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^{3}He and ^{3}H polarized structure functions, using the constituent quark model

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Abstract

We calculate polarized structure functions for ^{3}He and ^{3}H, using the convolution of the light cone momentum distribution with the polarized structure of the proton and neutron. The polarized structure function of the nucleon is computed using the constituent quark model. Hypergeometric orthogonal polynomials are employed to extract the unknown parameters in this phenomenological approach. These hypergeometric polynomials are placed at the third level of Askey scheme with two free parameters. The results obtained for the polarized nuclear structure functions and the ratio of the Bjorken sum rule for proton–neutron system to ^{3}He–^{3}H system are in good agreement with the available experimental data and some theoretical models. To improve the validity of the model at low x-values, the nuclear shadowing, antishadowing and Δ-resonance effects are also considered.

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1. Introduction

In contrast to most deep inelastic structure functions which correspond to spin averaged scattering, it is possible to extend the discussion to the situation where, for instance, the lepton beam and nucleon target are polarized in the longitudinal direction. The theoretical and experimental status of the spin structure of the nucleon has been discussed in great detail in several recent reviews [1–4]. During recent years several comprehensive analyses of polarized deep inelastic scattering (DIS) data, based on next-to-leading-order quantum chromodynamics, have appeared in Refs. [5–10]. In these analyses the polarized parton density functions (PPDFs) are either written in terms of the well-known parameterizations of the unpolarized PDFs or parameterized independently, and the unknown parameters are determined by fitting to the polarized DIS data. The nature of the short-distance structure of polarized nucleons is one of the central questions of present day hadron physics. For more than sixteen years, polarized inclusive deep inelastic scattering has been the main source of information on how the individual partons in the nucleon are polarized at very short distances. The extraction of the quark helicity distributions is one of the main tasks of the semi-inclusive deep inelastic scattering experiments (HERMES [11], COMPASS [12], SMC [13]) with a polarized beam and target. Very recently, experimental data have become available from the HERMES and SMC Collaborations for the spin structure function g_1, therefore there is enough motivation to study and utilize the spin structure and quark helicity distributions extracted via the phenomenological model.

One of the phenomenological models which we believe to be able to give us adequate parton distribution in the nucleon is the valon framework, and we use it in this paper. Hwa suggested valons to describe the deep-inelastic scattering data and applied it to a variety of phenomena [14–22]. He has also successfully formulated a treatment of low-P_T reactions based on a structural analysis of valons. Here a valon can be defined as a valence quark and associated sea quarks and gluons which arise in the dressing processes of QCD. In a bound state problem these processes are virtual, and a good approximation for the problem is to consider a valon as an integral unit whose internal structure cannot be resolved. In a scattering situation, on the other hand, the virtual partons inside a valon can be excited and be put on the mass shell. It is therefore more appropriate to think of a valon as a cluster of partons with some momentum distributions. The proton, for example, has three valons which interact with each other in a way that is characterized by the valon wave function, while they respond independently in an inclusive hard collision with a Q^2 dependence that can be calculated in QCD at high Q^2. Hwa and Yang refined the idea of the valon model and extracted new results for the valon distributions [23].

In this paper we use the idea of the polarized valon model to obtain the PPDFs in the LO and NLO approximations. The results of the present analysis are based on the hypergeometric orthogonal polynomials (HOPs) expansion of the polarized structure function. In the Askey scheme, the HOPs are divided into several levels [24]. Each level is specified by respectively, four, three, two and one free parameters. The HOPs at different levels can be converted to the next one via Mellin transforms in a hierarchical ladder approach.

Moreover, in recent years unpolarized and polarized nuclear structure functions have also been discussed from theoretical and experimental viewpoints [25–29]. The important issue in this subject is the different behavior of parton densities in free nucleons and bound nucleons, i.e. nuclei. Here the nuclear effects play an essential rule in calculating parton distribution functions in the nuclei [25,30]. To obtain the unpolarized and polarized nuclear structure functions from nucleon ones, we need to perform an integral in which the light cone momentum distribution of the nucleon in the nucleus is convoluted with nucleon structure functions (in both unpolarized and polarized cases) [31]. The main requirement for these calculations is well-behaved nucleon parton distributions. Consequently we are able to calculate the polarized structure function ^{3}He and ^{3}H. To increase the validity of the model at low x-values, the nuclear effects, including shadowing, antishadowing and Δ-resonance are also considered.

This paper is organized as follows. Section 1 starts with an introduction giving preliminary definitions of the constituent quark model, hypergeometric orthogonal polynomials and nuclear structure function. Section 2 describes the valon framework as a constituent quark model. In Section 3 the information required to extract polarized parton distributions and nucleon structure functions, based on the constituent quark model, is introduced. In Section 4, we employ the third level of hypergeometric orthogonal polynomials with two free parameters, to reconstruct the nucleon structure function in terms of its moments. This allows us to do the fitting to the available experimental data in a more convergent way. In Section 5 we deal with the spectral function and light cone distribution functions of nucleons to extract the nuclear structure function. In extracting this structure function one uses the nucleon structure function derived in the previous section. Section 6 contains the analytical results for $g_1^{^3\mathrm{He}}$ and $g_1^{^3\mathrm{H}}$. The nuclear shadowing, antishadowing and Δ-resonance effects are presented. A numerical result for ratio of the Bjorken sum rule in a proton–neutron system to ^{3}He–^{3}H system is also given. Comparison of the analytical results with available experimental data confirms the anticipated agreement with the data. Finally we give our conclusions in Section 7.

2. Valon model and polarized parton distributions

To describe the quark distribution function $q(x)$ in the valon model, one can try to relate the polarized quark distribution functions $q^{\uparrow}$ or $q^{\downarrow}$ to the corresponding valon distributions $G^{\uparrow}$ and $G^{\downarrow}$. The polarized valon can still have valence and sea quarks that are polarized in various directions, so long as the net polarization is that of the valon. When we have only one distribution $q(x, Q^2)$ to analyze, it is sensible to use the convolution in the valon model to describe the proton structure in terms of the valons. In the case that we have two quantities, unpolarized and polarized distributions, there is a choice of which linear combination exhibits more physical content. Therefore, in our calculations we assume a linear combination of $G^{\uparrow}$ and $G^{\downarrow}$ to determine, respectively, the unpolarized (G) and polarized (δG) valon distributions.

Polarized valon distributions were calculated by using an improved valon model in next-to-leading order approximation [6]. According to the improved valon model, the polarized parton distribution is related to the polarized valon distribution. On the other hand, the polarized parton distribution of a proton is obtained by convolution of two distributions: the polarized valon distributions in the proton and the polarized parton distributions in the valon.

$$\delta q_{i/p}\big(x, Q^2\big) = \sum_j \int_x^1 \delta q_{i/j}\left(\frac{x}{y}, Q^2\right) \delta G_{j/p}(y) \frac{dy}{y}, \tag{1}$$

where the summation is over the three valons. Here $\delta G_{j/p}(y)$ indicates the probability for the polarized j-valon to have momentum fraction y in the proton. $\delta q_{i/p}(x, Q^2)$ and $\delta q_{i/j}(\frac{x}{y}, Q^2)$ are, respectively, the polarized i-parton distribution in the proton and j-valon. As was noted in [6], the polarized quark distribution can be related to the polarized valon distribution. Using Eq. (1) we can obtain polarized parton distributions in the proton at different value of Q^2.

