol.66, no.3, 2005, pp.353-360.

[9] L. He, Z. Peng, B. Everding, X. Wang, C.Y. Han, K.L. Weiss and W.G. Wee, "A comparative study of deformable contour methods on medical image segmentation," *Image and Vision Computing*, vol.26, no.2, 2008, pp.141-163.

[10] D. Cremers, M. Rousson, and R. Deriche, "A Review of Statistical Approaches to Level Set Segmentation: Integrating Color, Texture, Motion and Shape," *International Journal of Computer Vision*, vol.72, no.2, 2007, pp. 195-215.

[11] J. Chen, T.N. Pappas, A. Mojsilovic, B.E. Rogowitz, "Adaptive perceptual color-texture image segmentation," *IEEE Transactions on Image Processing*, vol.14, no.10, 2005, pp.1524-1536.

[12] J. Ning, L. Zhang, D. Zhang, and C. Wu, "Interactive image segmentation by maximal similarity based region merging," *Pattern Recognition*, vol. 43, no. 2, 2010, pp.445–456.

[13] V. Reddy, C. Sanderson, B.C. Lovell, "An efficient and robust sequential algorithm for background estimation in video surveillance," *Proc. of IEEE International Conference on Image Processing*, 2009, pp.1109-1112.

[14] H. Farid H, and A.C. Popescu, "Blind removal of lens distortion," *Journal of the Optical Society of America A*, vol.18, no.9, 2001, pp.2072-2078.

[15] L. Alvarez, L. Gomez, and J.R . Sendra, "An algebraic approach to lens distortion by line rectification," *Journal of Mathematical Imaging and Vision*, vol.35, no.1, 2009, pp.36-50.

[16] J. Cai and R.Walker, "Height estimation from monocular image sequences using dynamic programming with explicit occlusions," *IET Computer Vision*, vol.4, no.3, 2010, pp.149-161.

[17] H. Finch, "Comparison of distance measures in cluster analysis with dichotomous data," *Journal of Data Science*, vol.3, 2005, pp.85-100.

[18] Y. Weiss, Bayesian motion estimation and segmentation, PhD thesis, Massachusetts Institute of Technology, May 1998.

[19] H.S. Lee and N.H. Younan, "DTM extraction of Lidar returns via adaptive processing," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 41, no. 9, September 2003, pp.2063-2069.

[20] O. Bernard, D. Friboulet, P. Thevenaz, M. Unser, " Variational B-Spline Level-Set: A Linear Filtering Approach for Fast Deformable Model Evolution," *IEEE Transactions on Image Processing*, vol.18, no.6, 2009, pp.1179-1191.

[21] C. Li, C.Y. Kao, J.C. Gore, Z. Ding, "Minimization of Region-Scalable Fitting Energy for Image Segmentation," *IEEE Transactions on Image Processing*, vol.17, no.10, 2008, pp.1940-1949.

[22] J. Cai, M. Golzarian and S. Miklavcic, "Novel Image Segmentation Using Gaussian Mixture Models -- Application to Plant Phenotypic Analysis," *Proc. of International Conference on Signal Acquisition and Processing*, vol.1, 2011, pp.308-312.

[23] M. Golzarian, J. Cai, R. Frick and S. Miklavcic, "A comparative evaluation of level set algorithms with applications for the segmentation of narrow leaf plant images," *Proc. of International Conference on Signal Acquisition and Processing*, vol.1, 2011, pp.282-286.

Jinhai Cai received the Ph.D. degree in computer science from the University of Melbourne, Melbourne, Australia, in 2000. Between 1999 and 2010, he was a Researcher in LH& Infotalk, Singapore, a Lecturer and Senior Research Fellow at the Queensland University of Technology, Brisbane, Australia. Since 2010, he has been a Senior Research Fellow at the University of South Australia. His research interests include computer vision, signal processing, pattern recognition, and bioinformatics. He has published a book and 17 papers in international journals. Dr. Cai has been a member of Technical Program Committee for many international conferences. Dr. Cai has been a Senior Member of IEEE since 2006.

Mahmood R. Golzarian received his BSc degree in Agricultural Engineering from the Ferdowsi University of Mashhad, Iran, in 1998 and the MSc degree in Mechanical Engineering of Agricultural Machines from the University of Tehran, Iran in 2002.
He received the PhD degree from the University of South Australia in August 2009. Dr Golzarian is currently a post-doctoral researcher at the Phenomics and Bioinformatics Research Centre, University of South Australia. He is also an academic member with the department of Agricultural Engineering-Agricultural Machinery, Faculty of Agriculture, Ferdowsi University of Mashhad, Iran. His research interests include the development and application of imaging and image processing methods for predicting the growth of plants, high-throughput plant phenotyping, plant detection and identification, and precision agriculture.

Stan Miklavcic received a BSc degree with 1st Class Honours in Applied Mathematics and Theoretical Mechanics from the University of New South Wales in 1985 and a Doctor of Philosophy from the Australian National University in 1989. Dr. Stanley J Miklavcic is Professor of Mathematics and Head of the School of Mathematics and Statistics at the University of South Australia. Professor Miklavcic has held Postdoctoral and Research Fellow appointments at the Universities of Lund and Linkoping in Sweden and the University of South Australia, including the prestigious Queen Elizabeth II Fellowship offered by the Australian Research Council. He is a lifetime Member of the Swedish Mathematical Society and a Fellow of the Australian Mathematical Society. He is currently Leader of the Phenomics and Bioinformatics Research Centre at the University of South Australia and a Specified Personnel of the Autralian Centre for Plant Functional Genomics. His research interests include mathematical modelling, signal processing, statistical mechanics and applied and numerical mathematics.