Optimal Cartel Penalty Regime with History Dependency*

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December 2011

Abstract

In this paper, we develop a model to study a dynamic enforcement game between an antitrust authority and several firms. Our main result is that the stylized European penalty legislation appears not to be as efficient as it could be, in the sense that fully compliant behavior is not the long-run steady-state equilibrium of the model. Furthermore, we suggest a penalty regime which depends both on the infringement's duration and the amount of law enforcement, and could prevent any collusion at all.

Keywords: Antitrust Policy, Cartel, Pricing Schema, Differential Game, Commitment.

Journal of Economic Literature Classification Numbers: L11 (Pricing, and Market Structure), L12 (Monopolization Strategies), L41 (Horizontal Anticompetitive Practices), L44 (Antitrust Policy)

1 Introduction

There seems to be a difference in the economic and legal approaches to the regulation of collusive conduct. In the economic approach, one first attempts to ascertain the existence of collusion and the extent of its outcomes and then considers which, if any, legal redress is apt. In this sense, collusion is defined as a market result, e.g., high prices, not as a specific process through which that particular outcome has resulted. However, under the legal approach, the first step is to find out whether a collusive agreement exists and, if so, then apply the punishment defined by law.

*I am grateful to Larry Karp and Evgenia Motchenkova for their invaluable comments. I am particularly indebted to Mehdi Fadaee for his insightful suggestions. Financial assistance from the IMFS is gratefully acknowledged. The usual disclaimer applies.

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For an antitrust authority, (AA hereafter), all practices used by firms to harmonize their actions overtly, in order to attain a collusive outcome, are illegal.

Compliance with antitrust regulations is, in general, achieved through the deterrent effects of penalties. The common factor for most competition law is that collusive fines are intended to deter engaging in illegal behavior, and to dissuade the forming of or joining in any anti-competitive conduct, rather than to compensate consumers whose welfare has been reduced. However, in some jurisdictions deterrence is not the only objective and the fining policy in cartel cases pursues further goals, such as retribution and the recovery of excess collusive profits.

Since in European competition law, monetary fines are the only possible sanction, they must carry the complete burden of deterrence, and a priori, may need to be higher than in jurisdictions, e.g., the US antitrust system, where they are combined with other sanctions such as individual fines, civil damages and imprisonment. The position of fines, as the only means of sanction against cartels in Europe, makes ascertaining the optimal amount of punishment even more crucial.

This paper studies the dynamic enforcement of competition law, analyzes the optimal policy for deterrence of violations of antitrust law and assesses whether the European regime of collusive fines can provide an outcome with complete deterrence. We incorporate specific features of European competition law enforcement into a dynamic model of intertemporal utility maximization by a firm and the AA. The firm, as regulated monopoly type or as participant in a cartel agreement, raises prices above the level of marginal costs, whereas the AA attempts to maximize consumer welfare.

Our main result confirms the stylized fact from the literature that proposes the sub-optimality of current fining structures. We demonstrate that the stylized European penalty legislation is not completely efficient, in the sense that it cannot provide the result of total deterrence. In particular, we illustrate that fully compliant behavior is not sustainable as a Nash Equilibrium over the planning period, and will never arise as the long-run steady-state equilibrium of the model. Then we suggest a new fining regime that could, in principal, completely deter cartels. This will enable us to view the effectiveness and adequacy of competition law in implementing antitrust policies and to develop policy implications as to how existing penalty schemes could be modified, in order to enhance their credibility, reputation and deterrence power against anti-competitive behavior.

The rest of this paper is organized as follows. In Section 2, we provide a review of related literature. In Section 3, we present detailed dynamic problem with model specifications and discuss the results of the proposed fining regime. And finally, Section 4 concludes.

2 The Related Literature

Several empirical papers, e.g., Wils (2006), and Connor (2006), assess the antitrust fines and private penalties imposed on the participants of cartels and demonstrate that the current fining structures lead to considerable under-deterrence. Cohen (1989) studies corporate fines handed down in the U.S. federal courts in the late 1980s. He concludes that the fines alone equaled only one-third of the harm caused by the companies. Therefore, the cartels' members can realistically anticipate incurring fines below their expected cartel profits. Even under the most optimistic assumptions about discovery, lenience and prosecution rates, the average cartel can sensibly expect to make a profit on a typical global price-fixing scheme and such fines cannot optimally deter them from infringements.

In fact, theoretical considerations also provide evidence for under-deterrence and seem to suggest that the fines for antitrust violations should be augmented. Souam (2001) proves how antitrust laws against price-fixing can be enforced efficiently in the presence of asymmetric information between the AAs and the industry and under different regimes of pecuniary punishment. The analysis illustrates that, since investigation is costly, it is optimal from a welfare point of view to accept some degree of collusion.