To extract polarized valon distribution, we need to unpolarized one. It is assumed in [20] a simple form for the exclusive valon distribution which facilitated the phenomenological analysis as follows

$$G_{UUD/p}(y_1, y_2, y_3) = g(y_1 y_2)^a y_3^b \delta(y_1 + y_2 + y_3 - 1), \tag{2}$$

where y_i is the momentum fraction of the i-th valon. The U and D type inclusive valon distributions in an unpolarized case, can be obtained by double integration over the unspecified variables

$$G_{U/p}(y) = \int dy_2 \int dy_3\, G_{UUD/p}(y, y_2, y_3) = gB(a+1, b+1)y^a(1-y)^{a+b+1}, \tag{3}$$

$$G_{D/p}(y) = \int dy_1 \int dy_2\, G_{UUD/p}(y_1, y_2, y) = gB(a+1, a+1)y^b(1-y)^{2a+1}. \tag{4}$$

The normalization parameter g has been fixed by

$$\int_0^1 G_{U/p}(y)\, dy = \int_0^1 G_{D/p}(y)\, dy = 1, \tag{5}$$

and is equal to $g = [B(a+1, b+1)B(a+1, a+b+2)]^{-1}$, where $B(m,n)$ is the Euler-Beta function. Authors in [23] have recalculated the unpolarized valon distribution in the proton with a new set of parameters. The new values of a, b are found to be $a = 1.76$ and $b = 1.05$.

To be more precise for the polarized valon model, let us start by defining unpolarized and polarized quark distribution as follows:

$$q(x) = q^{\uparrow}(x) + q^{\downarrow}(x),$$
$$\delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x).$$

For the polarized parton distributions, $|\delta f(x, Q^2)|$, and the unpolarized ones, $f(x, Q^2)$, positivity requirements at low values of Q^2 imply the constraint [33,34]

$$\left|\delta f\left(x, Q^2\right)\right| \leqslant f\left(x, Q^2\right), \tag{6}$$

where $f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$. Furthermore, we have the following sum rules [34]

$$\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d} = A_3 = 1.2573 \pm 0.0028, \tag{7}$$

$$\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2(\Delta s + \Delta\bar{s}) = A_8 = 0.579 \pm 0.025, \tag{8}$$

$$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q}) = A_8 + 3(\Delta s + \Delta\bar{s}) \equiv A_0, \tag{9}$$

where with $n = 1$, the first moment $\delta M(1, Q^2)$ is defined by

$$\Delta f\left(Q^2\right) \equiv \delta M\left(1, Q^2\right) = \int_0^1 dx\, \delta f\left(x, Q^2\right). \tag{10}$$

To determine the PPDFs, the main step is to relate the polarized input densities to the unpolarized ones [34] using some intuitive theoretical argument as the guideline. We employ the general ansatz for the PDF of Ref. [34], and introduce the following equations to relate the polarized and unpolarized valon distributions

$$\delta G_{j/p}(y) = \delta \mathcal{W}_j(y) \times G_{j/p}(y), \tag{11}$$

here j refers to U and D type valons. The unpolarized valon distributions, $G_{j/p}(y)$, have been defined by Eqs. (3), (4). The functions $\delta \mathcal{W}_j(y)$ play an essential role in constructing the polarized valon distributions $\delta G_{j/p}(y)$ from unpolarized ones. To improve the definition of polarized valon distribution functions, we introduce the following definitions:

$$\delta G_{j/p} \to \begin{cases} \delta \mathcal{W}'_j(y) \times G_{j/p}(y) & \text{for non-singlet case,} \\ \delta \mathcal{W}''_j(y) \times G_{j/p}(y) & \text{for singlet case.} \end{cases} \tag{12}$$

To define the actual y-dependence of the $\delta \mathcal{W}'$ function, we parameterize this function as

$$\delta \mathcal{W}'_j(y) = N_j y^{\alpha_j} (1-y)^{\beta_j} \big(1 + \gamma_j y + \eta_j y^{0.5}\big). \tag{13}$$

As before the subscript j refers to U and D-valons.

The motivation for choosing this functional form is that the term y^{α_j} controls the low-y behavior valon densities, and $(1-y)^{\beta_j}$ that at large values y. The remaining polynomial factor accounts for the behavior at intermediate y values.

For $\delta \mathcal{W}''_j(y)$ we choose the following form

$$\delta \mathcal{W}''_j(y) = \delta \mathcal{W}'_j(y) \times \sum_{m=0}^{5} \mathcal{B}_m y^{\frac{m-1}{2}}. \tag{14}$$

The additional term in the above equation ($\sum$ term) serves to control the behavior of the singlet sector at very low-y values in such a way that we can extract the sea quark contributions. Moreover, the functional form for $\delta \mathcal{W}'$ and $\delta \mathcal{W}''$ give us the best fitting χ^2 value. In these functions, all of the parameters are unknown. Using the experimental data for g_1^p [35,36] and the hypergeometric polynomials expansion as describe in next section, we can fit for the unknown parameters of Eqs. (13)–(14).

The first moment of the polarized u_v, d_v and Σ distribution functions obtained using the valon model, can help us to implement the constraint of Eqs. (7)–(9) for the improved polarized valon model with an SU(3) flavor symmetry assumption. These constraints play the same role as in the case of the unpolarized ones in controlling the parameter values which will appear in the polarized valon distributions.

3. The theoretical framework of QCD analysis

Let us define the Mellin transforms for any function $f(x, Q^2)$ as follows:

$$\mathcal{M}\{f\big(x, Q^2\big)\} = M\big(n, Q^2\big) = \int_0^1 x^{n-1} f\big(x, Q^2\big)\, dx, \tag{15}$$

here n is the order of moments.

In the QCD-improved quark parton model, i.e., at leading twist, and to leading logarithmic order in the running strong coupling constant $\alpha_s(Q^2)$ of quantum chromodynamics, the deep-inelastic scattering off the nucleon can be interpreted as the incoherent superposition of virtual-photon interactions with quarks of any flavor q. By angular momentum conservation, a spin-$\frac{1}{2}$

parton can absorb a hard photon only when their spin orientations are opposite. The spin structure function has then a probabilistic interpretation, which for the proton reads [37]

$$\begin{aligned}\mathcal{M}\{g_1^N(x,Q^2)\} \equiv \delta M^N(n,Q^2) &= \frac{1}{2}\sum_q e_q^2\big[\delta M_q(n,Q^2) + \delta M_{\bar{q}}(n,Q^2)\big] \\ &= \frac{1}{2}\langle e^2\rangle\big[\delta M^S(n,Q^2) + \delta M^{NS}(n,Q^2)\big],\end{aligned} \tag{16}$$

where the δ symbol denotes the moments in the polarized case and N, a nucleon, refers to the proton and neutron separately. In Eq. (16), e_q is the charge in units of the elementary charge $|e|$ of quarks of flavor q, $\langle e^2\rangle = \sum_q e_q^2/N_q$ is the average squared charge of the N_q active quark flavors, and $\delta M_q(n,Q^2)$ is the quark helicity distribution for quarks of flavor q. Correspondingly, $\delta M_{\bar{q}}(n,Q^2)$ are antiquark helicity distributions. Moreover the flavor singlet and flavor non-singlet quark helicity distributions are defined as

$$\delta M^S(n,Q^2) = \sum_q\big[\delta M_q(n,Q^2) + \delta M_{\bar{q}}(n,Q^2)\big], \tag{17}$$

and

$$\delta M^{NS}(n,Q^2) = \frac{1}{\langle e^2\rangle}\sum_q e_q^2\big[\delta M_q(n,Q^2) + \delta M_{\bar{q}}(n,Q^2)\big] - \delta M^S(n,Q^2). \tag{18}$$

For the analysis presented in this paper, only the three lightest quark flavors, $q = u, d, s$, are taken into account and the number of active quark flavors N_q is equal to three.

