Fuzzy-Stochastic Linear Programming in Water Resources Engineering

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Abstract

Linear programming (LP) is a popular method for optimization of a wide range of applications because of its simplicity and availability. However, LP, in its classic form, is not equipped to handle information with fuzzy uncertainty. This paper considers the situations that both stochastic as well as fuzzy information are available and therefore could be coupled for improved optimization. For this purpose, fuzzy linear programming is modified by considering the stochastic properties of variables; so the new problem, fuzzy-stochastic linear programming (FSLP), can be formulated. The proposed method can be used as a complement and/or replacement of linear programming when enough data is not available for crisp statistical optimization. FSLP is then applied to a classic problem in water resources engineering, in particular the storage-yield problem applied to numerical data from Saugatuck reservoir. Simulation results demonstrate that for the presented scenario, the FSLP presents a more realistic water management, and can be used for replacement and/or complement of crisp LP optimization.

1. Introduction

Optimization of management plans is a necessary step in evaluation and identification of water resources systems. The tasks of water resources planners and engineers can be divided in two general categories,

1. Identification and determination of water resources parameters such as reservoir capacity. and
2. Management planning for operation of water resources systems, and evaluation of their impacts such as the ecological, environmental, social, and economic.

The importance of optimized design is clear. Most proposed water resource plans involve large investment such as in land and reservoir, economic result of which may last for decades. Additionally, in both economic and ecological sense, these investments are often irreversible [Loucks et al. 1981].

Besides above complexities in water resources design and management, environmental and social parameters are continuously changed. Hence, a wide variety of parameters are vague and uncertain which have significant impact on system performance. Natural parameters such as precipitations and runoffs are dynamic and chaotic. So planners are forced to imagine the future state of the system with extrapolation of previous conditions. Some recent research topics such as "sustainable water resources management [Loucks et al. 2000]" have noted this problem.

A traditional approach for handling uncertainty is probability theory. In water resources engineering, probability often occurs in estimation of initial parameters such as amount of monthly precipitation and the inflow to the reservoir. This information is either derived from available data, or if available data is not adequate, various methods of curve fitting to a supposed probability distribution are used. Whether we use previously obtained data or extrapolate data, both methods have uncertainty. At best, there is no guarantee that future conditions have the same trend.

Fuzzy set theory (FST) is another method for dealing with uncertainty [Zadeh 1965]. One of the main goals of FST is dealing with the uncertainty and vagueness in measurements. Through the past four decades and in spite of challenges from scientists, especially probabilists, FST has matured and has found various engineering applications particularly in control. In water engineering, also, a wide range of applications can be found such as in infiltration modeling [Bardossy et al. 1990, 1993], circulation pattern prediction [Bardossy and Duckstein 1995], reservoir operation [Russel and Campbel 1996], pier scour prediction [Johnson and Ayuub 1996], and rainfall runoff modeling [Özelkan and Duckstein 2001].

The main characteristic of fuzzy models in contrast to probabilistic models is assuming error in model rather than measurements [Heshmaty and Kandel 1985]. Fuzzy models can be easily coupled. They can be transparent, adaptive, and robust; and imprecise information can be used which is an "extremely difficult task in probabilistic models" [Bardossy and Duckstein 1995]. In [Megdadi and Akbarzadeh, 2001], the authors argued that a unifying framework would be useful for handling complexity of both fuzzy nature as well as probabilistic nature. They applied a proposed unifying framework for modeling human behavior.

Considering above, traditional linear programming (LP) seems to be ill-equipped to handle uncertainties of a water resources management problem. In LP, the objective function and all of problem constraints are linear. In this paper, we compare the LP result of Saugatuck reservoir with fuzzy linear programming (FLP). The given problem is in both stochastic and fuzzy environments so the new problem fuzzy stochastic linear programming (FSLP) is introduced. In section 2 the over view on LP and FLP is provided. Consequently, in section 3, the conditions under which the two fuzzy and probability characteristics may occur is discussed, and FSLP is proposed. In section 4, the applicable and classic problem in water resources engineering is explained. In section 5 the results of FSLP are given and compared with LP. In section 6 conclusions are derived and various potential applications of FST in the field of water engineering is overviewed.