Schinkel (2007) offers an assessment of fines in the context of modern EU cartels. He assumes a 25% surcharge, five-year duration, fines at the level of single damages, a 30% annual depreciation rate, and the granting of full amnesty and cooperation discounts and explains that for the fine to be optimal, the probability of conviction must exceed 83% which is far above the average in reality. His exercise reveals that the European Commission's recent commitment to punish cartels are likely to remain insufficient to deter collusion.

Veljanovski (2007) scrutinizes the law, practice and evidence on fines for price-fixing under European competition law. Based on an analysis of 30 fully reported cartel decisions, and appeals against many of these, it suggests that the current method of imposing fines and their level are insufficient. The level of fines does not match the harm caused by the cartels nor are they likely to deter price-fixing. Moreover, actual and expected fines are not likely to reflect consumers' losses or deter price-fixing either.

Some papers, e.g., Caulkins (1993), suggest that the punishment should be modeled as a function, depending not only on the intensity but also on the offender's previous criminal record. Leung (1995) addresses this issue in a general dynamic deterrence model that incorporates recidivistic behavior. Motchenkova and Kort (2006) demonstrate that total deterrence with a fixed fining regime can be attained only at the cost of shutting down the firm. In contrast, the proportional fine scheme, which takes into account the history of the violation, can guarantee complete deterrence in the long run, even when penalties are moderate.

A branch of the literature on law enforcement has been devoted to the design of optimal antitrust law enforcement schemata. In his seminal paper, Becker (1968) examines the problem of how many resources and how much punishment are needed to implement different kinds of legislation with a minimum total social loss. He concludes that the optimal fine should be a multiple of the social cost of the crime and also be inversely related to the probability of detection. Hence, since the probability of detection is costly, the best policy for an authority is to set the maximum level of fine.

In some follow-up studies, Becker's static approach has been extended by including intertemporal aspects. They investigate the problem of optimal dynamic law enforcement by modeling the strategic interactions between the offender, who commits the crime, and the authority, whose aim is to prevent the crime. They analyze the intertemporal trade-off between the damages caused by the offense and the cost of enforcement.

Using differential games, Gradns (1989) takes into account the behavioral relationships between the government and firms within a dynamic environment. His main conclusion is that the credibility of a government's policy has a great influence on the market value of a firm. Feichtinger (1983) studies violations of criminal law by means of a differential game solution to a model of competition between a thief and the police.

Later, Feichtinger et al. (2002) present an intertemporal extension of Becker's static economic approach to determine the optimal dynamic trade-off between damages caused by offenders, law enforcement expenditures and the cost of imprisonment. It turns out that there exists a threshold, the so-called *Skiba point*, above which the optimal trade-off between the social costs implies a steady state with a high level of offenses, while below the threshold optimal law enforcement should eliminate crime.

The aim of Fent et al. (1999) is to find out the optimal intertemporal strategy of a profit maximizing criminal under a given static penalty policy in the model with only one agent. Fent et al. (2002) extend this framework to an intertemporal approach of utility maximization, allowing for two players with contradictory objectives, namely the authority and the offending individual.

Motchenkova (2008) analyze a differential game describing the interactions between a firm that might be violating competition law and the AA. It turns out that the current stylized penalty schemes are not capable of totally eliminating the illegal collusive gains. In particular, she proves that full compliance behavior is not sustainable as a Nash Equilibrium in Markovian strategies over the whole planning period, and will never occur as the long-run steady-state equilibrium. Moreover, a penalty system which completely dissuades cartel formation in a dynamic setting is an increasing function of the degree of offense and negatively related to the probability of law enforcement.

Kato (2010) proposes a dynamic dominant-firm type of model where the firm's use of market power, when it is discovered by an AA, will be penalized. Equilibrium entails a threshold market share above which the market tends toward monopoly and below which the market tends to competition. The size of region below this threshold depends on how fast market power depreciates.

Our analysis is technically close to Fent et al. (2002) and Motchenkova (2008). Compared to Motchenkova (2008), the main difference is that the offender's criminal record, as accumulated by the firm over the period of infringement, takes the role of a state variable in the dynamic game. Thus, a boost in this state variable is positively related to the degree of price fixing by the firm and amplifies the fine that the firm can expect in the case of being convicted. Moreover, our paper is different from these papers with regard to the penalty scheme. The base penalty is not only proportional to the gravity of the infringement but also to the duration of the infringement.

3 Model

Let us consider a group of firms which can engaged in a price fixing agreement and agree to increase prices above the marginal cost. Since the firms are symmetric, each of them has the same weight in the coalition. Therefore, the total cartel profits will be divided equally among them. We assume also that there is no strategic interaction between the cartel members, in the sense that we extrapolate from the possibility of self-reporting or any other non-cooperative behavior that would influence the internal stability of the cartel. Under these assumptions we can simplify the setting by considering not the whole cartel but only one representative firm and then apply similar sanctions to all the other cartel members.