The twist-2 contributions to the structure function $\delta M^N(n,Q^2)$ can be represented in terms of the polarized parton densities and the Wilson coefficient functions $\delta C_i(n)$ in the moment space by [2]

$$\begin{aligned}\delta M^N(n,Q^2) = \frac{1}{2}\sum_q e_q^2\bigg\{&\left(1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q(n)\right)\big[\delta M_q(n,Q^2) + \delta M_{\bar{q}}(n,Q^2)\big] \\ &+ \frac{\alpha_s(Q^2)}{2\pi}2\delta C_g(n)\delta M_g(n,Q^2)\bigg\}.\end{aligned} \tag{19}$$

In this equation the NLO running coupling constant is given by

$$\frac{\alpha_s(Q^2)}{4\pi} \cong \frac{1}{\beta_0\log\frac{Q^2}{\Lambda^2_{\overline{MS}}}} - \frac{\beta_1}{\beta_0^3}\frac{\ln(\ln\frac{Q^2}{\Lambda^2_{\overline{MS}}})}{(\ln\frac{Q^2}{\Lambda^2_{\overline{MS}}})^2}, \tag{20}$$

where β_0 and β_1 are given in [38]. To evolve the moments of parton distributions, we choose Q_0 as a fixed parameter and Λ is an unknown parameter which can be obtained by fitting to the experimental data.

In Eq. (19), $\delta M_q(n,Q^2) = \delta M_{q_v}(n,Q^2) + \delta M_{\bar{q}}(n,Q^2)$, $\delta M_{\bar{q}}(n,Q^2)$ and $\delta M_g(n,Q^2)$ are moments of the polarized parton distributions in a nucleon. $\delta C_q(n)$, $\delta C_g(n)$ are also the n-th moments of spin-dependent Wilson coefficients given by

$$\begin{aligned}\delta C_q(n) = \frac{4}{3}\bigg[&-S_2(n) + \big(S_1(n)\big)^2 + \left(\frac{3}{2} - \frac{1}{n(n+1)}\right)S_1(n) \\ &+ \frac{1}{n^2} + \frac{1}{2n} + \frac{1}{n+1} - \frac{9}{2}\bigg],\end{aligned} \tag{21}$$

and

$$\delta C_g(n) = \frac{1}{2}\left[-\frac{n-1}{n(n+1)}\big(S_1(n)+1\big) - \frac{1}{n^2} + \frac{2}{n(n+1)}\right], \tag{22}$$

with $S_k(n)$ defined as in Ref. [2].

According to the improved polarized *valon* model, the moments of the polarized parton distributions in a nucleon can be determined entirely from the moments of the polarized valon distributions. The moments of the polarized parton distribution functions are denoted respectively by $\delta M_{u_v}(n, Q^2)$, $\delta M_{d_v}(n, Q^2)$, $\delta M^S(n, Q^2)$ and $\delta M_g(n, Q^2)$ in the nucleon. Therefore, the moments of the polarized u and d-valence quark in a proton are the product of two moments [6]:

$$\delta M_{u_v}\big(n, Q^2\big) = 2\delta M^{NS}\big(n, Q^2\big) \times \delta M'_{U/p}(n), \tag{23}$$

$$\delta M_{d_v}\big(n, Q^2\big) = \delta M^{NS}\big(n, Q^2\big) \times \delta M'_{D/p}(n). \tag{24}$$

In the above equation $M'_{j/p}(N)$ is the moment of the $\delta G_{j/p}(y)$ distribution, i.e. $\delta M'_{j/p}(n) = \mathcal{M}\{\delta G_{j/p}(y)\}$. The moment of the non-singlet and singlet sectors for U and D valons are equal and we denote them respectively by $\delta M^{NS}(n, Q^2)$ and $\delta M^S_{(U,D)}(n, Q^2)$. Consequently the moment of the polarized singlet distribution is as follows:

$$\delta M^S\big(n, Q^2\big) = \delta M^S_{(U,D)}\big(n, Q^2\big)\big(2\delta M''_{U/p}(n) + \delta M''_{D/p}(n)\big). \tag{25}$$

Here $\delta M''_{j/p}(n)$ is the moment of the $\delta G_{j/p}(y)$ distribution [6] which is related to $\delta \mathcal{W}''_j(y)$ in Eq. (14).

For the gluon distribution we have

$$\delta M_g\big(n, Q^2\big) = \delta M^{gq}\big(n, Q^2\big)\big(2\delta M'_{U/p}(n) + \delta M'_{D/p}(n)\big). \tag{26}$$

The moments in Eqs. (23)–(26) including $\delta M^{NS}(n, Q^2)$, $\delta M^S_{(U,D)}(n, Q^2)$ and $\delta M^{gq}(n, Q^2)$ (quark-to-gluon evolution function), are defined in Ref. [6].

The unknown parameters which exist in $\delta M^N(n, Q^2)$ could be obtained by fitting to the available experimental data, making use of the hypergeometric orthogonal polynomials which will be explained in the next section.

4. Spin dependent analysis of structure function — Hypergeometric orthogonal polynomials

The evolution equations allow one to calculate the Q^2-dependence of the PPDFs which have been initiated at a certain reference point Q_0^2. These distributions are usually parameterized on the basis of plausible theoretical assumptions concerning their behavior near the end points $x = 0, 1$.

In the phenomenological investigations of the polarized and unpolarized structure functions, for example $xg_1^p(x, Q^2)$ or $xF_3^p(x, Q^2)$ for a given value of Q^2, only a limited number of experimental points, covering a partial range of values of x, are available. Therefore, one cannot directly determine the moments. To cover this deficiency, one is forced to use hypergeometric orthogonal polynomials [8,39].

In continuation of this line of research, we wish to discuss the Mellin integral transform of hypergeometric orthogonal polynomials from the Askey scheme [24,40]. There are many transform pairs among all hypergeometric orthogonal polynomials, ranging from Hermite polynomials up to the four parameter Wilson polynomials. There are several levels in the Askey scheme of hypergeometric orthogonal polynomials. The third one corresponds to the Meixner–Pollaczek, Jaccobi, Meixner and Kravchuk polynomials, which depends on two parameters.