2. Fuzzy linear programming

If objective function as well as all constraints is linear, then the optimization problem is a linear programming problem. The standard LP is formulated as,

$$\begin{aligned} & Optimize\, C^T X \\ & Subject\,to: \\ & AX \le b \\ & X \ge 0. \end{aligned} \tag{1}$$

In many cases, however, it can be argued that the '<' is not crisply defined, or that the crisp value for A, b, and C can't be found exactly. These conditions may be caused by a lack of adequate measurements and/or inherent ambiguity in system structure.

FLP problems can be expected to remedy above uncertainties. A FLP problem can be summarized in several forms such as the following [Wang 1997],

LP with fuzzy resources:

$$\begin{aligned} & \max C^T X \\ & Subject\,to: \\ & AX \le \tilde{b} \\ & X \ge 0 \end{aligned} \tag{2}$$

LP with fuzzy objective function

$$\begin{aligned} & \max \tilde{C}^T X \\ & Subject\,to: \\ & AX \le b \\ & X \ge 0 \end{aligned} \tag{3}$$

LP with fuzzy constraint coefficients:

$$\begin{aligned} & \max C^T X \\ & Subject\,to: \\ & \tilde{A}X \le b \\ & X \ge 0 \end{aligned} \tag{4}$$

Of course, the combinations of these formulations provide more types of fuzzy linear programming

Various solutions are introduced for above FLP problems. For example [Zimmermann 1985] proposed a general procedure. Alternatively, [Wang 1997] proposed a simple method as presented below.

Let's consider the LP problem with fuzzy constraint coefficients. For simplicity and without loss of much generality, we assume that A=$[\tilde{a}_{ij}]$ composes a triangular fuzzy number, that is

$$\begin{aligned} & \max \quad C^T X \\ & subject\,to: \\ & (A^- X, A^0 X, A^+ X) \le b \\ & X \ge 0 \end{aligned} \tag{5}$$

using the most-likely criterion [Lai & Hwang 1992], we can convert it in the following standard LP:

$$\begin{aligned} & \max \quad C^T X \\ & subject\,to: \\ & \frac{4A^0 + A^- + A^+}{6} X \le b \\ & X \ge 0 \end{aligned} \tag{6}$$

3. Fuzzy stochastic linear programming

Now, assume that in addition to the fuzzy nature, the coefficients have stochastic property as well. For this problem, we propose FSLP. Before describing the model, following definitions are introduced.

Definition 3.1: assume that the variable '*a*' has stochastic nature, but that not enough data is available. '*a*' is also fuzzy. Then we call it a "fuzzy-stochastic" variable.

Definition 3.2: assume a LP problem, with crisp objective function. If constraint coefficients are "fuzzy-stochastic" variable, our problem becomes a fuzzy-stochastic linear programming (FSLP) problem.

For FSLP formulation, the following steps is performed,

1- Estimated mean μ_{ij} and variance σ_{ij} of each constraint coefficient a_{ij} with available data.

2- Assume that each constraint coefficient a_{ij} is an independent random number and have a convenient probability distribution. Then the set of random numbers S_{ij} is defined as below by applying measured mean and variance:

$$S_{ij} = \{x_k \mid x_k \in \text{assumed distribution}; k = 1,...,N\} \quad (7)$$

Where x_k is *k*-th generated random number. N is chosen large enough to generated a set with enough number of random numbers to represent almost all states of the coefficient.

