A study on the relationship between students’ cognitive style and Mathematical word and procedural problem solving while controlling for students’ intelligent quotient and math anxiety

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The main purpose of the study is to investigate whether field dependency could predict students’ mathematical problem solving in word and procedural mathematical problems and to explore whether this association remains significant when students’ Mathematics anxiety and intelligent quotient (IQ) is controlled. So, we used data of 100 samples of guidance school girls and inferential statistical analysis (ANOVA and ANCOVA) for investigating the hypothesis of the study. Obtained results indicated that there were significant differences in students’ mathematical performance in word and procedural problems by the groups of field dependency. Moreover, this difference is still significant when IQ and Mathematics anxiety as covariate variables were considered. However, the amount of omega square for ANCOVA analysis decreased when covariate variables inserted to the model.

Key words: Cognitive style, Mathematical problem solving, Mathematics Anxiety, Intelligent quotient.

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The reviews of the literature find out a consensus that, generally, students’ cognitive style predicts mathematical performance. The majority of previous researches are correlational. That is, the effects of field dependency on mathematical performance were explored directly. It seems that the association between this factor and mathematical performance may be moderated by exogenous factors such as learners’ mathematics anxiety and IQ. Moreover, the comparison between the effects of field dependency on students’ problem solving in procedural and word problems hasn’t been considered in previous researches in Mathematics education. Therefore, it will be of interest to investigate the relationship between cognitive style with mathematical problem solving while controlling for students’ Mathematics anxiety and IQ. It seems to be more beneficial to describe the historical background of these variables before introducing research framework.

Field Dependent (FD) and Field Independent (FI)

Cognitive style is an individual approach to organizing and representing information, and the way in which a student seeks solutions to problems (Price, 2004; Riding & Al-Sanabani, 1998; Saracho, 1998). One of the Cognitive dimensions which are widely used for purposes of analyzing human activities is FD and FI introduced by Witkin and Fellows at 1977. FI/FD is the ability to separate an element from an embedding context. Individuals adept at locating a simple figure within a larger complex figure are referred to as field independent, while those at the opposite end of the continuum are referred to as field dependent (Witkin and Goodenough, 1977).

FD/FI specifies an individual’s mode of understanding, thinking, problem-solving, and remembering. Also, additional studies have found that, in contrast to FI individuals, FD people describe self and others more positively, have a greater preference for people oriented/humanistic professions learn social material more easily and show greater self-disclosure and cooperativeness (Oltman et al., 1975; Schleifer and Douglas, 1973; Sousa-Poza et al., 1973). Other researchers have shown that, in comparison to FD individuals, FI adolescents pay less attention to social problems and prefer professions that require high autonomous functioning and analytic thinking (Eagle et al., 1969; Witkin and Goodenough, 1981; Witkin et al., 1977).

In addition, it has extensively been studied by several scholars and has a wide application in educational studies (Alamolhodaei, 1996, 2002, 2009, 2009b; Mousavi, Radmehr & Alamolhodaei, 2012; Rollock, 1992; Saracho, 2003; Tinajero & Paramo, 1997). FI students are able to structure an analytical task, whereas FD students are better in a context where problems and learning is already structured and analyzed for them. Cassidy (2004) indicated that FI learners are characterized as operators with an internal frame of reference, intrinsically motivated with self-directed tools, structuring their own learning, and defining their own strategies; while, FD individuals are dependent more on
an external reference, are extrinsically motivated, response better to clearly defined problems which require structuring and guidance from instructor, and a desire to communicate with others.

Finally, several researchers have demonstrated the importance of field dependency in science education and mathematical problem solving, in particular word problems (e.g., Witkin and Goodenough, 1981; Talbi, 1990; Johnstone and Al-Naeme, 1991, 1995; Alamolhodaei, 1996; Sirvastava, 1997; Alamolhodaei, 2002, 2009; Mousavi et al., 2012). It was found that FI learners tend to get higher results than FD students in Calculus problem solving at university level. Moreover, school students with FI cognitive style achieved higher results than FD ones in mathematical problem solving (Alamolhodaei, 2009; Mousavi et al., 2012).