One of the two parameter orthogonal polynomials of the third level is defined by

$$P_n^{(\alpha,\beta)}(z) = \frac{(\alpha+1)_n}{n!}\,{}_2F_1\left(-n, n+\alpha+\beta+1, \alpha+1; \frac{1-z}{2}\right)$$
$$= \frac{(\alpha+1)_n}{n!}\sum_{k=0}^{n}\frac{(-n)_k(n+\alpha+\beta+1)}{2^k k!(\alpha+1)_k}(1-z)^k. \tag{27}$$

Here Pochhammer symbol $(m)_n$ is defined by

$$(m)_n = m(m+1)\cdots(m+n+1). \tag{28}$$

Using Eq. (27) and the integral representation of the Mellin transformation, it is not hard to show that

$$\int_0^\infty P_n^{(\alpha,\beta)}(1-2xt)t^{z-1}e^{-t}\,dt = \frac{(\alpha+1)_n}{n!}\,{}_3F_1(-n, n+\alpha+\beta+1, \alpha+1; x)\Gamma(z), \tag{29}$$

where $\Gamma(z)$ is the standard Gamma function. The inverse Mellin transform of Eq. (29) is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} t^{-z}\,{}_3F_1(-n, n+\alpha+\beta+1, \alpha+1; x)\Gamma(z)\,d\,\mathrm{Im}\,z$$
$$= \frac{n!}{(\alpha+1)_n}P_n^{(\alpha,\beta)}(1-2xt)e^{-t}. \tag{30}$$

Thus the Mellin transform equations (29) and (30) connect the third level in the hierarchical ladder of hypergeometric orthogonal polynomials [24] with the next one.

We should note that the polynomials defined in Eq. (27) with particular values of the parameters α and β are known to correspond to: the Gegenbauer polynomials $C_n^{(\lambda)}(x)$ when $\alpha = \beta = \lambda - \frac{1}{2}$; the Chebyshev polynomials of the first $T_n(x)$ and the second $U_n(x)$ kind, if $\alpha = \beta = -\frac{1}{2}$ and $\alpha = \beta = \frac{1}{2}$, respectively. The values $\alpha = \beta = 0$ corresponds to Legendre polynomials $P_n(x)$. In the following we chose the values of α and β as in [41] which corresponds to Jaccobi polynomials.

The evolution equation can be solved and QCD predictions for PPDFs obtain with the help of various methods. For example we can use the Bernstein polynomial to determine PPDFs in the NLO approximation to obtain some unknown parameters to parameterize PPDFs at Q_0^2 [6]. In this way, we can compare theoretical predictions with the experimental results for the Bernstein averages just in moment space. To obtain these experimental averages from the E143 and SMC data [35,36], we need to fit $xg_1(x, Q^2)$ for each bin in Q^2 separately.

One of the simplest and fastest possibilities in the PPDF reconstruction from the QCD predictions for its Mellin moments is the hypergeometric orthogonal polynomials expansion, defined in Eq. (27). These polynomials are especially suited for this purpose since they allow one to factor out an essential part of the x-dependence of the structure function into the weight function. The $P_n^{(\alpha,\beta)}(x)$ polynomials satisfy the following orthogonality relation with the weight $x^\beta(1-x)^\alpha$,

$$\int_{-1}^{1} P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\beta)}(x)(1-x)^{\alpha}(1+x)^{\beta}\,dx$$
$$= \frac{2^{\alpha+\beta+1}}{2n+\alpha+\beta+1}\frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(n+\alpha+\beta+1)}\delta_{n,m}. \tag{31}$$

If we change the weight function above to

$$\omega(x) = x^{\beta}(1-x)^{\alpha}$$

which can be done by a change of variable

$$x \longrightarrow 2x-1,$$

then we will obtain new polynomial

$$H_n^{(\alpha,\beta)}(x) = P_n^{(\alpha,\beta)}(2x-1), \tag{32}$$

which satisfies the following orthogonal condition:

$$\int_{-1}^{1} H_n^{(\alpha,\beta)}(x) H_m^{(\alpha,\beta)}(x) x^{\beta}(1-x)^{\alpha}\,dx$$
$$= \frac{1}{n!}\frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{(2n+\alpha+\beta+1)\Gamma(n+\alpha+\beta+1)}\delta_{n,m}. \tag{33}$$

If we now introduce new polynomials $\mathcal{P}_n^{(\alpha,\beta)}(x)$ where

$$\mathcal{P}_n^{(\alpha,\beta)}(x) = \sqrt{\frac{n!(2n+\alpha+\beta+1)\Gamma(n+\alpha+\beta+1)}{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}} H_n^{(\alpha,\beta)}(x), \tag{34}$$

the new orthogonality condition will be appeared as

$$\int_{0}^{1} dx\, x^{\beta}(1-x)^{\alpha}\mathcal{P}_k^{(\alpha,\beta)}(x)\mathcal{P}_l^{(\alpha,\beta)}(x) = \delta_{k,l}. \tag{35}$$

We can assume for $\mathcal{P}_k^{(\alpha,\beta)}(x)$ a series expansion of the form

$$\mathcal{P}_n^{(\alpha,\beta)}(x) = \sum_{k=0}^{\infty} d_n^{(k)}(\alpha,\beta) x^k. \tag{36}$$

Comparing this series expansion with Eq. (34) and considering the definition of $H_n^{\alpha,\beta}(x)$ in terms of $P_n^{(\alpha,\beta)}(2x-1)$ and ultimately using Eq. (27) as the series expansion for $P_n^{(\alpha,\beta)}(x)$, we can obtain the expansion coefficient for $d_n^{(k)}(\alpha,\beta)$,

$$d_n^{(k)}(\alpha,\beta) = \sqrt{\frac{n!(2n+\alpha+\beta+1)\Gamma(n+\alpha+\beta+1)}{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}}$$
$$\times \frac{1}{n!}\frac{1}{k!}(-n)_k(\alpha+\beta+n+1)_k(\beta+k+1)_{n-k}. \tag{37}$$

Thus, given the moments $\delta M(n, Q^2)$ of the polynomial in Eq. (34), a structure function $\delta f(x, Q^2)$ may be reconstructed in a form of the series [42]

$$x\delta f\big(x, Q^2\big) = x^{\beta}(1-x)^{\alpha} \sum_{n=0}^{\mu} \delta M\big(n, Q^2\big)\mathcal{P}_n^{(\alpha,\beta)}(x), \tag{38}$$

where μ is the number of polynomials needed for convergence of the summation.

We note that the Q^2 dependence is entirely contained in the moments

$$\delta M\big(n, Q^2\big) = \int_0^1 dx\, x\delta f\big(x, Q^2\big)\mathcal{P}_n^{(\alpha,\beta)}(x) = \sum_{k=0}^{n} d_n^{(k)}(\alpha,\beta)\delta f\big(k+2, Q^2\big) \tag{39}$$

obtained by inverting Eq. (38), using Eqs. (35)–(36) and also the definition of moments, $f(k, Q^2) = \int_0^1 dx\, x^{k-2} x\delta f(x, Q^2)$.

Using Eqs. (38)–(39) one can relate the PPDF with its Mellin moments

$$xg_{1;\mu}^{p}\big(x, Q^2\big) = x^{\beta}(1-x)^{\alpha} \sum_{n=0}^{\mu} \mathcal{P}_n^{(\alpha,\beta)}(x) \sum_{k=0}^{n} d_n^{(k)}(\alpha,\beta) g_1\big(k+2, Q^2\big), \tag{40}$$

where $g_1(k+2, Q^2)$ are the moments determined by Eq. (19). Here α, β and μ have to be chosen so as to achieve the fastest convergence of the series on the R.H.S. of Eq. (40) and to reconstruct xg_1 with the required accuracy. Obviously the Q^2-dependence of the polarized structure function is defined by the Q^2-dependence of the moments.

