Chaotic Attitude Synchronization of Multiple Spacecraft Using Distributed Predictive Control

Hossein Karimian, Naser Pariz
Department of Electrical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran
karimian.h@stu.um.ac.ir, n-pariz@um.ac.ir

Asad Azemi
Department of Electrical Engineering
Penn State University
Brandywine Campus, USA
azemi@psu.edu

Abstract—This paper investigates the problem of synchronizing the chaotic attitude dynamics of multiple spacecraft is investigated based on the distributed predictive control approach. Kinematic and dynamic equations are used to describe the spacecraft system according to the Modified Rodriguez Parameters representation. The respective method follows the sequential and cooperative structure of distributed algorithms in order to create the control input and apply it to any spacecraft. Finally, simulation results are presented to verify and demonstrate the effectiveness and feasibility of the proposed method.

Keywords—attitude synchronization; distributed control; predictive control; chaotic attitude; multiple spacecraft.

I. INTRODUCTION

Synchronization and control of chaotic attitude dynamics are important research topics in chaos theory which has been of interest to many researchers in recent years [1-8]. The chaos phenomenon has been observed in various problems such as meteorology, economics, military systems, space industries, and many engineering systems such as communication systems, electric circuits, radars, lasers, etc. In space industries, the synchronization concept has numerous applications including meteorological forecasting, prediction of natural occurrences, surveying of the earth’s gravitational fields, imaging and observation of the earth, and usage in synthetic aperture imaging and interferometry [3, 4]. Since the equations governing multiple systems are often highly nonlinear, these systems are very likely to show chaotic behavior when exposed to disturbances [9].

Just as the chaos phenomenon is extensively and deeply examined in different fields of science, chaotic dynamic motion has also been a focus of attention in single spacecraft. It has been demonstrated that chaotic motion exists in some spacecraft models such as spinning satellites in a circular orbit, gyrostabilized satellites in gravitational fields, tethered satellites and other complex satellites under the effect of gravitational and magnetic fields, or solar fluxes [5, 10]. So far various chaotic attractors have been found for the nonlinear dynamical system of spacecraft motion [7, 8]. In line with this, many control methods have been used for the synchronization and control of spacecraft chaotic dynamics [1, 2, 6, 7, 8]. In [8], synchronization of the chaotic dynamics of rigid body has been studied using the sliding mode control method and in [6], it has been investigated using adaptive control law. The satellite chaotic attitude exposed to external disturbances is investigated in [2] using impulsive control rules. In [1], the symmetric design method for the synchronization of two identical chaotic satellite systems is proposed by means of feedback predictive control in a drive/response manner.

A number of advantages such as flexibility and high performance of the problem, simplification of subproblems, having rigid bodies which are cheaper than the primary body, reduction in set-up and maintenance costs, and less strictness in constraints and the control algorithm applied to them have caused increasing interest. This has led research to be conducted on multiple systems and employment of decentralized or distributed control techniques for performance and stability improvement in them. If a local control agent fails in decentralized and distributed control for any reason, the failure will be merely limited to the region of the failed agent and it will not affect the whole system. Therefore, the stability of the entire system may still be preserved. In view of these reasons as well as the advantages of using multiple systems, usage of this type of control system is increasingly gaining interest. In [3], the leader-follower strategy is addressed for the synchronization of multiple spacecraft attitude. In spite of simple implementation in this method, reliance on one leader and the lack of feedback and connections from the followers to the group leader may lead to failure in the respective mission. In order to prevent this, decentralized control techniques such as the behavioral approach are presented for attitude synchronization [11-13]. In these schemes, the control laws pertaining to each spacecraft are functions of the state variables of the neighboring spacecraft. In [14], maintaining spacecraft formation using constrained robust model predictive control (MPC) with regard to linear Hill equations in presence of parametric uncertainty and actuators size uncertainty is addressed. To do so, the problem has been reduced to a convex optimization problem consisting of linear matrix inequalities. In [15], decentralized control schemes are proposed using virtual structure approach in order to control flight formation of multiple spacecraft. In [16], decentralized variable structure robust control scheme is explained by employing Lyapunov direct method in presence of model uncertainties and external disturbances as well as time-variant delays in internal communications. [17-
investigate the attitude control and synchronization problem for spacecraft formation using sliding mode control in the decentralized mode. [17] proves the robustness of the method for an indeterminate formation of spacecraft based on Modified Rodriguez Parameters (MRPs) representation and Lagrange-like model. [19] examines the problem of attitude tracking and synchronization of spacecraft formation under external disturbances as well as uncertainty in inertia parameters. In [20], two adaptive distributed controllers are proposed for the modes with and without a virtual leader for the attitude synchronization of multiple spacecraft. Distributed control laws are used without angular velocity measurement and they are based on graph theory approaches to attitude synchronization of multiple spacecraft by Euler parameters and MRPs in [11] and [21], respectively. Both papers [11] and [21] are applicable to undirected communication topologies. The cooperative distributed attitude tracking [22] for a spacecraft with a directed communication topology is investigated by a simplified graph including only one node.