The criminal offense: We define the variable q, following Motchenkova (2008) and, as $q := \frac{P-c}{P^m-c}$, a ratio of the realized price increase above the competitive level to the maximal price increase that is possible in case of a monopoly. This is also analogous to the *pilfering rate* in the model of Feichtinger (1983). Here P is the price level agreed by the firms, P^m is the monopoly price, and c is the marginal cost. Since the competitive price, c, leads to $q = 0$ and the monopoly price results in $q = 1$, q is in $[0, 1]$ interval and denotes the degree of price-fixing, illegal activities and the market power of the firm.

We observe that with linear inverse demand, $P = 1 - Q$, the monopoly price is $P^m = (1 + c)/2$, monopoly profit is $\Pi := (P^m - c)^2 = (1 - c)^2/4$, and the instantaneous illegal extra profits from price-fixing, i.e., the producer surplus, is determined according to

$$\begin{aligned} PS(q) &= (P-c)(1-P) \\ &= \left(\frac{P-c}{P^m-c}\right)\left(\frac{1-P}{P^m-c}\right)(P^m-c)^2 \\ &= q\left(\frac{1-c}{P^m-c}-\frac{P-c}{P^m-c}\right)(P^m-c)^2 \\ &= q(2-q)\Pi. \end{aligned}$$

Hence, the marginal profit for the firm is always positive and strictly declining in the interval $q \in [0,1]$. Moreover, for each positive level of offense, the profit is also positive, whereas the competitive price, $q = 0$, leads to zero profits.

Following the analysis above, the net loss in total social welfare is $NLSW(q) = \frac{1}{2}\Pi q^2$, and the consumer surplus is $CS(q) = \frac{1}{2}\Pi(2-q)^2$. These functions have been derived from the microeconomic model underlying the problem of price fixing. The consumer surplus is lower the higher the degree of collusion. The loss in consumer surplus is higher the higher the degree of collusion, while the rents from cartel for the firm are higher the higher the degree of collusion.

Records of crimes: The record of past crimes, $x(t)$, is the state variable of the model. It is influenced by the intensity of offenses, the control variable of the firm, as well as the law enforcement rate, $p(t)$, the control variable of the AA:

$$\dot{x}(t) = q(t)p(t) - \delta x(t), \;\; x(t_0) = x_0, \tag{1}$$

where $\dot{x}(t)$ stands for the change in the value of the state variable.

Equation (1) is similar to the dynamics of the criminal record of Fent et al. (1999) but since we have another player, the probability of being convicted here is a function of time as well. The first term in the system dynamics demonstrates that the state variable increases not only with the offense degree but also with the conviction probability. The second term describes the limitation for those cases, in which the records crimes diminishes with time so that earlier crimes have less impact. In other words, the AA would be more likely to count infringements that are in the distant past less seriously. $\delta = 0$ simply implies that each offense remains in the record forever.

Penalty schemes: There are strong legal and economic reasons for the introduction of a state variable in the form of the criminal record. It is related to the fact that, in juridical practices of many jurisdictions, e.g., both in the US and EU, the determination of the final amount of the fine to be paid by the firm in each particular case, is based on the degree of offense and its turnover throughout the entire duration of the infringement and thus the criminal record could serve as a proxy.

The economic analysis of optimal legal sanctions and criminal punishment is built upon the foundational observation that penalties should be sufficient to induce offenders to internalize the full social cost of their crimes. In a realistic setting in which the probabilities both of detection and of punishment are not perfect and enforcement is costly, optimal penalties must exceed the social cost of the crime, so that the expected sanction facing each potential violator is equal to the harm his violation will cause.

Since it is hard in real life to estimate illegal gains from price-fixing agreement, it is still common practice in most countries to use a percentage of turnover as a proxy for the gains from price-fixing activities. This is defined as the total sales of the cartelized product or service involved over the whole period of existence of the cartel. The advantage of such data is that it is rather easy to obtain, being normally collected, audited and kept in their records by the firms.

The European Union Competition Law is, in principal, similar to the US antitrust law and explicitly forbids cartels and restrictive vertical agreements, specifically in Article 101 of the Treaty on the Functioning of the European Union (TFEU). It reads "*all agreements between undertakings, decisions by associations of undertakings and concerted practices which may affect trade between member states and which have as their object or effect the prevention, restriction or distortion of competition within the common market, and in particular those that directly or indirectly fix purchase or selling prices or any other trading conditions, shall be prohibited as incompatible with the common market*".

