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Assessment of rain-gauge networks using a probabilistic GIS based approach

Mojtaba Shafiei, Bijan Ghahraman, Bahram Saghafian, Saket Pande, Shervan Gharari and Kamran Davary

ABSTRACT

Rain-gauge networks provide estimates of areal rainfall as a crucial input for hydrological applications. Hence, it is important to quantify the performance of a rain-gauge network and evaluate the contribution of each rain-gauge to the overall accuracy of areal rainfall estimation at basin scale. This paper evaluates the performance and augmentation of a rain-gauge network in a large basin in Iran. A probabilistic approach combined with a geographic information system (GIS) framework is applied, in order to assess the accuracy of point rainfall in terms of acceptance probability. A simple equation for calculating the acceptance probability is presented which facilitates the application of the probabilistic approach in a GIS environment. This approach analyzes the number and location of rain-gauges and quantifies each gauge's contribution to the accuracy of rainfall estimation over the study area. Results show that among 33 existing gauges, only 21 have significant effect on areal rainfall estimation while other 12 gauges have marginal contribution to the accuracy of the network. Also, by applying an augmentation algorithm, an optimal rain-gauge network with 28 gauges is formed.

Key words | acceptance probability, climatological variogram, kriging variance, rain-gauge network

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INTRODUCTION

Rainfall is the driving factor of most hydrologic designs. The estimation of average rainfall over a basin area based on measured data at several rain-gauges plays an important role in many hydrological applications (Chua & Bras 1982; Bastin *et al.* 1984). The design of rain-gauge networks is motivated by the need to accurately capture the areal average rainfall in basins. Additionally, in rainfall-runoff modeling, accurate knowledge of spatiotemporal rainfall is essential for accurately estimating discharge and determining other hydrological processes (Beven 2001). A large number of studies have revealed that rain-gauge network density and distribution can significantly affect the simulated discharge, sediment and other types of catchment responses (Seed & Austin 1990; Duncan *et al.* 1993; St-Hilaire *et al.* 2003; Chaplot *et al.* 2005; Bárdossy & Das 2008). For instance, Anctil *et al.* (2006) indicated that model performance diminishes rapidly when the areal average rainfall is computed by a number of rain-gauges less than a minimum threshold value. Furthermore, they found that some rain-gauge network combinations provide better estimation of areal rainfall than using all existing rain-gauges in the basin.

Nowadays, rain-gauge network optimization is considered rather out of date, as weather radars provide rainfall data with better spatial and temporal resolution. Nonetheless, there are several possible sources of errors in the measurement of rainfall by radars (Steiner *et al.* 1999; Jayakrishnan *et al.* 2004; Abdella & Alfredsen 2010). Several researches have shown the impact of radar rainfall estimation error on hydrological model outputs (e.g. Borga *et al.* 2006). Radar estimates can be biased (because of a bright band, for example). Such biases can lead to possibly

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large errors in hydrological simulation values (Berne & Krajewski 2013). Hence, the quality of radar precipitation data over the study area needs to be assessed using rain-gauge measurements before putting it to use. Moreover, complete coverage by weather radars is still limited to certain parts of the world. Thus, the evaluation and optimization of the rain-gauge networks is still an important issue that deserves attention.

A well-designed rain-gauge network can better estimate spatial and temporal variation of rainfall over a basin. Such information is useful for purposes such as management of water resources and reservoir operation. An optimum rain-gauge network varies with the study area and the purpose for which the data are collected (Kassim & Kottegoda 1991). Hence, the rain-gauge network evaluation should involve the analysis of the number and location of gauges at a specified spatio-temporal scale. In addition to achieving a desired level of accuracy, its design is also influenced by non-hydrological factors, such as available budget, accessibility, maintenance, etc.

A considerable amount of research has been carried out in evaluating and optimizing rain-gauge networks. In some cases, statistical methods such as coefficient of variance and the allowable percentage of error have been applied for rain-gauge network design (WMO 1994; Patra 2001).

The information entropy approach has also been used in the literature, including those by Krstanovic & Singh (1992), Al-Zahrani & Husain (1998), Kawachi *et al.* (2001), Chen *et al.* (2008) and Yoo *et al.* (2008). The kriging method has also been widely adopted for the optimal selection of sampling points in several hydrological network design problems. Kriging of the data has the advantage that the associated error variance at any location within the study area can be estimated. The associated uncertainty of the estimated areal rainfall based on the kriging variance can be used to better understand the behavior of areal rainfall over the basin. The well-known variance reduction method (Bras & Rodriguez-Iturbe 1976; Hughes & Lettenmaier 1981; Bastin *et al.* 1984; Bogardi & Bardossy 1985; Kassim & Kottegoda 1991; Papamichail & Metaxa 1996; Ghahraman & Sepaskhah 2001; Tsintikidis *et al.* 2002; Nour *et al.* 2006) is based on such an approach, which methodically searches for an appropriate number of rain-gauges and their locations in order to minimize the variance of the estimation error of areal average rainfall events. Furthermore, several researchers have combined the variance reduction method with optimization algorithms such as simulated annealing (e.g. Pardo-Igúzquiza 1998; Barca *et al.* 2008). Chebbi *et al.* (2011) combined the variance reduction method with simulated annealing to optimally extend a rain-gauge network in order to interpolate rainfall intensity and an erosivity index. Moreover, Chebbi *et al.* (2012) proposed a method for robust rain-gauge network optimization using intensity-duration-frequency data by minimizing the mean spatial kriging variance.