**Mathematics Anxiety**

Several definitions have been suggested for Mathematics anxiety. Some are as follows: Mathematics anxiety is defined as a feeling of tension, apprehension, or fear that interferes with math performance (Richardson and Suinn, 1972). Mathematics anxiety is a situation which shows itself with emotional stress and anxiety when the individual is faced with cases such as solving arithmetical problems or doing operations with numbers in either his school or everyday life. This anxiety state can cause amnesia and loss of self-confidence (Tobias, 1993). Math anxiety usually arises when students are faced with unknown or ambiguity and find it frightening rather than enjoyable challenging. In addition, several studies have shown that there is a negative relationship between students' mathematical anxiety and mathematical problem solving (Ashcraft & Kirk, 2001; Alamolhodaei, 2009; Hembree, 1990; Sherman & Wither, 2003; Kramarski et al., 2010; Pezeshki et al., 2011).

Mathematics Anxiety does not have a single cause. It maybe indication an inability to handle disappointment, ample of school absences, low self-concept, internalized negative parental and teacher attitudes toward mathematics, and an emphasis on learning Mathematics through drill without “real” understanding (Norwood, 1994; Singh & Broota, 1992). In addition, math anxiety disrupts cognitive processing by compromising ongoing activity in working memory and therefore, effects on any Mathematical processing that depend on it. Besides, according to Ashcraft (2002) students with high math anxiety demonstrated a smaller working memory span. Moreover, Alamolhodaei (2009) have studied the effect of field dependency, working memory, and mathematics anxiety on students' mathematical word problem solving. He found that FI students with low math anxiety achieved higher result than FD students with high math anxiety in mathematical problem solving. In addition, FD students tend to show higher math anxiety in comparison to FI learners.
Intelligence quotient

IQ is a numerical score based on standardised tests which attempt to measure intelligence. It was first introduced in 1912 by German psychologist, William Stern in reference to the intelligence tests created by psychologists Alfred Binet and Theodore Simon, who wanted to recognize learners that needed special help with the school curriculum. IQ is often measured using tests of visuospatial reasoning, including pattern analysis and visual display understanding (e.g., Raven’s Progressive Matrices) (Kyttala & Lehto, 2008). Jensen (1980) indicated the link between IQ and learning is greatest when the individual is learning new information. Moreover, learners with higher IQ make significantly greater academic progress in reading and writing than children with lower IQs (Shaywitz, Fletcher, Holahan & Shaywitz, 1992; Shinn, Ysseldyke, Deno & Tindal, 1986; Wise, Ring & Olson, 1999). Also, compared to average IQ students, high IQ children are better, more flexible, more adaptive, and efficient at choosing and utilizing effective strategies (Cho & Ahn, 2003; Jausovec, 1991; Muir-Broadus, 1995; Pressley & Hilden, 2006; Shore, 2000; Steiner, 2006). Therefore, according to causal relationship between IQ and achievement, it is beneficial to consider IQ when investigating predictors of achievement and relationships between scores (Watkins et al., 2007).

For a complex issue like mathematical achievement, many factors are likely to take parts such as psychological constructs, intelligence, motivation and social context (Winne & Nesbit, 2010). In regard of intelligence, Helmke (1992) and Spinath et al. (2006) reported that individual differences in intelligence only account for one quarter of the variance of mathematics achievement. And Flexer (1984) found that IQ is a significant predictor of eighth grade algebra scores after controlling for mathematics problem solving and prognosis test scores. Finally, Deary et al. (2007) stated that a general factor of intelligence has been recognized as the best predictor of academic achievement across a wide spectrum of domains and criteria. However, the correlation between intelligence and mathematics achievement is no more than 0.5 for primary school students.