From step 2, the constraint coefficient is defined as the following fuzzy number $\widetilde{A} = (A^-, A^0, A^+)_T$ as in [Bardossy and Duckstein 1995], where T stands for triangular membership function, and

A^0 : is the most credible value, which is assigned a membership value of 1, and is defined as average of the generated random set.

$$A^0 = \frac{\sum_{i=1}^{N} x_i}{N} \quad (8)$$

A^- : is assigned membership value 0, and is defined as minimum value of generated random set. A^- is expected to be almost certainly be exceeded by the actual parameter value.

$$A^- = \inf_{i=1,...,N} (x_i) \quad (9)$$

A^+ : is defined as maximum value of generated random set. A^+ equals or exceeds the actual parameter value.

$$A^+ = \sup_{i=1,...,N} (x_i) \quad (10)$$

For all constraint coefficients the above procedure is performed. Therefore, the uncertainty in all constraint coefficients is quantified by the triangular membership functions. The traditional LP model can be easily modified by adjusting crisp coefficient A with $[\widetilde{a}_{ij}]$ triangular membership function and can be solved with Eq. (6).

4. Application in water resources engineering

As mentioned in introduction, a wide range of water systems planning and management are in fuzzy environment. The first reason is a lack of adequate data, the second is extrapolation of previous conditions to the future states, and third is interactions of parameters that may not be obvious. So application of FLP as an optimization tool is intuitive and plausible. Below, the classic problem "storage-yield function" for design of water reservoir capacity is presented and discussed.

Storage-yield problem [Laucks et al 1981]: The 90% and 60% reliable reservoir storage yield function are produced by plotting the maximum value of yield r versus the reservoir capacity CP. for simplicity we ignore seepage losses and assume that the evaporation from the reservoir is linear function of the average reservoir storage during all months. It means that

$$E_k = C_k \frac{(S_k + S_{k+1})}{2} \quad (11)$$

Where k represents the month k={1,2,3…12}, S_k is the reservoir storage volume at the end of month k, E_k is the evaporation loss during each month k, and C_k is evaporation coefficient at month k.

The above problem can be formulated with LP. The objective is to maximize yield at steady state, and is constrained by the water available in each monthly period and by the reservoir capacity. Two sets of constraints are needed to define the relationships among the inflow I_k, the reservoir storage volumes S_k, the yield r, and the reservoir capacity CP. The first is the set of continuity equations, shown as:

$$\begin{aligned} S_{k+1} &= S_k + I_k - r - E_k \\ k &= 1,2,...,K \end{aligned} \quad (12)$$

Where K is either 12, 52, 365 for monthly, weekly and daily data sets, respectively. As indicated in Eq (12), due the annual periodicity of process, one can assume that period 1 follows period K. Therefore, it is not necessary to specify the value of both S_1 and S_{K+1}, since they are the same. The resulting "steady-state" solution is essentially based on the assumption that the entire inflow sequence will repeat itself again and again. Of course, this is very unlikely, but it is as good as the alternative assumptions that could be made. This assumption should have minor effect on the model's behavior [Laucks 1981].

If the stream flow sequence is relatively long, the second set of required constraints ensures that the reservoir storage volume S_k in each period k is not greater than the active reservoir capacity:

$$S_k \leq CP \quad (13)$$

To derive a storage-yield function, the model defined by above equations must be solved for various values of capacity CP.
The above problem was solved for Saugatuck reservoir in USA. The data consist of 10 years of monthly 90% and 60% probable inflows, demand for water and evaporation coefficient. The data and results are shown in Table (1), Figures (2) & (3) respectively.

Table (1): Data derived from 10 years data of Saugatuck reservoir (Bridge Port, CT.)

T	Month	Inflow P=0.9	Inflow P=0.6	Demand	C_k
1	Jan.	0.18	0.35	0.12	0.05
2	Feb.	0.20	0.40	0.10	0.05
3	March	0.30	0.50	0.10	0.05
4	April	0.85	1.25	0.10	0.06
5	May	1.00	1.30	0.15	0.07
6	June	1.10	1.40	0.20	0.09
7	July	0.55	0.70	0.25	0.10
8	Aug.	0.30	0.58	0.30	0.15
9	Sept.	0.35	0.42	0.25	0.10
10	Oct.	0.40	0.60	0.20	0.07
11	Nov.	0.42	0.64	0.18	0.05
12	Dec.	0.28	0.55	0.13	0.05