This paper is organized as follows. In section II, preliminaries, and the equations describing the system are explained. The structure of the distributed predictive algorithm which is used in the control and synchronization problem is investigated in section III. The example and simulation results are presented in section IV. In the end, the conclusion of the paper and recommendations for future work are expressed in section V.

II. PRELIMINARIES AND SYSTEM DESCRIPTION

A. Notation

The following notations are used throughout this paper. $\mathbf{R}$ and $\mathbf{Z}$ denote the set of real numbers and integers, respectively. $\mathbf{R}^{n \times n}$ signifies the set of all $n \times n$ real matrices, $\mathbf{I}_n$ is the N-dimensional identity matrix, and $\text{diag}(a_1, \ldots, a_n)$ indicates a diagonal matrix with $a_1, \ldots, a_n$ matrices on its diagonal. For each $n_1, n_2 \in \mathbf{Z} \cup \{\pm \infty\}$, the set $\mathbf{Z}^{n_1 \times n_2} = \{n \in \mathbf{Z} | n_1 \leq n \leq n_2\}$ is defined. For the vector $\mathbf{x} = [x_1 \ x_2 \ x_3]^T \in \mathbf{R}^3$, the cross-product operator $\mathbf{x} \times$ yields

$$\mathbf{S(x)} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix},$$

where $\mathbf{S(x)}$ is called a skew-symmetric matrix. Moreover, what meant by $x_i$ for $i = 1, \ldots, n$ is the variable vector of the $i$-th spacecraft.

B. Graph theory

Directed graphs are used to show the communication topology of the spacecraft. To this end, the directed graph $\mathbf{g} = (\mathcal{V}, \mathcal{E})$ consisting of the node set $\mathcal{V} = \{1, \ldots, N\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is defined, where node $i$ indicates the $i$-th spacecraft and the edge $(i, j)$ denotes transfer of data in relation to attitude from spacecraft $i$ to spacecraft $j$. A graph with the characteristic $(i, j) \in \mathcal{E} \Rightarrow (j, i) \in \mathcal{E}$ is called an undirected graph. The adjacency matrix $\mathbf{A} \in \mathbf{R}^{N \times N}$ of the directed graph $\mathbf{g}$ for $i, j = 1, \ldots, N$ is defined as follows $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$ and $a_{ii} = 0$. In undirected graphs, the adjacency matrix $\mathbf{A}$ is defined just the way the directed graph matrix is defined, except that $a_{ij} = a_{ji}, \forall i \neq j$. In this paper, the $a_{ij}$ values are considered to be 1 in case the corresponding spacecraft are connected.

C. Spacecraft attitude kinematics and dynamics

The MRPs are used to represent the spacecraft attitude with respect to the inertial frame [23]. MRP vector for the $i$-th spacecraft is defined as $\mathbf{\sigma}_i = \hat{\mathbf{e}} \tan(\phi_i / 4) \in \mathbf{R}^3$, where $\phi_i$ is Euler angles about principal Euler axes $\hat{\mathbf{e}}_i$ [24]. Thus, the dynamic and kinematic equations of the $i$-th spacecraft for $i = 1, \ldots, N$ are as follows

$$\dot{\mathbf{\sigma}}_i = \mathbf{G}(\mathbf{\sigma}_i)\mathbf{\omega}_i,$$  

$$\mathbf{J}_i \dot{\mathbf{\omega}}_i = -\mathbf{\omega}_i^\times \mathbf{J}_i \mathbf{\omega}_i + \mathbf{\tau}_i,$$

where $\mathbf{\omega}_i \in \mathbf{R}^3$ and $\mathbf{J}_i \in \mathbf{R}^{3 \times 3}$ signify angular velocity and inertia matrix, respectively. The input torque $\mathbf{\tau}_i \in \mathbf{R}^3$ is also a combination of the disturbance torque $\mathbf{\tau}_i^d$ and the control torque $\mathbf{\tau}_i^c$, and

$$\mathbf{G}(\mathbf{\sigma}_i) = \frac{1}{2}((1 - \frac{\mathbf{\sigma}_i^T \mathbf{\sigma}_i}{2}) \mathbf{I}_3 + \mathbf{\sigma}_i^\times + \mathbf{\sigma}_i \mathbf{\sigma}_i^T).$$