Research framework

Our research question is: What are the interactions between student’ filed dependency and students’ mathematical problem solving while controlling students' IQ and mathematics anxiety in word and procedural math problems? The review of the literature reveals a consensus that, generally, cognitive style (FD/I) and mathematics anxiety predict students' mathematical performance. As stated before, it was found that FI students tend to get higher results than FD students in Mathematical problem solving (e.g., Alamolhodaei, 2002, 2009; Mousavi et al., 2012). Besides, past researches have investigated the effects of mathematics anxiety on students’ mathematical problem.
solving. As mention above, several studies have shown that there is a negative relationship between students' mathematical anxiety and mathematical performance (Hembree, 1990; Ashcraft & Kirk, 2001; Sherman & Wither, 2003; Alamolhodaei, 2009; Kramarski et al., 2010; Pezeshki et al., 2011). On the other hand, the causal effect of IQ on future academic achievement was demonstrated in the previous researches (e.g., Watkins, Lei & Canivez, 2007) and employed for understanding individual differences in academic performance (Gaultney, Bjorklund & Goldstein, 1996). Therefore, it is important to consider IQ when investigating the relationships between psychological factors and mathematical performance.

Thus, in this study we investigate the relationship between Mathematical problem solving and field dependency while controlling students' IQ and mathematics anxiety; since, in recent studies the effect of students' IQ and mathematics anxiety were not considered in relationship between given psychological variable (Cognitive style) and mathematical performance. In addition, in this study we will investigate this association in procedural and word math problems, separately because the differences that exist between students' performance in these two tasks haven't been considered while diagnosing the relationship between field dependency and mathematical problem solving.

Therefore, our hypotheses are as follows:

Hypothesis 1. Given the strong theoretical support in the literature, cognitive style is expected to be significant predictors of mathematical performance.

Hypothesis 2. According to spark literature, an exploratory position is taken to examine the association between field dependency and Mathematical problem solving in procedural and word math problems, separately.

Hypothesis 3. As there is no evidence in literature, an exploratory position is taken to examine the interaction between field dependency and Mathematical problem solving in procedural and word math problems while the effects of students' IQ and Mathematics Anxiety are controlled.

**METHOD**

**Participants**

A total of 100 guidance school female students (aged 12-14 years old) from public schools of Khorasan Razavi province (City: Mashhad), participated in the study. For this purpose, randomly sampling design is used. This study was conducted during regular school hours in intact classes in 2011/2012 school year.

**Procedures**

The research instruments were:

(1) Group-Embedded Figures Test (GEFT).
(2) Raven Progressive Matrices Test.
(3) Mathematics Anxiety Test.
(4) Mathematics Exam (word and procedural).

*Group-Embedded Figures Test (GEFT)*

On the test, students are required to disembed a simple figure in each complex figure. GEFT consists of 8 simple and 18 complex figures. Each of the simple figures is embedded in several different complex ones. Students’ cognitive styles were determined based on a criterion used by (Alamolhodaei, 2009; Johnstone et al., 1993; Oltman et al., 1971; Scardamalia, 1977). Students who had a score less than ¼ standard deviation (SD) below the mean were classified as FD and those who had a score at least ¼ SD above the mean were classified as FI. In addition, students between (Mean ± ¼ SD) were labeled as field-intermediate (Fint) learners. An example of GEFT is shown below.

![GEFT Example](image)

*Raven Progressive Matrices Test*

Raven’s Progressive Matrices is a nonverbal test used in educational studies that was originally created by John C. Raven in 1936. It is a common test executed to groups ranging from 5-year-olds to the elderly and consists of 60 multiple choice questions, listed in order of difficulty. The purpose of designing this test is to measure the test takers reasoning ability or, ("meaning-making") component of Spearman’s g, which is often referred to as general intelligence. Moreover, researchers believed that this measure tapped components of fluid intelligence in children (see Klauer, Willmes & Phye, 2002).