In reconstructing the function $xg_{1;\mu}^{p}(x, Q^2)$ from its moments, there is no limitation on the x-values and it is valid for the whole range of x. The $g_1(k+2, Q^2)$ moments, eventually are extracted based on the constituent quark valon, model. Unknown parameters in the valon model, including the universal QCD dimensional transmutation parameter $\Lambda_{\overline{MS}}$ will be determined by fitting the available experimental data for $xg_{1;\mu}^{p}(x, Q^2)$, in the whole range of x-values to the analytical results for the moments $g_1(k+2, Q^2)$. We used the CERN subroutine MINUIT [43], and found an acceptable fit with minimum $\chi^2/\text{d.o.f.} = 0.936$ in the NLO approximation. The numerical values of the parameters which were obtained in this way, are tabulated in Table 1.

5. Nuclear polarized structure function

It has been suggested that the experimental data on deep inelastic lepton–nucleus scattering may reveal more information about the nucleon structure functions and nucleon correlations in nuclei and this can lead to better understanding of nuclear phenomena. The experimental data show nontrivial nuclear effects over whole range of Bjorken x. The nuclear effects which play an important role in the polarized and unpolarized DIS on nuclei can be divided into coherent and incoherent contributions. Incoherent nuclear effects result from the scattering of the incoming lepton on each individual nucleon, nucleon resonance. They are present at all Bjorken x. Coherent nuclear effects arise from the interaction of the incoming lepton with two or more nucleons in the target. They are typically concentrated at low values of Bjorken x. Nuclear shadowing and antishadowing are examples of coherent effects [44]. The treatment of nuclear effects is also divided into two regions: large x ($x \geqslant 0.3$) and small x ($x \leqslant 0.2$). The physical reason for such a division becomes apparent in the analysis of the DIS process. In the context of the multiple scattering framework, the large-x effects are described within the impulse approximation, in

Table 1
Numerical values of the parameter fit in the NLO approximation.

$\Lambda_{\overline{MS}}$; MeV	248 ± 53
N_U	0.0017 (fixed)
α_U	-2.3860 ± 0.025
β_U	-0.9588 ± 0.208
γ_U	22.6185 ± 0.417
η_U	-1.9873 ± 0.147
N_D	-0.0025 (fixed)
α_D	-1.6591 ± 0.033
β_D	-1.6196 ± 0.449
γ_D	13.1899 ± 0.939
η_D	0.8511 ± 0.139
$\mathcal{B}_0$	0.0051 ± 10^{-5}
$\mathcal{B}_1$	-3.0029 ± 0.239
$\mathcal{B}_2$	14.8701 ± 0.213
$\mathcal{B}_3$	-18.8123 ± 0.049
$\mathcal{B}_4$	8.4601 ± 0.296
$\mathcal{B}_5$	-0.3397 ± 0.049
χ^2/d.o.f.	$226.620/242 = 0.936$

which the virtual photon interacts with only one nucleon in the nuclei, while the other nucleon remains spectator to the interaction. The impulse approximation provides a natural framework within which effects from nuclear binding, Fermi motion, and nucleon off-shellness can be incorporated. At small x, on the other hand, there are important contributions from the rescattering of the probe from all nucleons in the target.

The calculation of the polarized or unpolarized structure function of nuclei usually take the convolution model as the starting point. The essence of this is that the parton distribution function inside a nucleus is sum of distributions inside each nucleon, corrected for the fact that nucleons are in motion relative to each other. We are interested in the polarized deep inelastic scattering (PDIS) process $\ell T \to \ell' X$, where T is a polarized nucleus (nucleon), ℓ is a polarized lepton and X is an unobserved hadronic state. The spin-dependent structure functions of the nucleon $g_1(x, Q^2)$ and $g_2(x, Q^2)$ are related to the antisymmetric hadronic tensor in the Bjorken limit [1],

$$W_{\mu\nu}^{(\mathcal{A})}(p, q; s) = \frac{2M}{p \cdot q} \varepsilon_{\mu\nu\alpha\beta} q^\alpha \left[s^\beta g_1\bigl(x, Q^2\bigr) + \left(s^\beta - \frac{s \cdot q}{p \cdot q} p^\beta \right) g_2\bigl(x, Q^2\bigr) \right], \tag{41}$$

where M is the nucleon mass and s denotes the spin polarization vector. The nucleonic contribution to the antisymmetric nuclear hadronic tensor for PDIS of the leptons off nuclei, is calculated by assuming the validity of the impulse approximation. Within the impulse approximation the virtual photon interacts with the nucleon and the final hadronic state X composed of the residual $(A-1)$-nucleon and incoherently scattering $\gamma^* N$ systems (A refers to nuclear mass number). So the following convolution formula for the spin-dependent hadronic tensor of nuclei is obtained [30];

$$W_{\mu\nu}^{(\mathcal{A}),T} = \sum_{i=p,n} \int d^4 p \, N_i \Delta\mathcal{S}_T^i(P_T, p; s) W_{\mu\nu}^{(\mathcal{A}),i}(p, q; s), \tag{42}$$

where T superscript refers to the nuclear target, N_i stands for number of protons and neutrons and finally $\Delta\mathcal{S}_T^i(P_T, p; s)$ denotes the invariant function describing the nuclear vertex with a struck nucleon (invariant spectral function). Thus, the polarized structure functions of nuclei can be written in terms of the polarized structure function of nucleons. In the Bjorken limit and defining the polarized light cone momentum distribution $\Delta f_T^i(y)$,

$$\Delta f_T^i(y) = \int d^4p\, \Delta \mathcal{S}_T^i(P_T, p; s) y\delta\left(y - \left(\frac{pq}{P_T q}\right)\left(\frac{M_T}{M_i}\right)\right), \tag{43}$$

here M_i and M_T denotes the masses of nucleon (proton and neutron) and target nuclei, respectively. Eq. (42) transforms to the usual convolution formula for spin-dependent structure functions,

$$g_{1,2}^T\left(x, Q^2\right) = \sum_{i=p,n} \int_{x\leqslant z} dy\, N_i \Delta f_T^i(y) g_{1,2}^i\left(z = \frac{x}{y}, Q^2\right) \equiv \sum_{i=p,n} N_i \Delta f_T^i \otimes g_{1,2}^i. \tag{44}$$

The polarized light cone momentum distribution in nucleus $\Delta f_T^i(y)$, is the spin-dependent probability to find the nucleon in the nucleus with a given fraction of the total momentum y of the nucleus on the light front and it can be related to the polarized spectral functions [30,31].