The previous studies have mainly focused on the accuracy of areal average rainfall estimation rather than on the accuracy of point rainfall estimation. In the majority of cases, the evaluation of the performance of a network was based on the estimation of the variance of areal rainfall, but not that of point rainfall across the study area (Cheng *et al.* 2008). More recently, Cheng *et al.* (2008) proposed a rain-gauge network evaluation and augmentation approach focusing on the accuracy assessment of point rainfall across the whole study area. It is a probabilistic approach that is based on variogram analysis and a criterion using ordinary kriging variance. It assesses the accuracy of rainfall estimation using the acceptance probability that is defined as the probability that estimation error falls within a desired range expressed in terms of the standard deviation of rainfall. Based on this criterion, the percentage of the total area with a prescribed acceptable accuracy in a certain network configuration can be calculated. They also presented a sequential algorithm to prioritize rain-gauges of the existing network and used this approach (hereafter acceptance probability (AP) approach) in northern Taiwan and showed that the identified base network, which comprised of approximately two-thirds of the total rain-gauges, can achieve almost the same level of performance as a complete network for hourly rainfall estimation.

In most parts of Iran, rain-gauge networks are the only source of rainfall data for evaluating the temporal and spatial variation of rainfall over a basin. Moreover, because of the crucial role of rainfall in assessing the water balance for water resources planning in basins, the task of evaluating the rain-gauge networks is of great importance. The objective of this research is to assess the number and location of the rain-gauges and to quantify the performance of an existing rain-gauge network in a large basin in Iran. A well-schemed rain-gauge network not only helps to better represent areal rainfall regionally, but also locally in parts of a basin. Another objective of this study is to identify the contribution of each rain-gauge to the overall network performance, as well as to increase the estimation accuracy of areal annual rainfall for any part of the basin. The paper extends the existing methodology of Cheng *et al.* (2008) to augment the existing network in the basin. It also simplifies the calculation of the acceptance probability criterion and implements the calculations in a geographic information system (GIS) environment for general use.

The remainder of this paper is organized as follows. First, a brief description of ordinary kriging method and climatological variogram analysis is presented. Then, the concepts of acceptance probability and acceptable accuracy are defined. After introducing the study area, the results of spatial characterization of annual rainfall, the performance evaluation of the network and its subsequent augmentation are presented and discussed.

MATERIALS AND METHODS

Ordinary kriging and variogram analysis

The ordinary kriging estimator $\hat{Z}(x_0)$ is a linear combination of weights and data representing variables at sample (observation) points in the vicinity of an estimated point:

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i) \tag{1}$$

where $\hat{Z}(x_0)$ is the estimate of Z at x_0, λ_i is the weight assigned to the ith observation, and n is the number of observations within the neighborhood. In ordinary kriging, the sum of weights is constrained to be one and the optimal weights are computed from the kriging system and are obtained by applying the Lagrange multipliers method (Webster & Oliver 2001). The kriging variance ($\sigma_k^2(x_0)$), which provides a measure of the error associated with the kriging estimator, is also obtained:

$$\sigma_k^2(x_0) = \mu + \sum_{i=1}^{n} \lambda_i \gamma(x_0, x_i) \tag{2}$$

where $\gamma(x_0, x_i)$ is the variogram between x_0 and x_i and μ denotes the Lagrange multiplier. The kriging estimation variance is a measure of the estimation accuracy of $\hat{Z}(x_0)$ and it is the basic tool of variance reduction techniques for optimal selection of sampling locations. The reason for this is that the estimation variance only depends on the variogram model, the number n of rain-gauges and its spatial location.

On the basis of the hypothesis of second-order stationarity, the kriging method assumes that the mean of the random field is constant and the variogram depends only on distance between points. The variogram is defined as one half of the variance between any two locations separated by h:

$$\gamma(h) = (1/2)\mathrm{Var}[Z(x) - Z(x + h)] \tag{3}$$

where h is the distance vector and x is the location vector. The variogram indicates how the dissimilarity between $Z(x)$ and $Z(x + h)$ evolves with the distance h. The influence range of a variogram is the minimum distance at which two random variables $Z(x_i)$ and $Z(x_j)$ become independent. For a second-order stationary random field, as the distance h increases, the variogram will reach an asymptotic value, known as the sill. The sill corresponds to zero correlation and it is equal to the variance of the random variable $Z(x)$. Experimental variogram is computed from data pairs of observations, for specific distance lags and directions (Webster & Oliver 2001).

For practical applications, Bastin *et al.* (1984) proposed an approach, that Cheng & Wang (2002) and Cheng *et al.* (2008) also used to compute the variogram by using dimensionless rainfall data. The experimental variogram is:

$$\gamma(m, h) = \alpha(m)\gamma^*(h) \tag{4}$$

where h is the Euclidian distance, $\alpha(m)$ a scaling factor and m is an index of time. The temporal non-stationarity is concentrated in the scaling factor $\alpha(m)$, yielding a time invariant scaled component $\gamma^*(h)$ called the climatological or dimensionless variogram. The scaling factor in Equation (4) is equivalent to the sill ω, or the variance of the rainfall field. The scaled estimation variance (based on Equation (2)) only depends on three factors; the climatological variogram, the number, n, and the location of the rain-gauge stations (Lebel *et al.* 1987).