D. The chaotic mode of spacecraft

Although certain factors such as gradient and gravitational torques, solar fluxes, and the spacecraft being located within the magnetic fields of heavenly bodies may give rise to complex dynamic behaviors such as periodic, pseudo-periodic, and chaotic behaviors in the spacecraft. The above equations should mathematically satisfy particular conditions. A number of modes leading to chaotic behavior in spacecraft attitude dynamics are presented in [8], [9], and [25].
Suppose $\mathbf{J}_i = \text{diag}(j_{i}^{(1)}, j_{i}^{(2)}, j_{i}^{(3)})$ and substitute it into (2), then the $i$-th spacecraft attitude dynamic equations are as follows

$$
\begin{align*}
\dot{\omega}_i^{(1)} &= \dot{\omega}_i^{(2)} \omega_i^{(3)} (j_{i}^{(22)} - j_{i}^{(33)}) + \tau_{i}^{(1)} \\
\dot{\omega}_i^{(2)} &= \dot{\omega}_i^{(1)} \omega_i^{(3)} (j_{i}^{(33)} - j_{i}^{(11)}) + \tau_{i}^{(2)} \\
\dot{\omega}_i^{(3)} &= \dot{\omega}_i^{(1)} \omega_i^{(2)} (j_{i}^{(11)} - j_{i}^{(22)}) + \tau_{i}^{(3)}
\end{align*}
$$

(4)

where $\omega_i^{(1)}, \omega_i^{(2)}, \omega_i^{(3)}$ are angular velocities, $j_{i}^{(11)}, j_{i}^{(22)}, j_{i}^{(33)}$ are principal moments of inertia, and $\tau_i^{(1)}, \tau_i^{(2)}, \tau_i^{(3)}$ are the torques exerted on the dynamics of the $i$-th spacecraft. Introducing

$$
a_i^{(1)} = (j_{i}^{(11)} - j_{i}^{(33)}) / j_{i}^{(11)}, \quad a_i^{(2)} = (j_{i}^{(33)} - j_{i}^{(11)}) / j_{i}^{(22)}, \quad a_i^{(3)} = (j_{i}^{(11)} - j_{i}^{(22)}) / j_{i}^{(33)}
$$

(4) is rewritten as follows

$$
\dot{\omega}_i = [a_i^{(1)} \omega_i^{(2)} \omega_i^{(3)}; a_i^{(2)} \omega_i^{(1)} \omega_i^{(3)}; a_i^{(3)} \omega_i^{(1)} \omega_i^{(2)}]^T + \mathbf{M}_i \dot{\omega}_i + \mathbf{U}_i,
$$

(5)

where

$$
\dot{\omega}_i = \mathbf{F} \dot{\omega}_i = \begin{bmatrix} \dot{\omega}_i^{(1)} & \dot{\omega}_i^{(2)} & \dot{\omega}_i^{(3)} \end{bmatrix}
$$

and

$$
\mathbf{M}_i \dot{\omega}_i = \begin{bmatrix} \dot{\omega}_i^{(1)} & \dot{\omega}_i^{(2)} & \dot{\omega}_i^{(3)} \end{bmatrix} \\
\mathbf{U}_i = \begin{bmatrix} \dot{\omega}_i^{(1)} & \dot{\omega}_i^{(2)} & \dot{\omega}_i^{(3)} \end{bmatrix}
$$

denotes the perturbation frequency matrix and $\mathbf{U}_i$ indicates the control torque.