The duty of ensuring the application of Article 101, forbidding cartels and other anti-competitive practices and Article 102, preventing the abuse of firms' dominant market positions, and of investigating suspected infringements of these Articles has been entrusted to the European Commission (EC). The EC's 1998 Penalty Guidelines set out the structure of fines for antitrust violations, which together with the 1996[1] and 2002 Leniency Notice[2] provide the basis for the fines for price fixing violations under Article 81 of the EC Treaty over the period of the study.

The 1998 Penalty Guidelines stipulate that its gravity is to be assessed by reference to the nature of the offense, the impact on the market, and the size of the relevant geographical market. In addition the basic amount can be increased to ensure sufficient deterrent. The 1998 Penalty Guidelines state that the basic amount should "*take account of the effective economic capacity of offenders to cause significant damage to other operators, in particular consumers, and to set*

[1] EC Notice on the non-imposition or reduction of fines in cartel cases, 96/C207/04 (1996 Leniency Notice)

[2] Commission Notice on immunity from fines and reduction of fines in cartel cases, 2002/C45/03 (2002 Leniency Notice)

the fine at a level which ensures that it has a sufficiently deterrent effect."

Three types of infringement are distinguished with a corresponding tariff of initial fines: minor offenses with fines between €1000 and €1 million; serious offenses with a fine of between €1 million and €20 million; and very serious offenses with a fine above €20 million. The basic amount is increased when there are aggravating circumstances, such as repeated infringements of the same type by the same firm, refusal to cooperate, or having a leading role in the infringement. It is decreased when there are attenuating circumstances, such as having a passive role in the firm, termination of the infringement as soon as the Commission intervenes, or effective cooperation by the firm in the proceedings.

After $q > 0$ has been observed, the AA might start a detailed investigation. In this case, it gets access to accounting documents and could observe all accumulated rents from cartel formation. According to *the Guidelines on the Method of Setting Fines Imposed for Violations of Competition Law in Europe*, the base penalty schemes for antitrust violations are based mainly on the gravity and duration of the violations in a linear manner. Accordingly, we relate the penalty not only to the current degree of offense, $q(t)$, but also to the criminal record, $x(t)$, which resembles the duration of crime:

$$S(t) = k\Pi q(t) + \varphi x(t). \tag{2}$$

The coefficient k is a constant, which captures this proportion and determines the steepness of the penalty scheme with respect to the cartel overcharge whereas $\varphi > 0$ demonstrates the importance of the duration of crime on the current fine.[3] This is basically a generalization of Fent et al. (2002) model to make the offender's cost function depending on the state $x(t)$ as well.

The fine the firm paid previously, independently of how many times the firm was caught in the past, will not be subtracted from the criminal record. Hence, the fine system implicitly takes into account that repeated offenders will be more heavily punished. In reality, the fine to be paid for the second conviction, compared to the fine for the first conviction, will be multiplied with a higher number as well.

In the US legislation, the Sherman Antitrust Act of 1890 prohibited all contracts and conspiracies that include cartel violations which unreasonably restrain domestic and foreign trade. The Clayton Act of 1914 was passed to supplement the Sherman Act and explicitly lists those types of business practices that distort fair competition, such as price discrimination, exclusive dealing, or mergers that substantially lessen competition. Both of these acts are now listed under Title 15 of the United States Code. The original version of the Sherman Act provided for double damages, but this was increased to treble damages.

[3]Note also that we assume that no additional costs arise after the firm has been caught. This is a reasonable assumption in the context of violations of antitrust law, since it is assumed that only a monetary fine can be imposed and this, contrary to imprisonment, is costless for the authority.

The purpose of the Green Paper of the European Commission was to identify the main obstacles to a more efficient system of damages claims and to set out different options for further reflection, as well as possible ways to improve damage recovery both in follow-on actions and in stand-alone actions, see Pheasant (2006). It suggests double damages for horizontal cartels.

The first part of the fine is a fraction of the profit, $\alpha\pi(q)$, which could be also read as the damage caused by the firm. We define the damage in pecuniary terms and assume that it equals the difference between what the consumers paid and what they would have paid if there had been no collusion. Therefore, $\alpha \geq 2$ and hence $k := \alpha(2-q)$ should also be greater than 2.

Objective functions: Under the above assumptions, the objective of the firm is to maximize the discounted summation of expected profit:

$$J_F = e^{-rt} \int_{t_0}^{T} \left[\Pi q(2-q) - p(k\Pi q + \varphi x)\right] dt, \tag{3}$$

subject to (1), where $r \geq 0$ denotes the discount rate, the first term reflects the instantaneous rents from collusion and the second term indicates the expected punishment for the firm. We drop the time index for the convenient.