To construct the climatological variogram, annual rainfall data are first preprocessed as:

$$R_i^*(j) = \frac{R_i(j) - R_{m,j}}{S(j)} \quad i = 1, 2..., n; j = 1, 2..., N \tag{5}$$

where $R_i(j)$ and $R_{m,j}$, respectively, represent the jth year's annual rainfall of rain-gauge i and the mean annual rainfall of the rain-gauges in jth year, and $S(j)$ is the standard deviation of the jth year's annual rainfalls of the rain-gauges (Cheng *et al.* 2008). The scaled rainfall ($R_i^*(j)$) is dimensionless and has zero mean and unit standard deviation and it is used to fit a climatological variogram. The VARIOWIN2.2 (Pannatier 1996) program is used to analyze and fit a variogram. A theoretical variogram model that has the highest Indicative Goodness of Fit (IGF) index with the experimental variogram is selected. The IGF index, which is computed by VARIOWIN, is a standardized weighted average of the squared difference between the experimental and modeled variogram values. Values of IGF close to zero indicate a good fit between the experimental variogram and the theoretical model (Pannatier 1996).

Acceptance probability concept and acceptable accuracy definition

Most of the rain-gauge network evaluation studies have focused on the estimation accuracy of areal average rainfall, while the estimation accuracy of point rainfall at ungauged sites have not been considered. The estimation accuracy of point rainfall varies within a study area and depends on the number and location of the rain-gauges. An efficient rain-gauge network should provide acceptable accuracy for most points within the study area (Cheng *et al.* 2008). We describe the acceptance probability criterion, introduced by Cheng *et al.* (2008), in the following.

Assume that annual rainfalls, $Z(x,t)$, are measured by a network of n rain-gauges at location x_i, $i = 1, \ldots, n$, for a period of time $t_1 \leq t \leq t_p$. Rainfall at an ungauged site x_0, i.e. $Z(x_0,t)$, is estimated using measurements $Z(x,t)$, $i = 1, \ldots, n$, and Equation (1). The estimation accuracy is given by the ordinary kriging variance in Equation (2). Since the estimation uses same-time measurements, the time dependence of rainfall $Z(x,t)$ is dropped hereafter. Intuitively, an estimation is considered acceptable only if the estimation falls within a given range of the 'true' value, so that:

$$\left|\tilde{Z}(x_0)\right| = \left|\hat{Z}(x_0) - Z(x_0)\right| < r \tag{6}$$

where $r > 0$. Even so, at the location x_0, the estimation accuracy varies from time to time and from event to event; thus, it should be evaluated on an ensemble basis. The given range r is specified by using the variance of the rainfall field $Z(x)$, i.e. σ_z^2. Equation (6) can then be given by:

$$P\left[\left|\hat{Z}(x_0) - Z(x_0)\right| < k\sigma_z\right] \geq \alpha \tag{7}$$

The acceptable range of the estimation error (i.e. $\hat{Z}(x_0) - Z(x_0)$) in Equation (7) can be expressed in terms of standard deviation of the random variable $Z(x)$, while the multiplier k and the minimum probability α are chosen according to factors such as available budget for installation of gauges and maintenance costs and the desired level of estimation accuracy (Cheng *et al.* 2008). In this study we choose $k = 1$ and $\alpha = 0.8$.

Since the ordinary kriging estimator is unbiased, the estimation error at x_0 has zero mean and variance $\sigma_k^2(x_0)$. If the estimation error at x_0 is assumed to be normally distributed, then the probability for the estimation error $\tilde{Z}(x_0)$ to fall within the desired range ($-\sigma_z$, σ_z) can be determined using the cumulative probability of the standard normal distribution:

$$\begin{aligned} P\left[\left|\tilde{Z}(x_0)\right| < \sigma_z\right] &= P\left[\frac{\left|\tilde{Z}(x_0)\right|}{\sigma_k(x_0)} < \frac{\sigma_z}{\sigma_k(x_0)}\right] \\ &= P\left[\left|\tilde{Z}^*(x_0)\right| < \frac{\sigma_z}{\sigma_k(x_0)}\right] = p_A(x_0) \end{aligned} \tag{8}$$

In Equation (8), $\tilde{Z}^*(x_0)$ is the standardized estimation error and has a standard normal distribution, i.e. $\tilde{Z}^*(x_0) \sim N(0, 1)$. Additionally, $p_A(x_0)$ is termed the *acceptance probability* at x_0, and it is the probability that the estimation error at x_0 is less than σ_z. The estimation accuracy at an ungauged point is acceptable only if the associated acceptance probability is no less than α. The estimation at that point is then said to have an acceptable accuracy (Cheng *et al.* 2008). Here, σ_z is the sill value of the climatological variogram. It is notable that points associated with higher kriging variances correspond to lower acceptance probabilities.