*Mathematics Anxiety Test*

Students’ Mathematics anxiety was determined by the score obtained from the Math Anxiety Rating Scale (MARS). This questionnaire has been developed in the school of Mathematical sciences of Ferdowsi University of Mashhad (Alamolhodaei, 2009; Amani et al., 2012) and consists of 25 items. Each one presented an anxiety arousing situation and the participant decided the degree of anxiety and abstraction anxiety aroused using a five rating scale ranging from very much to not at all (5-l).
five items were hypothesized to measure a new component of math anxiety distinct from those already identified by (Suinn, 1970; Richardson and Suinn, 1972). These items were used to identify abstraction anxiety, according to Ferguson (1986). Cronbach’s alpha, the degree of internal consistency of MARS for the research, was 0.92.

Mathematics exam word and procedural

Both Word and procedural mathematics problems consisted of 5 questions that students should answer as many questions as they can in limited time (25 minutes for word problems and 20 minutes for procedural ones) and each question has 2 points. This exam is designed with cooperation and monitoring students’ teachers by the researchers.

Data analysis

Data of the present study were analyzed by descriptive and inferential statistics. Table 1 presents the means (M), standard deviations (SD) and score ranges for all variables in the study. Hypotheses of the study were analyzed by one-way ANOVA and one-way Analysis of Covariance (ANCOVA) with the Statistical Package for the Social Sciences (SPSS). Table 1 presents the means (M), SD and score ranges for all variables in the study.

| Table 1. Means, standard deviations and ranges of variables for the sample |
|-----------------------------|-------|-------|----------------|
| 1.Mathematical problem solving | 8.14  | 4.13  | 17.5 |
| 2.Word Math problem solving   | 3.21  | 2.04  | 8.5  |
| 4.Mathematics Anxiety         | 81.52 | 20.18 | 88   |
| 5.Intelligence Quotient       | 108.75| 9.48  | 48   |
| 6.GEFT                        | 6.87  | 4.12  | 19   |

RESULTS

Pearson Correlations among variables (see table 2) for the total sample showed that GEFT score was positively correlated with students' Mathematical problem solving in word and procedural problems. Moreover, it positively correlated with IQ and negatively correlated with students' Mathematics anxiety. Concern to Mathematics anxiety, it negatively correlated with students' Mathematical problem solving in word and procedural problems. However, it negatively correlated with students' IQ but this relationship wasn't significant (P-value=.136). In regards of students' IQ, the results of Pearson correlation showed that it positively correlated with students' mathematical problem solving in word and procedural problems at the 0.001 level. Correlations found among variables generally conformed the study hypotheses.
Table 2. Correlations among study variables

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<tr>
<td>1. Mathematical problem solving</td>
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<td>2. Word Math problem solving</td>
<td>.859a</td>
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<td>3. Procedural Math problem solving</td>
<td>.915a</td>
<td>.579a</td>
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<tr>
<td>4. Mathematics Anxiety</td>
<td>-.356a</td>
<td>-.359a</td>
<td>-.284b</td>
<td>1</td>
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<tr>
<td>5. Intelligence Quotient</td>
<td>.491a</td>
<td>.338a</td>
<td>.515a</td>
<td>-.150</td>
<td>1</td>
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</tr>
<tr>
<td>6. GEFT</td>
<td>.353a</td>
<td>.368a</td>
<td>.272a</td>
<td>-.250</td>
<td>.309b</td>
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*Correlation is significant at the 0.001 level. † Correlation is significant at the 0.01 level. ‡ Correlation is significant at the 0.05 level.