The systematic calculations of the nucleon momentum distributions in nuclei follow from nuclear ground state theory [32]. An exact investigation of the invariant spectral function and nucleon momentum distributions would require complete information about the nuclear states (wave functions), which for heavy nuclei is a complicated matter, and for that reason this calculation has been limited to the light nuclei such as ^{3}He and ^{3}H and nuclear matter. The three-nucleon wave function obtained using the Faddeev decomposition and Schrödinger equation is given by,

$$|\Psi\rangle = |\varphi_\varsigma\rangle + |\varphi_\xi\rangle + |\varphi_\zeta\rangle \equiv G_0(E)V|\Psi\rangle \equiv \sum_{i=1}^{3} G_0(E)V_i|\Psi\rangle. \tag{45}$$

In this equation $|\varphi\rangle$ refers to the Faddeev component of the wave function and the indices run from 1 to 3, where $G_0(E)$ is the three-body Greens function and V_i is the two body interaction potential. Using the symmetry group properties of the three-nucleon wave function, one can derive a set of coupled integral equations for the Faddeev components,

$$|\varphi_\varsigma\rangle = G_0(E)T_\varsigma(E - \epsilon_\varsigma)\big(|\varphi_\xi\rangle + |\varphi_\zeta\rangle\big). \tag{46}$$

T_ς is the usual T-matrix defined by the Lippmann–Schwinger equation and ϵ_ς is the energy of the ς particle in the three-body center of mass system. The exact solution of these coupled equations is complicated and has been dealt with using different techniques by different authors [31].

As a special case $g_1^{^3\mathrm{He}}$ and $g_1^{^3\mathrm{H}}$ can be represented as the convolution of the neutron (g_1^n) and proton (g_1^p) spin structure functions with the spin-dependent nucleon light-cone momentum distributions $\Delta f_{^3\mathrm{He}}^N(y)$ and $\Delta f_{^3\mathrm{H}}^N(y)$, where y is the ratio of the struck nucleon to nucleus light-cone plus components of the momenta

$$g_1^{^3\mathrm{He}}\left(x, Q^2\right) = \int_x^3 \frac{dy}{y} \Delta f_{^3\mathrm{He}}^n(y) g_1^n\left(x/y, Q^2\right) + 2\int_x^3 \frac{dy}{y} \Delta f_{^3\mathrm{He}}^p(y) g_1^p\left(x/y, Q^2\right), \tag{47}$$

$$g_1^{^3\mathrm{H}}\left(x, Q^2\right) = 2\int_x^3 \frac{dy}{y} \Delta f_{^3\mathrm{H}}^p(y) g_1^n\left(x/y, Q^2\right) + \int_x^3 \frac{dy}{y} \Delta f_{^3\mathrm{H}}^n(y) g_1^p\left(x/y, Q^2\right). \tag{48}$$

The motion of the nucleons inside the nucleus (Fermi motion) and their binding are parameterized through the distributions $\Delta f_{^3\mathrm{He}}^N$ ($\Delta f_{^3\mathrm{H}}^N$), which was readily calculated using the ground-state

Table 2
Numerical values of the parameters in Eq. (49).

$\Delta f^p_{^3\mathrm{He}}(y)$		$\Delta f^n_{^3\mathrm{He}}(y)$	
ap	0.00203	an	0.03682
bp	−4.01660	bn	−1.99756
cp	−0.01385	cn	−0.01201
dp	6.06288	dn	1.00815
ep	0.02688		
fp	−4.07592		
gp	−0.02057		
hp	1.02974		
ip	0.00550		

wave functions of ^{3}He (^{3}H). We should note that due to isospin symmetry, the light cone momentum distribution $\Delta f^p_{^3\mathrm{He}}(y)$ is equal to $\Delta f^n_{^3\mathrm{H}}(y)$. So to complete the calculation we just need two $\Delta f^p_{^3\mathrm{He}}(y)$ and $\Delta f^n_{^3\mathrm{He}}(y)$ functions.

To extract the polarized light cone momentum distributions in nuclei, $\Delta f^p_{^3\mathrm{He}}(y)$ and $\Delta f^n_{^3\mathrm{He}}(y)$, we used the numerical results of [31] and fitted the following functions over the related data points:

$$\Delta f^p_{^3\mathrm{He}}(y) = \frac{ap + cp\,y + ep\,y^2 + gp\,y^3 + ip\,y^4}{1 + bp\,y + dp\,y^2 + fp\,y^3 + hp\,y^4},$$
$$\Delta f^n_{^3\mathrm{He}}(y) = \left(\frac{an + cn\,x}{1 + bn\,x + dn\,x^2}\right)^2. \tag{49}$$

The numerical values of the parameters in Eq. (49) which are obtained in this way, are tabulated in Table 2.

The plots for $\Delta f^p_{^3\mathrm{He}}(y)$ and $\Delta f^n_{^3\mathrm{He}}(y)$ functions, using Eq. (49) are depicted in Fig. 1. Since the polarized nucleon structure functions $g_1^p(x/y, Q^2)$ and $g_1^n(x/y, Q^2)$ are known in Section 4, we can follow the calculation to obtain $g_1^{^3\mathrm{He}}(x, Q^2)$ and $g_1^{^3\mathrm{H}}(x, Q^2)$ structure functions.

It has been known for a long time that non-nucleonic degrees of freedom, such as pions, vector mesons, the $\Delta(1232)$-isobar, can play an important role in the calculation of some low-energy observable in nuclear physics. In particular, the analysis of Ref. [45] demonstrated that the two body exchange currents involving a $\Delta(1232)$-isobar increase the theoretical prediction for the axial vector coupling constant of the triton which makes it consistent with experiment.

The contribution of the $\Delta(1232)$ to $g_1^{^3\mathrm{He}}$ realized through Feynman diagrams involving the non-diagonal interference transitions $n \to \Delta^0$ and $p \to \Delta^+$. This requires new spin structure functions, $g_1^{n\to\Delta^0}$ and $g_1^{p\to\Delta^+}$, as well as the effective polarizations $P_{n\to\Delta^0}$ and $P_{P\to\Delta^+}$. Taking into account the interference transitions, leads to a correction to the polarized structure function of $A = 3$ mirror nuclei, δg_1^Δ:

$$\delta g_1^\Delta = \pm\big[2P_{n\to\Delta^0} g_1^{n\to\Delta^0} + 2P_{p\to\Delta^+} g_1^{p\to\Delta^+}\big] \tag{50}$$

where the $\pm$ signs correspond to ^{3}He (^{3}H), respectively [31].

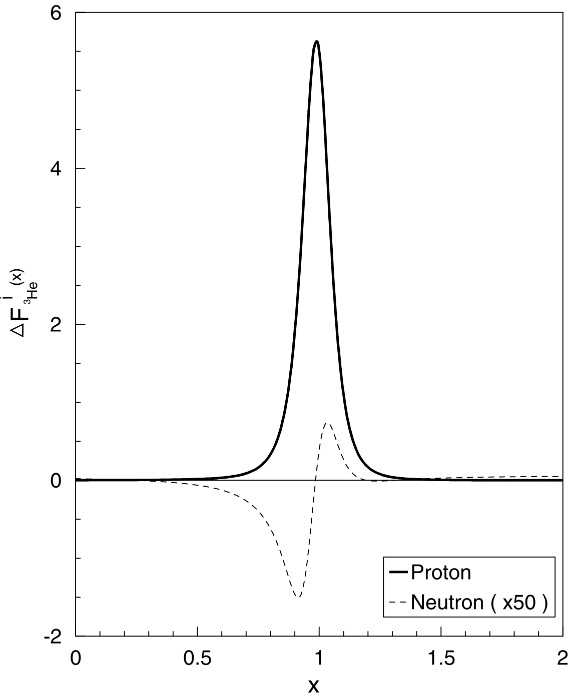

Fig. 1. The polarized light cone distribution functions for the proton and neutron in the ^{3}He, based on the results of Ref. [31].