The acceptance probability ($p_A(x_0)$) in Equation (8) is assumed to be cumulative standard normal distribution. The cumulative standard normal distribution function (i.e. $F(x)$) is given as:

$$F(x) = P(Z < x) = \frac{1}{2}\left[erf\left(\frac{x}{\sqrt{2}}\right) + 1\right] \tag{9}$$

The error $erf(y)$ does not have a closed form, thereby inhibiting the implementation of an acceptable probability approach in a GIS environment. Thus an approximation of the error function is used to calculate the cumulative probability of the standard normal distribution function (Winitzki 2003):

$$erf(y) \approx \left[1 - \exp\left(-y^2 \frac{(4/\pi) + 0.14y^2}{1 + 0.14y^2}\right)\right]^{1/2} \tag{10}$$

Following Equations (8) and (9) and substituting Equation (10) into (9), the acceptance probability can now be expressed as:

$$p_A(x_0) = 1 - \left[1 - \left[1 - \exp\left(-\tau^2 \frac{(4/\pi) + 0.14\tau^2}{1 + 0.14\tau^2}\right)\right]^{1/2}\right], \quad \tau = \frac{k\sigma_z}{\sqrt{2}\sigma_k(x_0)} \tag{11}$$

Rain-gauge network evaluation and augmentation

As discussed in the previous section, the estimation accuracy for each point in the study area can be evaluated using the acceptance probability (Equation (11)). The performance of a rain-gauge network is evaluated based on the percentage of area with a certain acceptable accuracy (hereafter expressed by A_p, defined as the fraction of the study area above a certain acceptable probability). Also, because the acceptance probability is computed at each point in the study area, a raster (contour) map of acceptance probability is produced to assist in the evaluation of an existing rain-gauge network. For example, if the minimum probability α is taken as 0.8 then parts of the study area which have $p_A(x_0) \geq \alpha$ are said to have *acceptable accuracy* and a corresponding A_p is calculated.

Cheng *et al.* (2008) proposed a sequential algorithm for assessing the contribution of each rain-gauge to the accuracy of areal rainfall estimation of a network. The augmentation of a rain-gauge network with a certain acceptable accuracy by adding new gauges or relocating existing gauges can also be assessed. The sequential algorithm that is described below prioritizes the existing rain-gauges and evaluates sequentially the joint performance of a subset of rain-gauges.

- Step 1: Calculate the A_p for the network by removing one gauge from the existing rain-gauge network at a specified level of accuracy (i.e. α).
- Step 2: Return the removed gauge to the network, select another gauge and recalculate the A_p value. This step is repeated until all the gauges in the network have been chosen. A corresponding set of values of A_p are thus obtained.
- Step 3: Remove the gauge associated with the highest value of A_p in step (2). Reduce the number of remaining gauges by one and repeat steps (1) and (2). Step (3) is repeated until there is only one gauge remaining.

After finishing the algorithm, all rain-gauges are prioritized based on their order of removal in step (3). Furthermore, at each stage when a gauge is removed in step (3), a raster map of acceptance probability for annual rainfall and its corresponding A_p value is also determined using only the remaining gauges. Finally, an illustrative figure is constructed based on the number of removed gauges against A_p to show the prioritized order of rain-gauges and performance of a subset of rain-gauges (Cheng *et al.* 2008).

For practical application of the above sequential algorithm, a tool is also developed within ArcGIS® system. This tool uses Equation (11) and the kriging toolbox facilities of the ModelBuilder™ in ArcGIS® software (Allen 2011).

STUDY AREA AND DATA

Although the dominant climate in Iran is characterized as arid and semi-arid, the northern part of the country along the southern Caspian Sea coastline receives high to moderate precipitation. Annual precipitation, however, decreases from west to east. The Gorgan-Rud river basin is located in the eastern part of the southern Caspian Sea coastline (Figure 1). The climate of this area is characterized as mild and the annual precipitation drops from 450 to 250 mm in a west–east direction (Saghafian *et al.* 2008). The evaluation and optimization of rain-gauge networks is one important assessment in Water Resources Management Research (WRMR) at basin scale. As a part of WRMR in the Gorgan-Rud river basin, the AP method is applied to evaluate and augment its rain-gauge network.

The area of the Gorgan-Rud basin is about 114,000 km^2. The highest elevation, located in the south, is 3,900 meters above mean sea level and the lowest elevation is near the coast, in the west of the basin. Annual rainfall data of 33 gauges in the Gorgan-Rud river basin from 1988 to 2008 are used in this study (Figure 1). Table 1 shows rain-gauges accompanied by their elevation and average annual rainfall over 20 years of recorded data.

RESULTS AND DISCUSSION

Characterization of the spatial variation of annual rainfall

Long-duration rainfall, especially annual rainfall, is of major concern for assessing potential water resources in large basins. In this section the spatial variation of annual rainfall in the Gorgan-Rud river basin is characterized using climatological variogram analysis.

Elevation and orographic influences have a significant effect on the variogram analysis, especially for annual rainfalls. If orographic effects exist, then the random field $Z(x)$ is not stationary and the variogram may increase without approaching a sill. We therefore check for orographic effect or existence of a trend in average annual rainfall prior to experimental variogram fitting. Average annual rainfall depths are calculated for each of the 33 rain-gauges using 20 years of annual rainfall data and plotted against the elevation of each rain-gauge (Figure 2). Results based on Figure 2 demonstrated the