For calculating the students' mathematical problem solving, their score on the word and procedural problems were added together. As to the first objective of this study, a one-way ANOVA was conducted to determine if there is statistically significant difference in students’ mathematical problem solving among the students who have FD/Fint/FI styles. The result of one-way ANOVA showed that all were significantly different in terms of mean scores obtained by students in math exam at p-value less than 0.001 ($F_{(2,97)}=9.469, \omega^2=0.16$). The omega square ($\omega^2$) ranges in value from 0 to 1, and is interpreted as the proportion of variance of the dependent variable related to the factor (independent variable, holding constant the covariate. Therefore, 16% of the total variance in mathematical performance was accounted for by the three groups of student field dependency and the results were in line with the hypothesis 1. Concern to second hypothesis, a one-way ANOVA was conducted for procedural math problem and word math problem, separately. The result of one-way ANOVA for three groups of field dependency showed that there are significant differences in terms of mean scores obtained in math procedural problems ($F_{(2,97)}=7.731, p\text{-value}=0.001, \omega^2=0.12$) and math word problems ($F_{(2,97)}=7.299, p\text{-value}=0.001, \omega^2=0.12$). Superiority of the mathematical performance was in FI, Fint and then FD style respectively as shown in figure 1. In addition, in both word and procedural math problems, 12% of total variance in mathematical performance was accounted for groups of cognitive style. Therefore, the results comfort the study hypothesis 2.

Figure 1. Field dependency and performance
The last objective of the study was to discover if there is significant difference in students’ mathematical performance among those who have different cognitive style after adjusting for students’ mathematics anxiety and IQ. Thus, ANCOVA was executed, with covariates Mathematics anxiety and IQ at the significance level 0.05. The ANCOVA was performed after checking prerequisite for running ANCOVA were attained.

First, IQ was considered as a covariate, and analysis were performed for Mathematical problem solving as a whole and for word and procedural problems. The ANCOVA reported there were significant differences in students’ performance by the groups of cognitive styles in all models (Model 1: Mathematical problem solving: $F_{(2, 96)}=5.071$, $p$-value=0.008, $\omega^2=0.03$; Model 2: Word math problem: $F_{(2,96)}=4.340$, $p$-value=0.016, $\omega^2=0.06$; Model 3: Procedural Math problem: $F_{(2,96)}=3.983$, $p$-value=0.022, $\omega^2=0.05$). However, as it can be seen the amount of $\omega^2$ decreased for them. In addition, in these three analyses, FD students had the lowest performance in comparison to Fint and FI students as shown in figure 2.

![Figure 2. Field dependency and performance while controlling for IQ](image)

At the next stage, mathematics anxiety considered as a covariate and analyses were executed for mathematical problem solving as a whole and for word and procedural problems, separately. The ANCOVA reported there were significant differences in students’ performance by the groups of field dependency in all models (Model 1: Mathematical problem solving: $F_{(2, 96)}=7.048$, $p$-value=0.001, $\omega^2=0.11$; Model 2: Word math problem: $F_{(2,96)}=4.791$, $p$-value=0.010, $\omega^2=0.07$; Model 3: Procedural Math problem: $F_{(2,96)}=6.151$, $p$-value=0.003, $\omega^2=0.09$).

Moreover, it can be seen that the amount of $\omega^2$ a little decreased in each situation in contrast to analysis without covariate. Besides, based on figure 3, in all models, FD student had the lowest performance in comparison to Fint and FI student.
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Finally, both mathematics anxiety and IQ were considered as a covariate and ANCOVA analysis were performed for students’ mathematical problem solving as a whole and for word and procedural problems. According to the results obtained, there were significant differences in students' performance in these three models by the groups of cognitive style (Model 1: Mathematical problem solving: $F_{(2,95)}=3.823$, $p$-value=0.025, $\omega^2=0.04$; Model 2: Word math problem: $F_{(2,95)}=2.774$, $p$-value=0.067, $\omega^2=0.03$; Model 3: Procedural Math problem: $F_{(2,95)}=3.425$, $p$-value=0.037, $\omega^2=0.04$). It should be mentioned that the relationship for word math problem is significant at 0.1 levels and for other models they were significant at 0.05 levels. In addition, $\omega^2$ for all model decreased in contrast to ANOVA and ANCOVA with just one variable as a covariate. Moreover, based on figure 4, the lowest performance occurred for FD student in all of the models similar to previous analyses.