Additionally the physics of the nuclear shadowing in deep inelastic scattering can be most easily understood in the laboratory frame using the Glauber–Gribov picture [46]. The dynamics of nuclear shadowing in DIS at high energies is most transparent in the target rest frame where the virtual photon–nucleus scattering, $\gamma^* T \to X$, is a time-ordered three-stage process:

(i) the photon may be considered to fluctuate into a linear superposition of hadronic components,
(ii) the hadronic configurations interact strongly with the nucleus,
(iii) the hadronic final state X is formed [31,46].

The nuclear shadowing–antishadowing correction, may be expressed as follows for ^{3}He

$$\delta g_1^{\text{sh--ash}} = \mathcal{C}(x) g_1^n + \mathcal{D}(x) g_1^p \tag{51}$$

where $\mathcal{C}$ ($\mathcal{D}$) coincide with $\mathcal{C}^{\text{sh}}$ ($\mathcal{D}^{\text{sh}}$) in the nuclear shadowing region of Bjorken x and model antishadowing at larger x [31]. Thus, including the $\Delta(1232)$ isobar and nuclear shadowing–antishadowing corrections, the spin structure function of ^{3}He reads

$$g_1^{^3\text{He}}\left(x, Q^2\right) = \int_x^3 \frac{dy}{y} \Delta f^n_{^3\text{He}}(y) g_1^n\left(x/y, Q^2\right) + 2\int_x^3 \frac{dy}{y} \Delta f^p_{^3\text{He}}(y) g_1^p\left(x/y, Q^2\right) + \delta g_1^{\Delta} + \delta g_1^{\text{sh--ash}}. \tag{52}$$

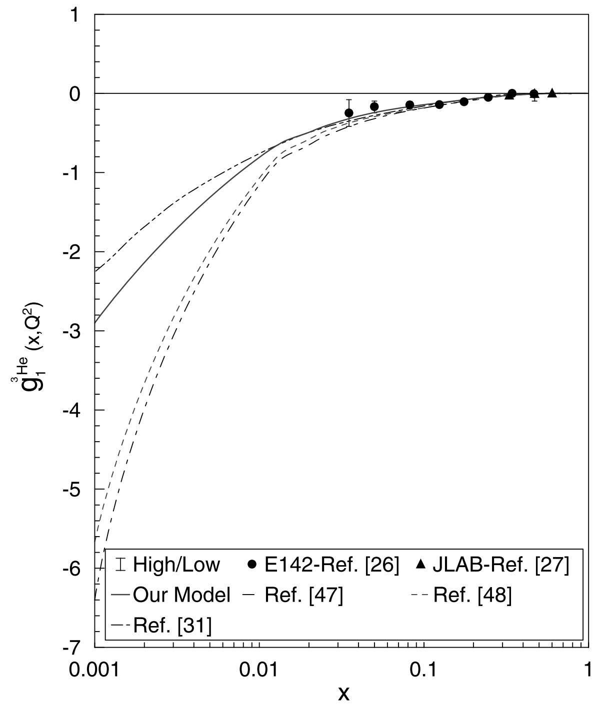

Fig. 2. Analytical result for the polarized ^{3}He structure function in our model and [31] which has been compared with experimental data [26,27] and plotted against the *AAC* and *BB* theoretical models [47,48].

6. Results and discussions

Now we return to Eqs. (47), (48) and restrict ourselves to $g_1(x, Q^2)$ nucleon and nuclear structure functions. The nucleon $g_1^N(x, Q^2)$ structure function is completely known to us, using the results of Sections 3 and 4. In Eqs. (47), (48), the polarized light cone momentum distribution in the nucleus, $\Delta f_T^i(y)$ is obtained from [31], using Eq. (49). The results for the polarized structure functions of ^{3}He and ^{3}H are indicated in Figs. 2 and 3 and compared with available experimental data and some theoretical models [26,27,31,47,48]. Our results are in good agreement with experimental data and theoretical model of Thomas et al. in [31]. The other *AAC* and *BB* theoretical models [47,48] do not indicate such good agreement at small x-values as our model and the theoretical model in [31] where there are not any experimental data in the range of the low-x values. Theoretical argument to deal with the behavior of polarized nuclear structure function at low values of x can be found in [31,46,49].

Determination of the parton distributions in a nucleon in the framework of quantum chromodynamics (QCD) always involves some model-dependent procedure like *AAC* and *BB* models. In the *AAC* model, the polarized parton distribution functions are determined by using world data from the longitudinally polarized deep inelastic scattering experiments. A new parametrization of the parton distribution functions is adopted in the kinematical range $x \sim [10^{-9}, 1]$ and $Q^2 \sim [1, 10^6]$ GeV^2 by taking into account the positivity and the counting rule. $Q_0^2 = 1$ GeV^2 is chosen in their global fit [50]. Also they investigate PPDFs and their uncertainties by using

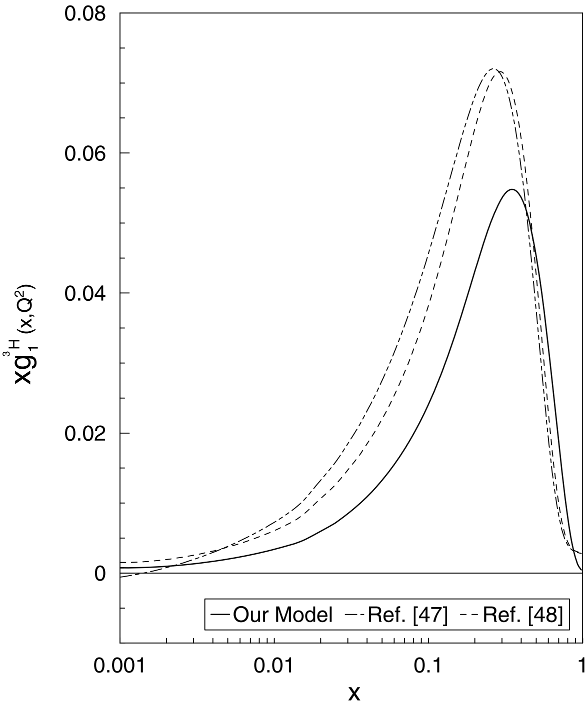

Fig. 3. Analytical result for the polarized ^{3}H structure function in our model, compared with the *AAC* and *BB* models [47,48].

world data on the spin asymmetry A1. The uncertainties of PPDFs are estimated by the Hessian method. In addition, gluon-distribution uncertainties are reduced due to the correlation with the antiquark distributions [47]. In the *BB* model, a QCD analysis of the world data on polarized deep inelastic scattering is presented in leading and next-to-leading order. New parameterizations are derived for the quark and gluon distributions for the kinematical range $x \sim [10^{-9}, 1]$, $Q^2 \sim [1, 10^6]$ GeV^2. $Q_0^2 = 1$ GeV^2 is chosen in their global fit. The values of $\Lambda_{\overline{MS}}$ and $\alpha_s(Mz^2)$ are determined. Finally they perform a factorization-scheme invariant QCD analysis based on the observable $g_1(x, Q^2)$ and $dg_1(x, Q^2)/d\log(Q^2)$ in next-to-leading order, which is compared to the standard analysis.