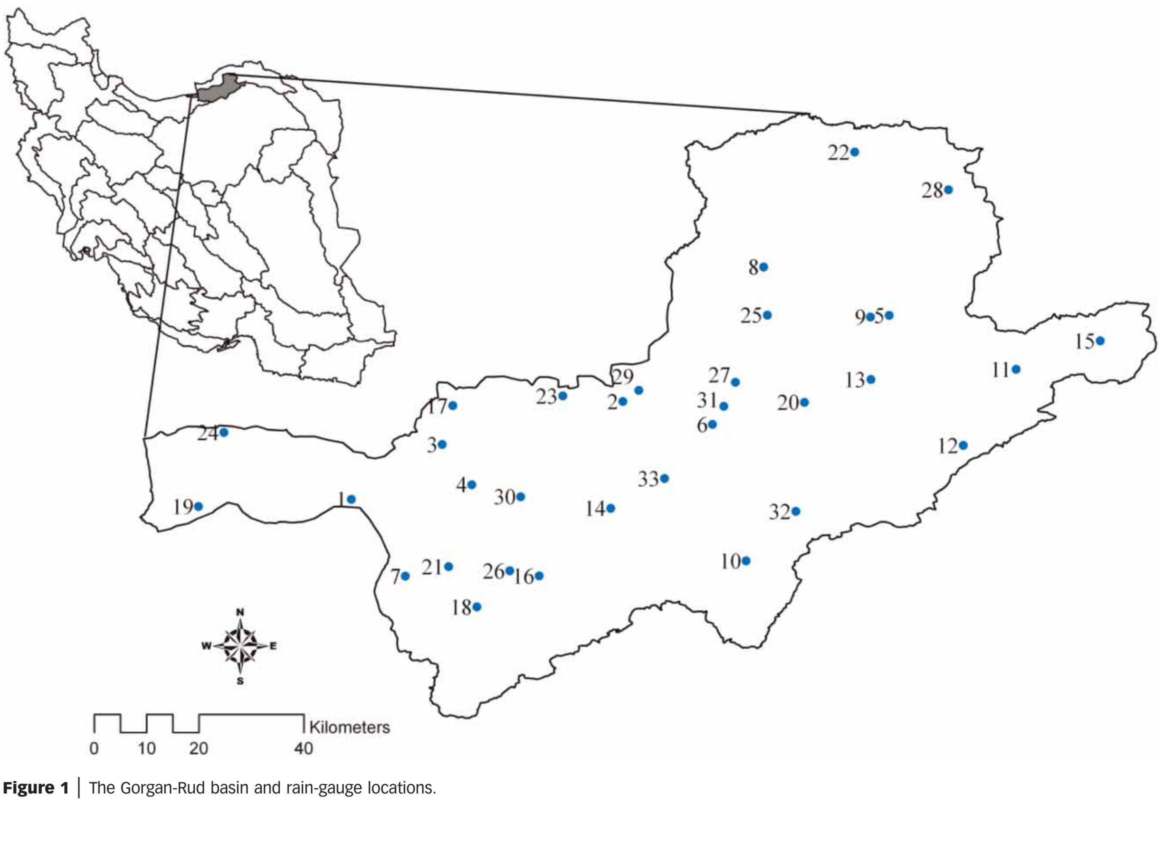

Figure 1 | The Gorgan-Rud basin and rain-gauge locations.

Table 1 | Characteristics of rain-gauges over the Gorgan-Rud basin

No.	Name	Elevation (m)	Average annual rainfall (mm)	No.	Name	Elevation (m)	Average annual rainfall (mm)
1	Aghala	–12	411	18	Sermu	500	762
2	Arazkuseh	34	444	19	Ghaffar	–22	435
3	Bagh Salian	20	373	20	Farsian	900	705
4	Behlekeh	24	392	21	Fazelabad	210	674
5	Park Melli	460	808	22	Ghernagh	500	492
6	Pasposhteh	182	934	23	Ghezagli	30	364
7	Taghiabad	148	565	24	Ghale	–33	361
8	Tamar	132	573	25	Ghuchmaz	160	753
9	Tangrah	330	748	26	Kabudal	200	615
10	Tilabad	1,000	232	27	Galikash	250	782
11	Cheshme	1,250	227	28	Golidagh	1,000	758
12	Hagholkhajeh	1,200	198	29	Gonbad	36	442
13	Dashtshad	1,450	408	30	Lalehbagh	31	472
14	Ramian	200	858	31	Lozureh	155	813
15	Robat	1,450	193	32	Narab	1,500	341
16	Zeringel	282	800	33	Nudeh	280	874
17	Sade Gorgan	12	332				

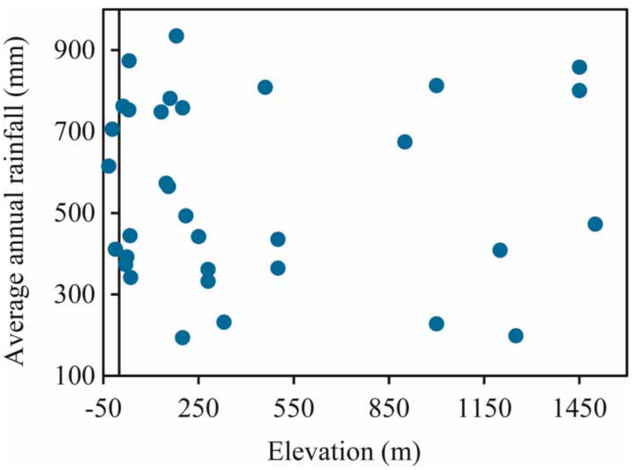

Figure 2 | Relationship between average annual rainfall depth and elevation.