Stochastic nature of the model is concentrated in inflow amounts (60% and 90% level of probability), demands and evaporation coefficients. These values, however, are often obtained from inadequate measurements; additionally due to chaotic nature of the natural system, extrapolating these data to the future state of the system is challenging. Each measurement has average (as shown in Table (1)), and variance that could be obtained from data of various years. So alternatively, the model can be transformed to FSLP.
FSLP can be used to solve the optimization problem, or sensitivity analysis of the model when a fuzzy scenario is proposed. For this purpose, we must consider the set of possible scenarios that may happen in the future. A simple and plausible scenario is given as follow:
Presented scenario: Assume data of Table (1); It is imagined that mean of evaporation coefficient and demand are increased by 10% in average values. So the reservoir must be sustainable for 10% increase in demand without any change in inflow or capacity. It is also assumed that variances of evaporation coefficient and demand have not changed. Gamma distribution is considered as proper probability distribution.

5. Results and discussion

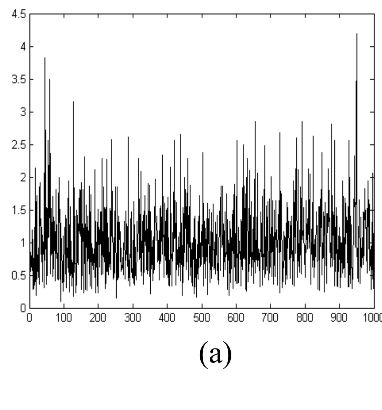
First the membership function of constraints coefficients must be built. For each parameter, a set of *N=1000* random variables was generated in the form of Eq. (7) with the assumption of gamma distribution with respect to their mean and variance. For this purpose we use following transformation:

$$\mu = \alpha \cdot \beta$$
$$\sigma^2 = \alpha \cdot \beta^2 \quad (14)$$

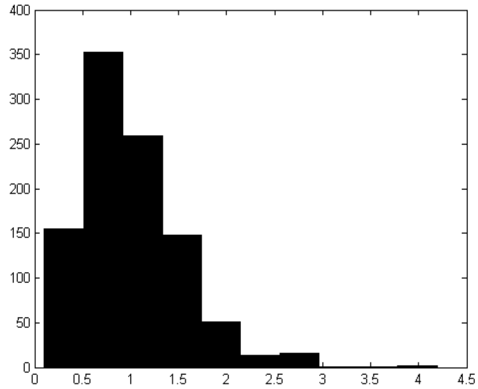
Where α and β are gamma distribution parameters.
Results for 90% probable inflow are shown in Table (2). Figure (1) shows the procedure for constructing membership function of fuzzy-stochastic coefficients in the model.
After construction of membership functions FSLP can be easily solved in the form of LP with Eq. (6). The results are shown in Figures (2) & (3). It can be seen that in both 60% & 90% probable inflows FSLP gives a higher value of yield with a softer curve. In LP model for both 60% & 90% probable inflows, the optimized capacity is 2, but FSLP yields 2.2. It can be also seen that in FSLP model infeasibility decrease more moderately than LP as capacity increases.
For presented scenario the results of FSLP can be used for sensitivity analysis of storage volumes in each month. Comparing FSLP results with LP, a measure of difference of LP optimization can be introduced as follow:

$$D^p{}_k = \frac{S_{LP\,k}{}^p - S_{FSLP\,k}{}^p}{S_{FSLP\,k}{}^p} \quad (15)$$

Where $D^p{}_k$ is a reliability measure of LP optimization in probability level p and month k, $S_{LP\,k}{}^p$ is storage volume at the probability level p and month k based on LP optimization, and $S_{FSLP\,k}{}^p$ is storage volume at the probability level p and month k based on FSLP optimization. The results given by this measure for presented scenario are shown in Figure (4).