III. CONTROLLER DESCRIPTION

Definition 1. Consider the nonlinear discrete-time system comprising $N$ subsystems which is characterized with the directed graph $\mathbf{g}^S = (\{1, \ldots, N\}, \mathcal{E}^S)$. The $i$-th system from this set is described as follows

$$
x_i[k+1] = f_i(x_i[k], p_i[k]; u_i[k])
$$

(6)

where $x_i[k] \in X_i \subseteq \mathbb{R}^{n_i}$ is the state vector with the initial value $x_i[0] \in X_i$, and $u_i[k] \in U_i \subseteq \mathbb{R}^{m_i}$ is the control input for $n_i, m_i \geq 1$. Furthermore, $p_i[k] = (x_j[k])_{(j,i) \in \mathcal{E}^S} \in \mathbb{R}^{\sum_{j \neq i} n_j}$ indicates the state vectors of the entire systems which can affect the $i$-th system through their dynamics. The mapping $f_i : X_i \times \prod_{(j,i) \in \mathcal{E}^S} X_j \times U_j \rightarrow X_i$ for each $1 \leq i \leq N$, indicates the trajectory of system $i$.

Definition 2. The following cost function with the directed graph $\mathbf{g}^C = (\{1, \ldots, N\}, \mathcal{E}^C)$ is used as the control objective in order to be minimized in the finite-horizon optimization problem with the conditions mentioned in Definition 1 [26]

$$
\mathbf{J}_i((x_i[k])_{k=0}^\infty; (u_i[k:k+T])_{k=0}^\infty) = \sum_{t=0}^{T} \sum_{i=1}^{N} l_i(x_i[t], q_i[t]; u_i[t]),
$$

(7)

where $k \in \mathbb{Z}_{\geq 0}$ is the sampling time, $T \in \mathbb{Z}_{\geq 0}$ indicates the prediction horizon, and $q_i[k] = (x_j[k])_{(j,i) \in \mathcal{E}^C} \in \mathbb{R}^{\sum_{j \neq i} n_j}$ denotes the vector of the states of all of the systems affecting the cost of the $i$-th system.

Note 1. Since other spacecraft do not affect the dynamics of the $i$-th spacecraft in here, $p_i[k]$ variables are not taken into account.

Note 2. According to [27], the cost optimization problem (7) may be solved by arbitrary accuracy under the assumption of the essential convexity of $T$. It can be proven that with this assumption, the problem has a unique global minimum for $T \in \mathbb{Z}_{\geq 0} \cup \{\infty\}$.

A. Sequential Distributed Model Predictive Control

In this section, the distributed model predictive control (DMPC) approach with sequential architecture introduced in [28] is addressed in order to propose an appropriate control law for the attitude synchronization of the multiple systems that are mentioned in Definition 1.
This procedure is summarized in Procedure I.

In the centralized mode of the predictive system is rewritten according to the above Definitions

\[ \mathbf{u}_i(k : k + T)_{j=1}^{N} = \arg \min_{(\mathbf{u}_i(k : k + T))_{j=1}^{N}} J^{[T]}(\mathbf{x}_i(k))_{j=1}^{N}; (\mathbf{u}_i(k : k + T))_{j=1}^{N} \]

s.t. \[ \hat{x}_i(t + 1) = f(\hat{x}_i(t), \hat{\mathbf{u}}_i(t)), 1 \leq i \leq N \]
\[ \hat{x}_i(k) = x_i(k), 1 \leq i \leq N \]
\[ \hat{x}_i(t) \in X_i, \hat{\mathbf{u}}_i(t) \in U_i, \forall t \in Z_{2k}^{d+T}, \forall i \]

where

\[ J^{[T]}(\mathbf{x}_i(k))_{j=1}^{N}; (\mathbf{u}_i(k : k + T))_{j=1}^{N} = \sum_{t=1}^{T} \sum_{j=1}^{N} l_i(\hat{x}_i(t), \hat{\mathbf{u}}_i(t)), \]

and subject to

\[ \hat{x}_i(t + 1) = f(\hat{x}_i(t), \hat{\mathbf{u}}_i(t), \hat{\mathbf{u}}_i(t)) \]
\[ \hat{x}_i(t) = x_i(t), 1 \leq i \leq N, \forall t \in Z_{2k}^{d+T} \]

and convex constraints

\[ \hat{x}_i(t) \in X_i, \hat{\mathbf{u}}_i(t) \in U_i \]
\[ \hat{\mathbf{u}}_i(t) \in \prod_{(j,l) \in E^2} X_j, \hat{\mathbf{u}}_i(t) \in \prod_{(j,l) \in E^2} X_j, 1 \leq i, j \leq N, \forall t \in Z_{2k}^{d+T}. \]

For this purpose, the finite-horizon optimal control problem used in local controllers corresponding to each system is rewritten according to the above Definitions

\[ (\mathbf{u}_i(k : k + T)_{j=1}^{N}) = \]

\[ \arg \min_{(\mathbf{u}_i(k : k + T))_{j=1}^{N}} J^{[T]}(\mathbf{x}_i(k))_{j=1}^{N}; (\mathbf{u}_i(k : k + T))_{j=1}^{N}, \]