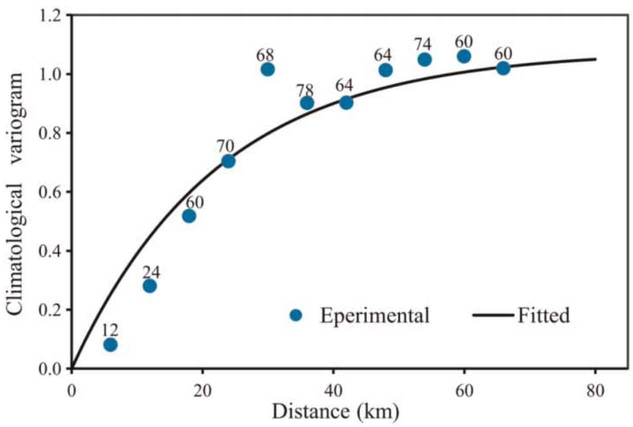

Figure 3 | Experimental and fitted climatological variograms of annual rainfall (the number of pairs for variogram derivation is also given).

absence of any orographic effect in average annual rainfall of the study area.

Before constructing the experimental climatological variogram, the annual rainfall data are processed using Equation (5). The VARIOWIN2.2 (Pannatier 1996) program is then used to fit a variogram to the experimental climatological variogram (Figure 3). The exponential variogram model is chosen as the best fit, which validation test gave an IGF value of 0.038 (–). The influence range, sill and nugget effect are about 67 (km), 1.08 (–) and zero (–), respectively. Zero nugget shows strong spatial correlation between the rain-gauges in the network.

Rain-gauge network performance evaluation and augmentation

The estimation accuracy at every point within the study area is evaluated based on acceptance probability. The acceptance probability ($p_A(x_0)$) is calculated over the study area, based on the tool in ArcGIS® software that is developed as part of this study. The tool can calculate acceptance probability and determine the percentage of the study area with a certain acceptable accuracy level. Figure 4 shows the spatial distribution of $p_A(x_0)$ over the Gorgan-Rud basin. It is notable that the $p_A(x_0)$ value always equals unity at any rain-gauge location, since ordinary kriging with zero nugget is an exact estimator and yields zero estimation error at the measurement points (Webster & Oliver 2001). Additionally, as it can be seen near the boundaries, the values of $p_A(x_0)$ are less than other parts of the study area. As illustrated by Figure 4, at $\alpha = 0.8$ about 88% of the total area has acceptable accuracy, i.e. $A_p = 88\%$. It can also be seen that A_p value is about 40% at $\alpha = 0.9$ which is very low. Thus, $\alpha = 0.9$ may be an expensive choice for the study area. Similar results have been provided by Cheng et al. (2008) in the Danshuei river basin in Taiwan. They found an A_p of approximately 36% at $\alpha = 0.9$ for annual rainfall.

If a threshold value of A_p, say 80%, is set as the network evaluation and augmentation criterion, then at $\alpha = 0.8$, the current network meets the criterion. Even so, if the threshold is set at a higher level, say 100%, then the current network fails the evaluation test and network augmentation is required. In this case study we decided to have a rain-gauge network with 100% A_p and augment the network by adding, relocating or removing of rain-gauges.

Figure 5 is constructed using the sequential algorithm. It demonstrates the prioritization of rain-gauges and corresponding values of A_p. About 12 gauges (gauges 2, 3, 4, 6, 9, 20, 21, 25, 26, 30, 31 and 33) need to be removed or relocated at $\alpha = 0.8$ since without these gauges the remaining 21 gauges achieve almost the same level of A_p as the complete network of 33 gauges (Figure 6). The remaining gauges form the *base network* and are not relocated in the network augmentation process.

The rain-gauges that are not included in the base network are redundant and contribute little to the network. These 12 rain-gauges can either be subtracted to reduce the maintenance cost or be relocated to achieve higher