On the other hand, instead of relying on mathematical simplicity as a guide, we take a viewpoint in which the physical picture of the nucleon structure is emphasized. That is, we consider the model for the nucleon which is compatible with the description of the bound state problem in terms of three constituent quarks. We adopt the view that these constituent quarks in the scattering problems should be regarded as the valence quark clusters rather than point-like objects. They have been referred to as *valons*. It is defined as a cluster of valence quarks accompanied by a cloud of sea quarks and gluons. It can be considered as a bound state in which for instance a proton consists of three valons, two U-valons and one D-valon. We have used the valon model to describe deep inelastic scattering. The model bridges the gap between the bound state problem and the scattering problem for hadrons. In contrast to this model people usually use the GLAP equations to evolve the parton distributions from an initial value Q_0, as was described before. The valon model which was first introduced by Hwa [14,22], gives us a clear insight as to how to

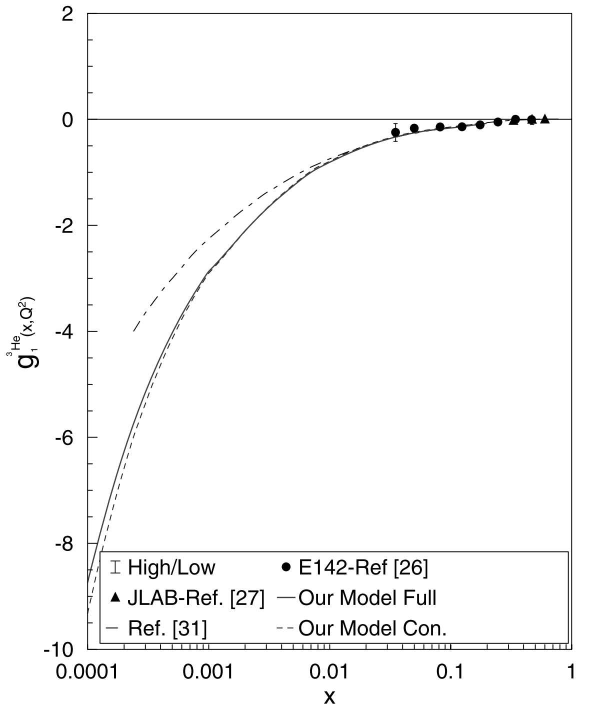

Fig. 4. Analytical result for the polarized ^{3}He structure function in our model, by employing the shadowing and anti-shadowing and also Δ-resonance effects (solid curve). These effects are also considered in plotting the result of Ref. [31] (dashed-dotted curve). The experimental data has been adopted from [26,27]. The dotted curve indicates our result without including nuclear effects.

construct the hadron structure from the parton distributions. This model is valid in a range of x and Q^2 values that are comparable with global fitting of the *AAC* and *BB* models.

The authors in [31] have recently studied and considered in more detail the subject of polarized nuclear structure functions, our model is also in good agreement with their results. Considering the shadowing, antishadowing and Δ-resonance effects as indicated by Eq. (52) we can obtain new result for $g_1^{^3\mathrm{He}}$ as depicted in Fig. 4. It can be seen that due to these effects the ^{3}He polarized structure function at low-x values has been modified as expected.

We calculate the Bjorken sum rule to confirm the accuracy of the model. The Bjorken sum rule [51] relates the difference of the first moments of the proton and neutron spin structure functions to the axial vector coupling constant of the neutron β decay g_A, where $g_A = 1.2670 \pm 0.0035$ [52],

$$\int_0^1 \left(g_1^p\left(x, Q^2\right) - g_1^n\left(x, Q^2\right)\right) dx = \frac{1}{6} g_A \left(1 + O\left(\frac{\alpha_s}{\pi}\right)\right). \tag{53}$$

Here "$O(\alpha_s/\pi)$" is the QCD radiative corrections. This sum rule can be generalized to the ^{3}He–^{3}H system:

$$\int_0^3 \left(g_1^{^3\mathrm{H}}\left(x,Q^2\right)-g_1^{^3\mathrm{He}}\left(x,Q^2\right)\right)dx=\frac{1}{6}g_A\bigg|_{\mathrm{triton}}\left(1+O\left(\frac{\alpha_s}{\pi}\right)\right), \tag{54}$$

where $g_A|_{\mathrm{triton}}$ is the axial vector coupling constant of the triton β decay, $g_A|_{\mathrm{triton}} = 1.211 \pm 0.002$. Taking the ratio of Eqs. (54) and (53), one obtains [51]

$$\frac{\int_0^3 (g_1^{^3\mathrm{H}}(x,Q^2)-g_1^{^3\mathrm{He}}(x,Q^2))\,dx}{\int_0^1 (g_1^p(x,Q^2)-g_1^n(x,Q^2))\,dx}=\frac{g_A|_{\mathrm{triton}}}{g_A}=0.956\pm 0.004. \tag{55}$$

The numerical value of this ratio in our calculation is 0.951384 and it is in good agreement with the reported experimental data in Eq. (55). So once again, the reliability of the phenomenological model used to construct $g_1^{^3\mathrm{He}}$ and $g_1^{^3\mathrm{H}}$ is confirmed.

7. Conclusion

We used the constituent quark framework (valon model) to extract polarized parton distributions and hence polarized nucleon structure functions. Due to limited experimental data which do not cover the whole range of x and Q^2 values, and to increase the reliability of the fitting, we employed Jaccobi polynomials which are placed at the third level of hypergeometric orthogonal polynomials in the Askey scheme. Having the results for $g_1^N(x,Q^2)$ and using the light cone moment distribution of nucleon in the nucleus, it was possible to extract polarized structure functions for ^{3}He and ^{3}H. The results are displayed in Figs. 2 and 3 and indicate good agreement with available experimental data and the theoretical models in [31]. The effect of nuclear shadowing, antishadowing and Δ-resonance which are dominate at low-x values were also considered. Fig. 4 was plotted, based on these effects. A good agreement between the experimental value and analytical result for the ratio of the Bjorken sum rule in proton–neutron system to ^{3}He–^{3}H system, once again confirmed the anticipated reliability of the phenomenological model we have used.

There is a very precise data of the CLAS Collaboration [53] for the $\frac{g_1}{F_1}$ ratio. We can parameterize g_1 and f_1 structure functions to obtain the unknown parameters, using the fit to the available experimental data of CLAS Collaboration. Putting back the fitted values in g_1 structure function, the g_1 plot will be obtained which can be compared with the experimental data that exists independently for the spin proton structure function [35,36]. By comparing the analytical result for g_1 which is obtained by this way with the analytical result, obtained by fitting to experimental data in [35,36], the influence of data in [53] for nucleon spin structure function can be determined. This can be done as a new research job in future.

To improve the results one could consider the massive effects in extracting the polarized parton distribution inside the nucleon and finally nuclei and it is hoped to do this in future work. Using other levels of the Askey scheme and the related hypergeometric orthogonal polynomials which have three or four free parameters, would be a challenging exercise and might lead to more reliable results. An extension of the calculation using the approach of complete renormalization group improvement [54] would also be valuable.

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