(a)



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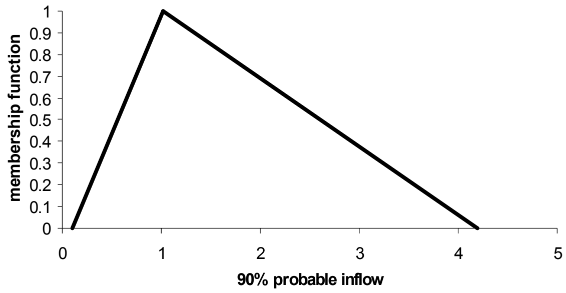
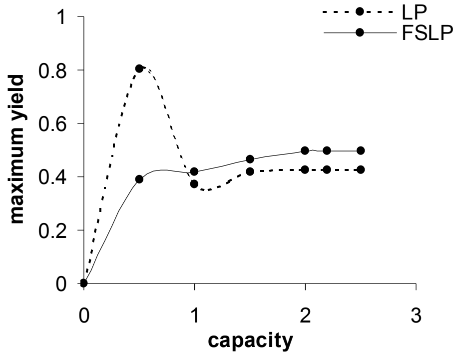


(c)

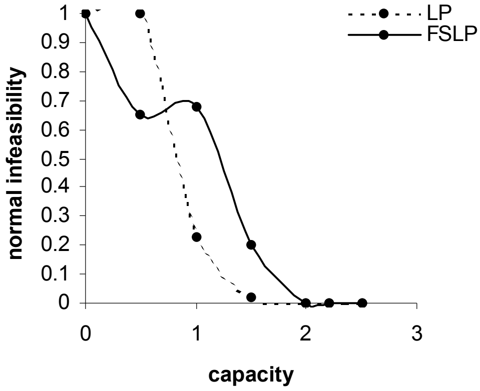
Figure (1): Membership function generation for May 90% inflow: (a) generated random set; 1000 random number with gamma distribution [mean=1, std=0.5] (b) histogram of the random set (c) generated membership function

Table (2): membership function of constraint coefficients of 90% probable inflow

Month	Inflow (P=0.9)	Stan. dev	C^0	C^-	C^+
Jan	0.18	0.05	0.18	0.04	0.39
Feb	0.2	0.1	0.20	0.02	0.83
March	0.3	0.15	0.29	0.02	1.06
April	0.85	0.3	0.85	0.09	2.19
May	1	0.5	1.01	0.10	4.19
June	1.1	0.48	1.07	0.22	3.05
July	0.55	0.2	0.55	0.11	1.57
Aug.	0.3	0.11	0.30	0.05	0.79
Sept.	0.35	0.11	0.35	0.09	0.87
Oct.	0.4	0.19	0.40	0.06	1.18
Nov.	0.42	0.14	0.42	0.08	0.94
Dec.	0.28	0.12	0.27	0.05	0.89

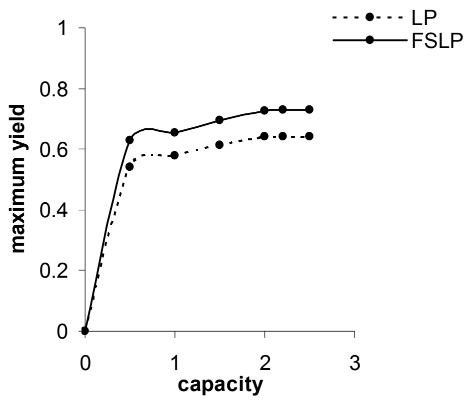


(a)

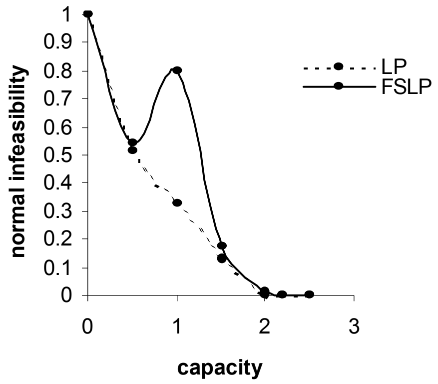


(b)

Figure (2): LP & FSLP results comparisons for storage-yield problem: (a) LP & FSLP results for 90% probable inflow, (b) comparisons between LP & FSLP infeasibility for 90% probable inflow.



(a)



(b)

Figure (3): FSLP & LP results comparisons for storage-yield problem: (a) LP & FSLP result for 60% probable inflow, (b) comparison between LP & FSLP infeasibility for 60% probable inflow.