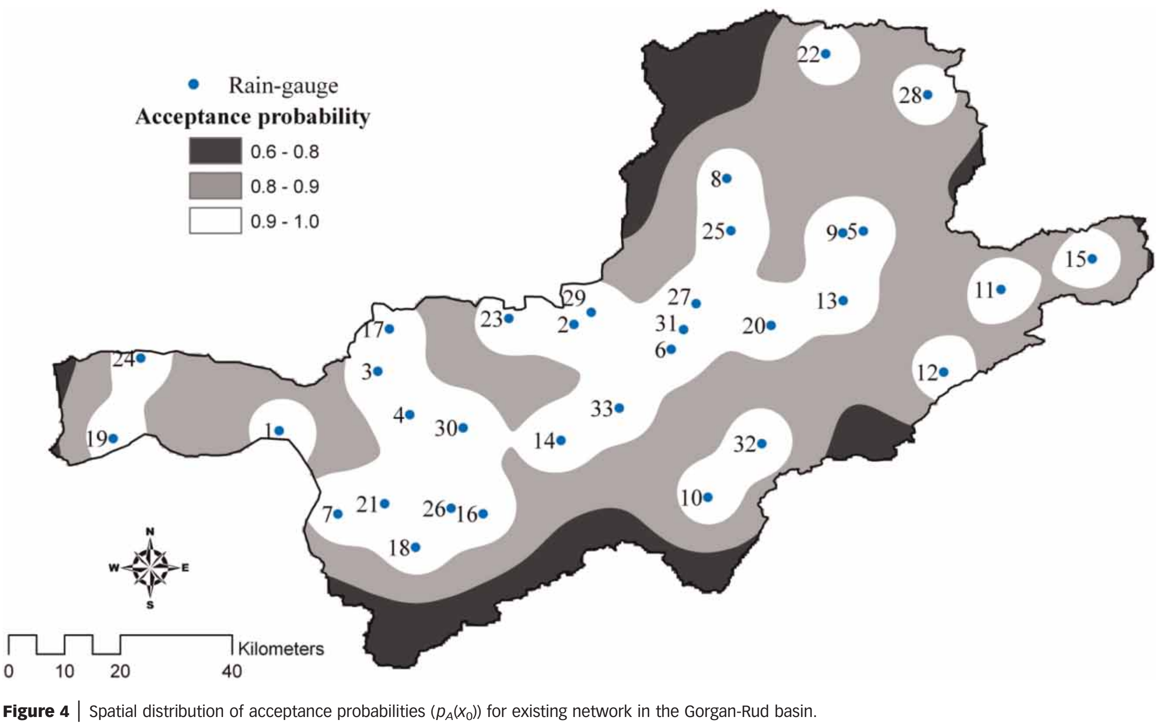

Figure 4 | Spatial distribution of acceptance probabilities ($p_A(x_0)$) for existing network in the Gorgan-Rud basin.

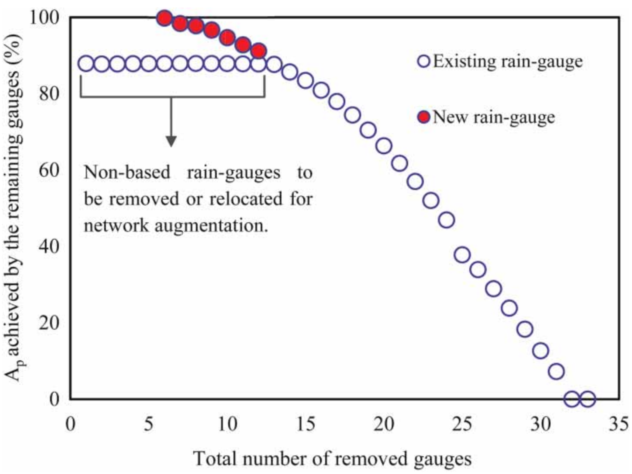

Figure 5 | Rain-gauge prioritization and the corresponding A_p values for rain-gauge network evaluation and augmentation in the Gorgan-Rud basin.

level of A_p. For achieving a more efficient rain-gauge network and meeting the 100% A_p in the study area, relocation of some of the redundant rain-gauges is conducted based on the sequential algorithm. A candidate location for augmentation is obtained by searching one point (gauge) among all points with $p_A(x_0) < \alpha$ (white region in Figure 6) such that it, along with the base network, yields the highest value of A_p. Figure 5 illustrates the level of A_p that is achieved by sequentially adding (relocated) gauges to the base network. By relocating only seven gauges out of the 12 non-base gauges in the Gorgan-Rud basin, 100% A_p is achieved. Figure 7 shows the augmented network including base rain-gauges plus relocated gauges. Thus, an 'optimal' network with 28 gauges is more efficient than the existing network of 33 gauges.

The benefits of the AP approach when compared with other approaches such as the variance reduction approach are as follows:

1. It focuses on the accuracy of point rainfall across the whole study area rather than aiming for greater accuracy of areal rainfall estimation (Cheng *et al.* 2008). By contrast, the variance reduction method minimizes the average kriging estimation variance over the whole study area. The AP approach not only provides an optimal rain-gauge network over a basin but it also estimates the level of accuracy of the spatial distribution of rainfall for any part of a basin.
2. It is highly flexible in parameters that are related to accuracy assessment such as k, α and the percentage of area with acceptable accuracy (A_p).