(8)

where

\[ J^{[T]}(\mathbf{x}_i(k))_{j=1}^{N}; (\mathbf{u}_i(k : k + T))_{j=1}^{N} = \sum_{t=1}^{T} \sum_{j=1}^{N} l_i(\hat{x}_i(t), \hat{\mathbf{u}}_i(t)), \]

and subject to

\[ \hat{x}_i(t + 1) = f(\hat{x}_i(t), \hat{\mathbf{u}}_i(t), \hat{\mathbf{u}}_i(t)) \]
\[ \hat{x}_i(t) = x_i(t), 1 \leq i \leq N, \forall t \in Z_{2k}^{d+T} \]

and convex constraints

\[ \hat{x}_i(t) \in X_i, \hat{\mathbf{u}}_i(t) \in U_i \]
\[ \hat{\mathbf{u}}_i(t) \in \prod_{(j,l) \in E^2} X_j, \hat{\mathbf{u}}_i(t) \in \prod_{(j,l) \in E^2} X_j, 1 \leq i, j \leq N, \forall t \in Z_{2k}^{d+T}. \]

Note that \( \hat{x}_i \) and \( \hat{\mathbf{u}}_i \) are also the predicted variables yielded as a result of employing the system model.

In the DMPC scheme used (Fig. 1), the local controllers corresponding to systems initially take all of the state measurements from sensors. Then, each controller receives the predicted input trajectories pertaining to its previous controllers. In view of this information, each controller evaluates the future input trajectory value and sends it to its actuators as well as the subsequent controllers. This process continues with the measurement of new state values (\( k \rightarrow k + 1 \)). This procedure is summarized in Procedure I.

Note 3. In the centralized mode of the predictive controller, the control signal applied to all systems is calculated by once solving the finite-horizon problem (8) in each sampling time \( k \).

IV. NUMERICAL SIMULATION

Simulation results are presented in this section in order to illustrate the effectiveness of using the distributed predictive controller scheme introduced in section III for control and synchronization the chaotic attitude dynamics of multiple spacecraft.
TABLE I. SPACECRAFT SPECIFICATIONS.

<table>
<thead>
<tr>
<th>Systems</th>
<th>Perturbing Frequency Matrices (M)</th>
<th>Parameters (a) (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacecraft 1</td>
<td>[-6,6,0;0,3,0;0,0,0,0]</td>
<td>[10,-10,10]</td>
</tr>
<tr>
<td>Spacecraft 2</td>
<td>[-4,0,4;0,0,1.75,0;0,-2.4,0,0,0,0]</td>
<td>[3,3,3]</td>
</tr>
<tr>
<td>Spacecraft 3</td>
<td>[-7,7,-6;6,0,0,0,-1,-1,-33;5,50,0,0]</td>
<td>[0,1,1]</td>
</tr>
<tr>
<td>Spacecraft 4</td>
<td>[-8,8,-0,6;0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]</td>
<td>[4,1,2]</td>
</tr>
</tbody>
</table>

TABLE II. INITIAL VALUE OF ANGULAR VELOCITIES.

<table>
<thead>
<tr>
<th>Initial Value</th>
<th>Spacecraft 1</th>
<th>Spacecraft 2</th>
<th>Spacecraft 3</th>
<th>Spacecraft 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>w(0)(rad/s)</td>
<td>[-0.1;+0.1;0.1]</td>
<td>[+3;+3;3]</td>
<td>[+0.2;−0.2;−1]</td>
<td>[+5;−8;+5]</td>
</tr>
</tbody>
</table>

To this end, 4 spacecraft with the communication topology of Fig. 2 are considered. The adjacency matrix corresponding to this structure is

\[
A = \begin{bmatrix}
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The parameters proportional to the elements of the inertia matrix \(a^{(a)}, a^{(b)}\), the perturbation frequency matrix \(\mathbf{M}\), and also the states of initial angular velocities \(w_i(0)\) for \(i = 1,\ldots,4\) are demonstrated in Table I and Table II for each spacecraft in order to show chaotic dynamic behavior [7, 8]. Also, the initial attitude states \(\sigma(0)\) are chosen [0.2553,0.2553,0.2553] for all the spacecraft.

Figure 3. Chaotic attractors of spacecraft 1: (a) Phase portrait of the angular velocities. (b) Phase portrait of the attitude angles.