For much of the history of competition policy, there have been extensive debates and disagreement over the goals of antitrust law. Nevertheless, the modern consensus among economists and antitrust practitioners is that antitrust law should exist primarily to achieve efficient resource allocation and advance consumer welfare through the promotion of effective competition. In our model, the AA's aim is also to maximize consumer welfare at the lowest possible cost.

It has one instrument, which is the rate of law enforcement or the probability of auditing, denoted by $p(t)$. The analysis of the game will be conducted for the case when the costs of law enforcement, e.g., the number of inspections and salaries for auditors, are quadratic, i.e., $Np^2(t)$. The objective functional of the AA is then,

$$J_A = e^{-rt} \int_{t_0}^{T} \left[\frac{1}{2}\Pi(2-q)^2 - Np^2 + p(k\Pi q + \varphi x)\right] dt, \tag{4}$$

subject to (1). The first term reflects the consumer surplus which decreases with q. We have assumed that if the cartel is discovered, the fine would go to the AA.

The corresponding differential game with two players, one state variable $x(t)$, and two control variables, $q(t)$ and $p(t)$, is represented by the expressions (1)-(4). In our Nash Equilibria, we will find $\phi : [0,T] \to [0,1]$ and $\psi : [0,T] \to [0,1]$ as the fixed steady state strategies for the AA and the firm, respectively. Hence, ϕ corresponds to the control variable $p(t)$, and ψ corresponds to the control variable $q(t)$. The concavity of the consumer and producer surplus and fine regime allows obtaining the expressions for an interior solution of the game.

3.1 The Stylized European Fine Regime

We have supposed that each player takes all his opponents' choices as given, in which case the Nash equilibrium concept can be defined in the usual manner. Each player's decision in an optimal strategy is an optimal control problem in which the player takes into consideration the influence of his actions on the state, both directly and indirectly through the influence of the state on the strategies of the player's opponents.

Lemma 1 *Given $p(t) := \phi$ and $q(t) := \psi$, the steady state price and probability of auditing are*

$$q^*(t) = \begin{cases} \frac{(r+\delta)(2-k\phi)\Pi-\varphi\phi^2}{2\Pi(r+\delta)} & \text{if} \quad (r+\delta)(2-k\phi)\Pi > \varphi\phi^2 \\ 0 & \text{otherwise} \end{cases}, \tag{5}$$

$$p^*(t) = \begin{cases} \frac{k\Pi\delta\psi(\delta+r)}{2\delta N(r+\delta)-\varphi\psi(r+2\delta)} & \text{if } 2\delta N(r+\delta) > \varphi\psi(r+2\delta) \\ 1 & \text{otherwise} \end{cases}. \tag{6}$$

Proof. If the AA has chosen to play $p(t) := \phi$ in the steady state, the current value Hamiltonian for the firm problem is

$$H(q,x,\lambda) = \Pi q(2-q) - \phi(k\Pi q + \varphi x) + \lambda(q\phi - \delta x), \tag{7}$$

which is strictly concave with respect to q. The costate variable $\lambda(t)$ associated with $x(t)$, representing the its shadow price. The associated transversality condition is $\lim_{t\to\infty} e^{-rt}\lambda(t)x(t) = 0$. The Hamiltonian is well-defined and differentiable for all non-negative values of x and q. From (7), we derive the adjoint equation,

$$\dot{\lambda} = r\lambda - \partial H(q,x,\lambda)/\partial x = \varphi\phi + (\delta+r)\lambda, \tag{8}$$

and the first order condition $\partial H(q,x,\lambda)/\partial q = 2\Pi - 2q\Pi - k\Pi\phi + \lambda\phi = 0$, which leads to

$$\lambda = \Pi(2q + k\phi - 2)/\phi, \tag{9}$$

and

$$q = (2\Pi - k\Pi\phi + \lambda\phi)/2\Pi. \tag{10}$$

Differentiating (10), using (8) and (9) yields

$$\dot{q} = \frac{\dot{\lambda}\phi}{2\Pi} = \frac{\varphi\phi^2 + k\phi\Pi(\delta+r)}{2\Pi} - (\delta+r)(1-q).$$

Therefore a stationary solution is the intersect of the locuses $\dot{q} = 0$ and $\dot{x} = 0$,

$$q = \frac{(r+\delta)(2-k\phi)\Pi - \phi^2\varphi}{2\Pi(r+\delta)} = 1 - \frac{k\phi\Pi(r+\delta) + \phi^2\varphi}{2\Pi(r+\delta)}.$$

Note that for a given $p(t) := \phi$, the best response price of the firm is always less than the monopoly price, $q^m = 1$.