DISCUSSION AND CONCLUSION

Educational researchers have long recognized the unique differences among individuals and the impact these differences can have on learning and performance. Concern for these differences led to research on the cognitive variables that individuals possess. The cognitive style, field dependence/independence, (Alamolhodaei, 2009; Nicolaou & Xistouri, 2011; Mousavi et al., 2012) have been recognized as having
widespread implications for mathematics education. Therefore, the main purpose of this study was 1) to investigate whether field dependency could predict mathematical problem solving in procedural and word mathematical problems and 2) to explore whether this association remains significant when students’ mathematics anxiety and IQ were controlled. So, we used data of 100 samples of guidance school girls and inferential statistical analysis (ANOVA and ANCOVA) for investigating the hypothesis of the study.

The result of one-way ANOVA showed that FD students tend to perform significantly lower in comparison to Fint and FI learners in procedural and word mathematical problems. The results of ANCOVA were similar to ANOVA analysis and students with FD style had lower performance in word and mathematical problems in contrast to Fint and FI style when we adjust for students’ mathematics anxiety and IQ. However, the amount of omega square decreased especially when we consider IQ as a covariate variable. Findings of this study were in line with previous claims that student’s cognitive style (Alamolhodaei, 2002, 2009; Johnstone & Al-Naeme, 1991, 1995; Mousavi et al., 2012; Nicolaou & Xistouri, 2011; Talbi, 1990; Witkin & Goodenough, 1981) could predict mathematical performance and also have important implications for the educational process.

In regards of two categories of students’ mathematical problem solving (i.e., word and procedural problems), for word problems, in all of the models, FI learners had better performance in comparison to Fint and FD learners in ANOVA and ANCOVA analysis while concern to procedural problems without considering covariates, Fint and FI learners, had shown the same mathematical performance. However, when we consider covariates, Fint learners had better mathematical performance in contrast to other groups (i.e., FD and FI).

Moreover, in regards of Omega square, without considering covariates, field dependency predict the same variance of mathematical problem solving in word and procedural problems. But when we consider IQ as a covariate there is a slight difference between them (1 percent in favour of word problems). In regards of Math anxiety as a covariate, like what we obtained for IQ, there is a little different between them (2 percent in favour of procedural problems). Finally, when we considered both covariates (i.e. Math Anxiety and IQ), there is one percent different in their omega square in favour of procedural problems. Therefore, this study shown that students’ cognitive style could predict their mathematical performance in procedural problems. Thus, it extends Alamolhodaei (2009) claims that student field dependency is a significant predictor of Mathematical word problem solving.

In a real problem-solving situation, where signal and noise are both present, FD learners suffer a drop in performance. Therefore, it would seem important from a teaching point of view to filter out signal from noise to allow the student to use the
potential space fully for useful processing. It is safe to suggest that teaching style and mathematical tasks should be adapted to students’ cognitive styles (Alamolhodaei, 2009). Therefore, Mathematics teachers should pay more attention to such a task. It important that Mathematics teachers are made aware of the role played by cognitive and affective factors as predictor variables in determining student success. In addition, this study has found that students who score higher on cognitive style test not only have a better chance of solving ordinary mathematics problems, but they have also shown better results in solving word problems, even when we control for students’ IQ and Mathematics Anxiety.

Limitations

The present research has certain limitations. For instance, the study sampled students from a restricted age range (12-14 years old) and is focused on guidance school student. Moreover, all samples were drawn schools in one city of Iran and the present study was restricted to mathematics lesson. The findings of the present study are based upon female student samples. Consequently, further experiments are necessary perhaps under more specific conditions for finding more information, in particular for male students.

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