Figure 4. Chaotic attractors of spacecraft 2: (a) Phase portrait of the angular velocities. (b) Phase portrait of the attitude angles.

Figure 5. Chaotic attractors of spacecraft 3: (a) Phase portrait of the angular velocities. (b) Phase portrait of the attitude angles.

Figure 6. Chaotic attractors of spacecraft 4: (a) Phase portrait of the angular velocities. (b) Phase portrait of the attitude angles.
The chaotic behavior of the four above-mentioned spacecraft is depicted in Figs. 3-6, respectively. In order to create an appropriate control signal to apply to each spacecraft, the optimization problem (8) is solved along the prediction horizon $T = 3$ for each spacecraft by the distributed sequential architecture mentioned for predictive control. For this purpose, spacecraft minimize the following cost function in each time step $k \in \mathbb{Z}_{\geq 0}$ given the defined communication topology,

$$J_{k}^{(i)}((x_{i}[k])_{m_{i}^{k}}^{n_{i}^{k}},(u_{i}[k : k + T])_{m_{i}^{k}}^{n_{i}^{k}}) =$$

$$\sum_{i=1}^{n_{i}} \left[ \sum_{j \neq i}^{n_{i}} ((x_{j}[t] - x_{i}[t])^{T}Q_{mm}(x_{j}[t] - x_{i}[t])) + \sum_{i=1}^{n_{i}} u_{i}[t]^{T}R_{uu}[u_{i}[t]] \right],$$

where $x_{i}[k] = [\omega_{i}^{(1)}[k], \omega_{i}^{(2)}[k], \omega_{i}^{(3)}[k], \sigma_{i}^{(1)}[k], \sigma_{i}^{(2)}[k], \sigma_{i}^{(3)}[k]]^{T} \in \mathbb{R}^{6}$ are the states of the $i$-th spacecraft, $u_{i}[k] = [\omega_{i}^{(1)}[k], \omega_{i}^{(2)}[k], \omega_{i}^{(3)}[k]]^{T} \in \mathbb{R}^{3}$ are the control torques applied to chaotic dynamics, and $Q_{mm}$ and $R_{uu}$ are positive definite matrices, and $i = 1, \ldots, 4$. The values diag(1,1,100,100,100) and diag(1,0,0) for matrices $Q_{mm}$ and $R_{uu}$ are respectively chosen in relation to each cost function of spacecraft.

Simulation results are demonstrated in Figs. 7(a)-(d). Fig. 7(a)-(c) show attitudes, angular velocities, and the control torques of the four presumed spacecraft, respectively; and Fig. 7(d) illustrates the error caused by synchronization. It needs to be mentioned that the difference between the angular velocity values of each two neighboring systems are defined as the error

$$e_{ij}[k] = [e_{ij}^{(1)}[k], e_{ij}^{(2)}[k], e_{ij}^{(3)}[k]]^{T} = [\omega_{i}^{(1)}[k] - \omega_{j}^{(1)}[k], \omega_{i}^{(2)}[k] - \omega_{j}^{(2)}[k], \omega_{i}^{(3)}[k] - \omega_{j}^{(3)}[k]]^{T}$$

It may be observed in the figures that the synchronization error pertaining to the dynamic state variables of the neighboring spacecraft quickly converges to zero under the sequential distributed predictive control used here. This is indicative of achieving acceptable and perfect synchronization for the assumed chaotic systems.

V. CONCLUSION AND FUTURE WORK

The use of DMPC for the control and synchronization of chaotic dynamics of multiple spacecraft has been investigated in this paper. In order to create appropriate control law, the controller corresponding to each spacecraft solves a finite-horizon optimization problem in a sequential distributed algorithm. The control law resulting from the implementation of the mentioned algorithm may be used for synchronization and tracking a reference trajectory, while eliminating the chaotic behavior of the systems. Finally, an illustrative example for four spacecraft with a hypothetical topology is provided in order to demonstrate the effectiveness and feasibility of the selected scheme. In the example, each spacecraft is under the effect of a number of neighboring spacecraft introduced in the hypothetical topology in terms of performance criterion. Investigating the effect of delay on multiple spacecraft and using delayed distributed algorithms could be addressed in future research.

REFERENCES


Figure 7. Attitudes, angular velocities, control torques, and synchronization errors of 4 spacecraft: (a) Attitudes. (b) Angular velocities. (c) Control torques. (d) Absolute errors.