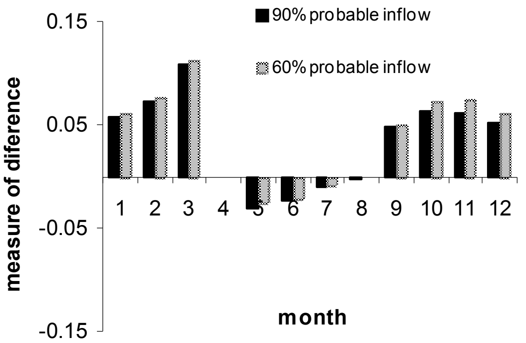


Figure (4): Sensivity analysis for storage volume in presented scenario: 90% probable inflow, 60% probable inflow

6. Conclusion

In this paper simple and intuitive method for optimization in both fuzzy and stochastic environments is introduced. FSLP can be used as a complement of LP optimization. In many cases that adequate measures are not available, FSLP can be applied for optimization. The FSLP was applied for classic problem storage-yield in water resources optimization and compared with LP results. It is shown that FSLP is more realistic than LP with respect to inherent uncertainties of parameters. Many improvements can be applied in FSLP, such as accounting for non-stationary condition and/or another probability distribution.

Water engineering is dynamic and fuzzy. FST can be adequately applied to the complex problems in this field. In recent years the cooperation of different methodologies in solving problems has become popular. This hybrid approach, soft computing, is playing a more and more important role in modern day water engineering and finds new and complex problems to solve.

7. Acknowledgement

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8. References

[Bardossy, et. Al., 1990] Bardossy, A., Bogardi, I., Duckstein, L.,: "Fuzzy regression in hydrology"; *Journal of water resources research*. VOL. 26, NO. 7, pp. 1497-1508, 1990.

[Bardossy, 1993] Bardossy, A.,: "Fuzzy rule-based models for infiltration"; *Journal of water resources research*. VOL. 29, NO.2, pp. 373-382, 1993 .

[Bardossy and Duckstein, 1995] Bardossy, A., Duckstein, L.,: *Fuzzy rule-based modeling with applications to geophysical, biological & engineering systems*. CRC Press, Boca Raton, 1995.

[Johnson and Ayyub, 1996] Johnson, P. A., Ayyub, B. M.,: "Modeling uncertainty in prediction of pier scour". *Journal of hydraulic engineering*. VOL.122, NO. 2, pp. 66-72, 1996.

[Heshmaty and Kandel, 1985] Heshmaty, B., Kandel, A.,: "Fuzzy linear regression and its applications to forecasting in uncertain environments"; *Fuzzy sets and systems*, VOL. 15, pp. 159-191, 1985.

[Loucks, et. Al, 1981] Loucks, D. P., Stedinger, J. R. , Haith, D. A.,: *Water resources system planning and analysis*, 1981.

[Loucks, et. Al. 2000] Loucks, D. P., Stakhiv, E. J., "Sustainable water resources management", *Journal of water resources planning and managements*, Div ASCE , March & Aprill . 2000.

[Meghdadi and Akbarzadeh, 2001] A.-H. Meghdadi and M.-R. Akbarzadeh-T., "Probabilistic Fuzzy Logic and Probabilistic Fuzzy Systems," Proceedings of the 10th IEEE International Conference on Fuzzy Systems, Melbourne, Australia, December, 2001.

[Özelkan, Duckstein, 2001] Özelkan, E. C., Duckstein L.,: "Fuzzy conceptual rainfall-runoff models", *Journal of hydrology* VOL. 253, pp. 41-68, 2001.

[Russel and Campbel, 1996] Russel, S. O., Campbel P. F.,: "Reservoir operating rules with fuzzy programming", *Journal of water resources planning and management*, VOL. 122, No. 3, pp. 165-170, 1996

[Wang, 1997] Wang X. J.: *A course in fuzzy systems and control*, Prentice Hall Inc. , Upper Saddle River, NJ, 1997

[Zimmermann, 1985] Zimmermann, H. J.,: *Fuzzy set theory and its applications*, Kluwer-Nijhoff Publishing 1985.