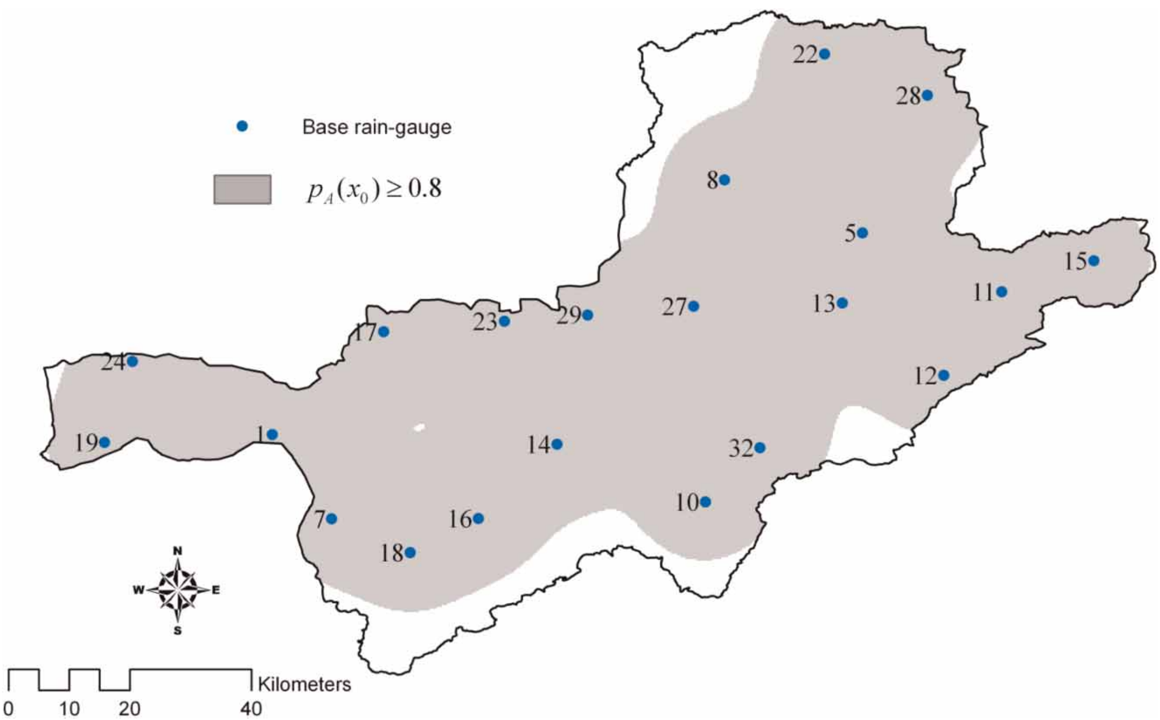

Figure 6 | Spatial distribution of acceptance probabilities ($p_A(x_0)$) for base network in the Gorgan-Rud basin.

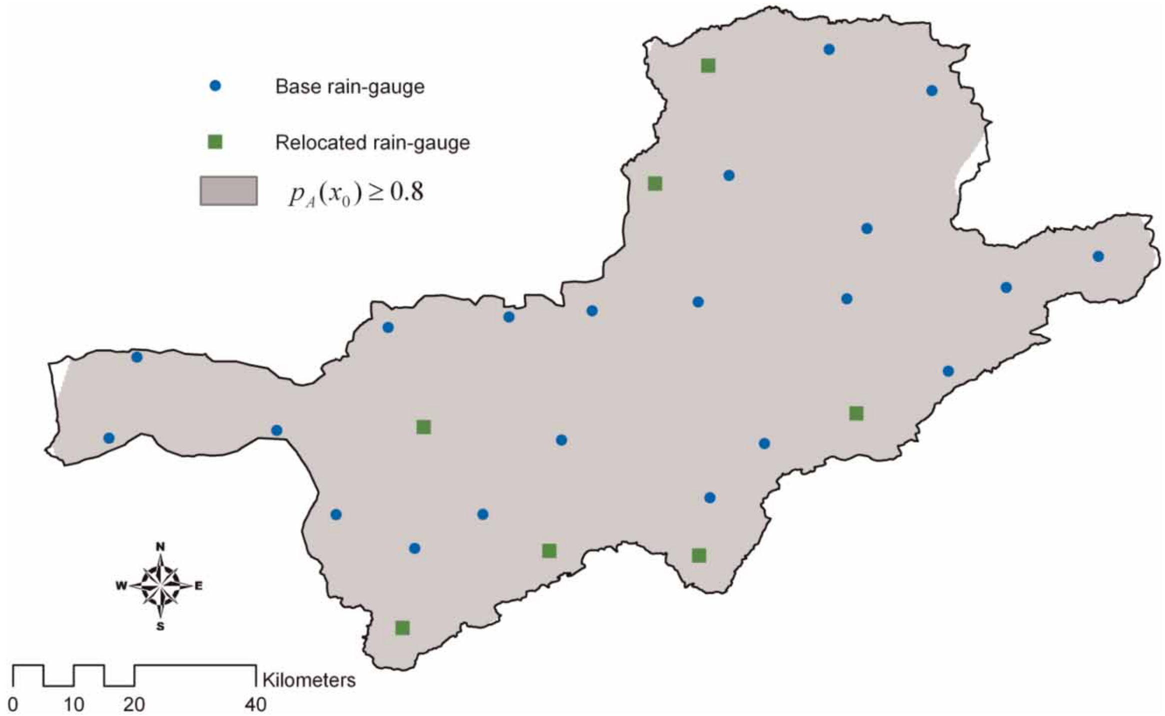

Figure 7 | Spatial distribution of acceptance probabilities ($p_A(x_0)$) for augmented network in the Gorgan-Rud basin.

3. The most important parameter in AP approach is calculated easily using Equation (11). This facilitates its practical implementation in a GIS framework.

CONCLUSIONS

The estimation of areal rainfall is still a true challenge for hydrological applications. An efficient rain-gauge network that can accurately provide required rainfall spatial characteristics in a basin is desirable. Rain-gauge network evaluation involves the analysis of the number and location of gauges necessary for achieving the required accuracy. The goal of this paper was to evaluate the performance of an existing rain-gauge network in a large basin. An acceptance probability approach was adopted which is based on the accuracy assessment of point rainfall estimation and uses ordinary kriging variance.

A core of 21 rain-gauges in the study area achieved almost the same level of performance (A_p equal to 88%) as the whole network of existing 33 rain-gauges for areal annual rainfall estimation. Also, by relocating only seven gauges out of the 12 remaining gauges, an acceptance probability of at least 0.8 was achieved. The threshold value of 0.8 (or $\alpha = 0.8$) for acceptance probability was also found to be a suitable criterion for evaluating rain-gauge networks. An approximation for acceptance probability was also introduced that efficiently facilitated the implementation of acceptance probability approach in a GIS. In future research, it is proposed to study the trade-off between cost and accuracy in rain-gauge networks through application of AP approach.

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