Given the firm's choice of $q(t) = \psi$, we could also define the current value Hamiltonian of the AA as

$$H(p,x,\mu) = \frac{1}{2}\Pi(2-\psi)^2 - Np^2 + p(k\Pi\psi + \varphi x) + \mu(\psi p - \delta x), \tag{11}$$

where $\mu(t)$ is the current value adjoint variable with the associated transversality condition $\lim_{t\to\infty} e^{-rt}\mu(t)x(t) = 0$. We derive from (11), the adjoint equation

$$\dot{\mu} = r\mu - \partial H(p,x,\mu)/\partial x = (\delta + r)\mu - \varphi p, \tag{12}$$

and the necessary optimality condition

$$\partial H(p,x,\mu)/\partial p = -2Np + k\Pi\psi + \varphi x + \mu\psi = 0,$$

which leads to

$$\mu = (-k\Pi\psi - \varphi x + 2Np)/\psi, \tag{13}$$

$$p = (k\Pi\psi + \varphi x + \mu\psi)/2N. \tag{14}$$

By differentiating (14) with respect to t, using (12) and (13), we obtain

$$\dot{p} = \frac{\dot{\mu}\psi}{2N} = \frac{-\psi\varphi p + (\delta + r)(-k\Pi\psi - \varphi x + 2Np)}{2N}.$$

Consequently, the intersect of the locuses $\dot{p} = 0$ and $\dot{x} = 0$ give rise to

$$p = \frac{k\Pi\delta\psi\,(r+\delta)}{2N\delta\,(r+\delta) - \psi\varphi\,(r+2\delta)}.$$

Q.E.D. ■

According to (5) and (6), if there is no threat of investigate at all, $\phi = 0$, the firm will set the monopoly price, $q = 1$ and if the firm set the competitive price, $\psi = 0$, there will be no auditing either, $p = 0$. Condition (5) says that if the auditing probability is high enough, the firm will be watchful and will not take the risk of setting a price higher than the competitive one, $q = 0$ whereas condition (6) reads when the firm's price is sufficiently high, the AA will audit for sure, $p = 1$.

The optimal degree of price-fixing by the firm increases when the maximal gains from collusion, Π, raises whereas it declines with an augmention of the expected penalty through higher k. On the other hand, the rise in the absolute value of the penalty, $k\Pi$, causes a rise in the rate of law enforcement, since it becomes more beneficial for the AA to discover more violations. At the same time, it holds that an increase in the marginal costs of law enforcement, N, lessens the equilibrium rate of law enforcement.

These behaviors have been also confirmed in previous studies of optimal deterrence strategies. In particular, the fact that law enforcement is also declining in the fine parameter is in line with the law and economics literature, which emphasizes the trade-off between probability and severity of punishment and suggests that detection probability and punishment are substitutes.

Proposition 1 *If the penalty schedule has the form of (2), then the outcome with no collusion, i.e., $q(t) = 0$ for all $t \in [0, T]$, cannot arise as equilibrium strategy of the firm.*

Proof. From (6), it is clear that $p^*(t) = 0$ if and only if $q(t) = 0$. But this opposes the optimal path for the steady state of firm given by (5), which implies that, when $p(t) = 0$, $q^*(t)$ must be equal to 1. Therefore, $p^*(t) = 0$ and $q^*(t) = 0$ do not form a Nash Equilibrium of the game in the steady state. As a result, the strategy $q(t) = 0$ for all t cannot be sustained as a Nash Equilibrium in open-loop or Markovian strategies. **Q.E.D.** ■

This result basically points out the inability of the European sentencing regime for violations of antitrust law to accomplish complete deterrence.

3.2 Proposed Penalty Schedule

Becker (1968) and Landes (1983) persuasively suggest that in order to achieve optimal deterrence, the sanction should takes into account the probability of detection as well. Following the same intuition, we propose a new fine regime that depends not only on the degree of offense by the firm, but in addition it is inversely related to the rate of law enforcement by the AA,

$$S^p(t) := \frac{k\Pi}{p} q(t) + \varphi x(t). \tag{15}$$

The proposed penalty regime requires that a law enforcer should have knowledge about the probability of law enforcement. This is, in fact, the case in Europe, where competition law is imposed by the European Commission, which is itself a court as well. The perceived probability of detection from a firm perspective is not assessable, but one could develop some proxies for that. For instance, as Connor and Miller (2009) mentioned, cartels facing many buyers or with large members face a high probability of law enforcement, whereas asymmetry among members indicates that it is low.

Moreover, an estimate of the probability of detection from the AA perspective is needed for the calculation of a fine. In general, determining the probability of detection of infringements is difficult and depends on a multitude of factors, such as the firm's subjective expectation of being caught, the industry and the type of anti-competitive behavior. However, Bryant and Woodrow (1991) examine a sample of price-fixing cases from 1961 to 1981 and find that the probability of getting caught is in the range of 13% to 17% in a given year. The authors' results are based on approximate conspiracy durations calculated from data reported for a large sample of the US Department of Justice price-fixing indictments.

We establish that equilibrium with zero degree of collusion can be sustained as an ε-equilibrium in open-loop or Markovian strategies. The proposition 2 implies that in the long run full compliance behavior occurs. Furthermore, it is the saddle point equilibrium of the model. We conclude that the outcome with $q^* = 0$ and $p^* \to 0$ is the unique long run stable steady state equilibrium of the model.

Proposition 2 *If the penalty schedule has the form (15), then the unique and stable ε-equilibrium is given by $q^*(t) = 0$ and $p^*(t) = \rho$ for all $t \in [0, T]$ where $\rho > 0$, and $\rho(\varepsilon) \to 0$ if $\varepsilon \to 0$.*

Proof. If the AA has chosen to play $p(t) := \phi$ in the steady state, we could define the current value Hamiltonian:

$$H(q, x, \eta) = \Pi q(2 - q) - \phi(\frac{k\Pi}{\phi}q + \varphi x) + \eta(\phi q - \delta x), \tag{16}$$

where again $\eta(t)$ is the costate variable with the associated transversality condition of $\lim_{t\to\infty} e^{-rt}\eta(t)x(t) = 0$. We derive from (16), the adjoint equation as

$$\dot{\eta} = r\eta - \partial H(q, x, \eta)/\partial x = \phi\varphi + \eta\,(r + \delta)\,, \tag{17}$$

and the necessary optimality condition as

$$\partial H(q, x, \eta)/\partial q = 2\Pi - 2q\Pi - k\Pi + \eta\phi = 0,$$

which leads to $\eta = \Pi\,(2q + k - 2)\,/\phi$, and $q = (2\Pi - k\Pi + \eta\phi)\,/2\Pi$. Differentiating q, using (17), we obtain

$$\dot{q} = \frac{\dot{\eta}\phi}{2\Pi} = \frac{1}{2\Pi}\left[\varphi\phi^2 + (r + \delta)\,(2q\Pi + k\Pi - 2\Pi)\right].$$

A stationary point can be obtained by $\dot{q} = 0$,

$$q = \frac{(r + \delta)\,(2\Pi - k\Pi) - \varphi\phi^2}{2\Pi\,(r + \delta)},$$

which is always negative. Therefore, a stationary point given by $q = 0$, since by definition it cannot be negative.

Given the firm's choice of $q(t) = \psi$, the current value Hamiltonian for the AA is

$$H(p, x, \gamma) = \frac{1}{2}\Pi(2 - \psi)^2 - Np^2 + p(\frac{k\Pi}{p}\psi + \varphi x) + \gamma(\psi p - \delta x), \tag{18}$$

where η is the costate variable. The necessary optimality conditions,

$$\partial H(p, x, \gamma)/\partial p = -2Np + \varphi x + \gamma\psi = 0,$$

$$\dot{\gamma} = r\gamma - \partial H/\partial x = (\delta + r)\gamma - p\varphi, \tag{19}$$

give us $\gamma = (2Np - \varphi x)/\psi$, and $p = (\varphi x + \gamma\psi)/2N$. The derivative of p with respect to t is

$$\dot{p} = \frac{\dot{\gamma}\psi}{2N} = \frac{(\delta + r)(2Np - \varphi x) - p\varphi\psi}{2N}.$$

A stationary point can be obtained by intersecting the locuses $\dot{p} = 0$ and $\dot{x} = 0$, that is given by $p = 0$. We can conclude that $q^*(T) = 0$ and $p^*(T) = 0$ could be sustained as an open-loop or Markovian Nash Equilibrium at the end of the planning horizon. Similar arguments hold for $p^*(t)$ and $q^*(t)$ at each instant of time, $t \in [0, T)$.

The problem here is that, based on (15), the penalty and, as a result, the objective functions turn out to be indeterminate when $p(t) = 0$. In order to resolve this problem, we utilize the notion of ε-equilibrium. i.e., almost equilibrium and consider $q^*(t) = 0$ and $p^*(t) = \rho$ for all $t \in [0, T)$, as a candidate for it, where $\rho > 0$ and $\rho(\varepsilon) \to 0$ if $\varepsilon \to 0$. An ε-equilibrium of a strategic-form game, following Myerson (1991), is a combination of strategies such that no player could expect to gain more than ε by switching to any of his feasible strategies, instead of following the strategy specified for him.

In order to demonstrate that $p^*(t) = \rho$ and $q^*(t) = 0$ for all $t \in [0, T)$ can be sustained as an open-loop or Markovian Nash equilibrium of this game, we should confirm that this solution satisfies the necessary conditions for optimality,

$$\lim_{\rho\to 0^+} \partial H(q, x, \eta)/\partial q = \lim_{\rho\to 0^+} [2\Pi - 2q\Pi - k\Pi + \eta p]_{(p=\rho, q=0)} = 0,$$

$$\lim_{\rho\to 0^+} \partial H(p, x, \gamma)/\partial p = \lim_{\rho\to 0^+} \left[2Np - \varphi\frac{pq}{\delta} + \gamma q\right]_{(p=\rho, q=0)} = 0.$$

Note that the first equality is satisfied when $k = 2$. However, when $k > 2$, the point with $p = \rho$, $q = 0$ is a boundary optimum and hence also solves the optimization problem.

In order to illustrate the stability of the result, we should check the Jacobian matrix for the differential equations of each player. In general, with arbitrary values of the parameters and arbitrary equilibrium values, the Jacobian matrix has two real eigenvalues of opposite sign and the steady state has the local saddle-point property. This means that there exists a manifold containing the equilibrium point such that, if the system starts at the initial time on this manifold and at the neighborhood of the equilibrium point, it will approach the equilibrium point at $t \to \infty$.

We write the dynamic system for the firm's problem in matrix form as follows:

$$\begin{bmatrix} \dot{q} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} r+\delta & 0 \\ \phi & -\delta \end{bmatrix} \begin{bmatrix} q \\ x \end{bmatrix} + \begin{bmatrix} \left(\phi^2\varphi - (r+\delta)(2\Pi - k\Pi)\right)/2\Pi \\ 0 \end{bmatrix}.$$

Since the determinant of the Jacobian matrix is negative, the solution for q is a saddle point. For the problem of the AA, the dynamic system could be rephrased as

$$\begin{bmatrix} \dot{p} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} r+\delta - \frac{1}{2N}\psi\varphi & -\frac{1}{2N}\varphi(r+\delta) \\ \phi & -\delta \end{bmatrix} \begin{bmatrix} p \\ x \end{bmatrix}.$$

The determinant of this Jacobian matrix is also negative in limit,

$$\lim_{\rho\to 0^+} \Delta = \lim_{\rho\to 0^+} \left[\frac{\varphi}{2N}(rp+\delta p+\delta q) - \delta(r+\delta)\right]_{(p=\rho, q=0)} = -\delta(r+\delta) < 0.$$

Hence, the solution for p is a saddle point as well. ■

Hence, we conclude that the outcome with no collusion $q(t) = 0$ for all $t \in [0, T]$ can occur as an open-loop or Markovian Nash Equilibrium solution of the game and this equilibrium is unique. It turns out that the proposed penalty scheme is more efficient than the current EU penalty schemes, since it leads to the complete deterrence outcome and the AA can achieve a no price-fixing outcome at the lowest possible cost. A no price-fixing outcome, i.e., $q(t) = 0$, can be sustained, but it occurs only at the steady state. The dynamics of the optimal behavior of the firm is such that, given the parameters of the penalty system, the firm gradually declines price to the marginal cost and then no more collusion will take place.

Under the proposed penalty scheme, the firm gradually reduces the degree of violation to zero. The parameters of the penalty system have an effect on the optimal behavior of the firm and thereby on the deterrence power of the penalty system. The parameter k affects only the rate of convergence to the steady state, not its value. Clearly, a higher k induces the firm to stop the violation earlier.

Note that this outcome has a lot in common with the result of Becker (1968). Intuitively, this penalty structure guarantees that, whatever the decision by AA is, it always provides enough motivation for the firms to avoid violating competition law. On the one hand, when the rate of law enforcement is high, the firm is deterred since its probability of being caught is high. On the other hand, when the rate of law enforcement is low, the firm is deterred because of an excessively high penalty in the case that the violation is discovered.

4 Conclusion

In this paper, we develop a model which can be used to study dynamic optimal enforcement of competition law. In particular, we study a game between firms, which maximize their collusive profit, and the antitrust authority, which attempts to maximize consumer surplus.

We illustrate that European penalty legislation regarding monetary penalties applied to cartels appears not to be completely efficient in the sense that it cannot provide the outcome of complete deterrence. In particular, we prove that fully compliant behavior is not sustainable as a Nash Equilibrium and will never arise as the long-run steady state equilibrium of the model. The reason could be that fines for antitrust violations do not depend in any way on the probability of law enforcement, which should be an important determinant of the efficiency of penalty schemes.

Furthermore, we suggest a penalty system, which is efficient from the point of view of the possibility of complete deterrence of cartel formation in a dynamic setting. We find that there is a possibility to achieve the socially desirable outcome, i.e., the outcome with no price-fixing. The amount of the fine should not only be an increasing function of the degree of offense and its duration but also be negatively related to the probability of law enforcement.

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