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ORIGINAL ARTICLE

# Dust-ion acoustic waves modulation in dusty plasmas with nonextensive electrons

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**Abstract** The nonlinear amplitude modulation of dust-ion acoustic wave (DIAW) is studied in the presence of nonextensive distributed electrons in dusty plasmas with stationary dust particles. Using the reductive perturbation method (RPM), the nonlinear Schrödinger equation (NLSE) which governs the modulational instability (MI) of the DIAWs is obtained. Modulational instability regions and the growth rate of nonlinear waves are discussed. It is shown that the wave characters are affected by the value of nonextensive parameter and also relative density of plasma constituents.

**Keywords** Dust ion acoustic waves · Dusty plasma · Nonextensive electrons · Modulational instability

## 1 Introduction

Theoretical investigation of nonlinear structures in astrophysical and laboratory multicomponent plasmas has received significant interests in plasma physics. Different types of nonlinear structures have been studied in these fields, such as solitons, envelope structures, shocks, vortices and so on. Nonlinear localized defects associated with the dust ion acoustic waves (DIAW) (Fechting et al. 1979; Rao et al. 1990; Mendis and Rosenberg 1994), especially dust ion-acoustic envelope solitary waves (Nakamura and Sharma 2001; Amin et al. 1998) have received a great deal of attention in plasma physics because of their importance in the environment of space and in laboratory (Mendis and Horanyi 1991; Pieper and Goree 1996; Shukla 2000; Rosenberg and Merlino 2007; Goertz 1989; Whipple et al. 1989). The first theoretical investigation of conservation of equilibrium charge density in dusty plasmas has been presented by Shukla and Silin (Shukla and Silin 1992). They have shown that dusty plasmas (with negatively charged static dust) support low-frequency DIAWs with phase speed much smaller (larger) than the electron (ion) thermal speed. The DIAWs have also been observed in many laboratory experiments (Merlino and Goree 2004; Barkan et al. 1996). Mamun and Shukla (2002, 2005) have theoretically investigated DIASWs in an unmagnified dusty plasmas containing cold ion fluid, isothermal electrons and negatively charged static dust particles. The propagation of nonlinear one-dimensional DIASWs in unmagnetized dusty plasmas containing adiabatic ions and electrons, and negatively charged static dust grains has been discussed by Mamun (2008). The nonlinear wave propagation is generically subject to an amplitude modulation due to the carrier wave self-interaction which is a well-known harmonic generation mechanism. The standard multiple scale technique (Taniuti and Yajima 1969; Asano et al. 1969) has been employed in the study of this mechanism. Investigations lead to a nonlinear Schrodinger-type equation (NLSE), describing the evolution of the wave envelope. Under certain conditions, the wave may undergo a Benjamin–Feir-type (modulational) Instability (MI). In this situation its envelope may collapse under the influence of external perturbations. In addition, the analysis of the NLSE reveals the possibility of the existence of local-

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ized excitations (envelope solitary waves) whose characteristics depend on criteria similar to the ones necessary for the MI to occur (Remoissenet 1994; Sulem and Sulem 1999; Hasegawa 1989). These structures are created by the mutual compensation of dispersion and nonlinearity and may be the result of energy localization in the evolution stage following the wave amplitude collapse and propagate in the nonlinear medium for long times. The dynamics of modulated dust-acoustic wave packets in dusty plasma with Boltzmann distributed electrons have been studied in Amin et al. (1998), Xue and He (2003), Kourakis and Shukla (2003). It is interesting to study MI in plasmas out of their thermal equilibrium states which are widely observed in space plasmas. Over the last few years, a great deal of attention has been paid to nonextensive statistical mechanics based on the deviations of Boltzmann–Gibbs–Shannon (BGS) entropic measure. A suitable nonextensive generalization of the BGS entropy for statistical equilibrium was first proposed by Renyi (1955) and subsequently explained by Tsallis (1988). This situation is a suitably extending the standard additively of the entropies to the nonlinear, nonextensive case where one particular parameter (the entropic index $q$) characterizes the degree of system nonextensivity. This nonadditive entropy of Tsallis and the ensuing generalized statistics have been successfully employed in a wide range of phenomena characterized by nonextensivity (Lima et al. 2000; Leubner and Vörös 2005; Leubner 2008; De Almeida 2008; Hanel and Thurner 2009). It may be noted that the Maxwellian distribution in Boltzmann–Gibbs statistics is believed valid universally for the macroscopic ergodic equilibrium systems. However, for the systems with the long-range interactions Maxwellian distributions might be inadequate for the description of the systems which are in nonequilibrium stationary states. Plasmas and gravitational systems are well-known situations with long range interactions. Because of a lack of formal derivation, a nonextensive approach to kappa distributions has been suggested recently (Leubner 2002, 2004a, 2004b). It has been shown that distributions very close to kappa distributions are a consequence of the generalized entropy favored by nonextensive statistics. Indeed one can find similarities between the Tsallis distribution with $q > 1$ and the so-called Kappa distribution. However there is no simple transformation between the Tsallis distribution and the Kappa-one as the argument and the power do not fit the same transformation. It may be noted that some recent theoretical work focused on the effects of suprathermal particles on different types of linear and nonlinear collective processes in plasmas (Mace et al. 1999; Hellberg and Mace 2002; Aoutou et al. 2008; Saini and Kourakis 2008; Baluku and Hellberg 2008; Tribeche et al. 2009; Saini et al. 2009; Tribeche and Boubakour 2009; Boubakour et al. 2009) too.

Saha and Chatterjee (2014a) have studied dust ion acoustic waves in magnetized dusty plasmas with nonextensive distributed electrons. They have shown that, amplitude of the solitary waves (compressive and rarefactive) increases when $q$ increases, But the width of the solitary waves (compressive and rarefactive) decreases with an increasing values of $q$. It is interesting that, both amplitude and width of the periodic travelling waves increase wit $q$ and the amplitude of the kink and antikink waves are decreasing functions of the parameter $q$. Saha and Chatterjee (2014b) also have investigated dust acoustic solitary waves and periodic waves in an unmagnetized dusty plasma with nonextensive ions using non-perturbative approach. Also the cylindrical Zakharov–Kuznestov equation has been derived for ion-acoustic waves in plasmas comprising of ions and electrons featuring nonextensive distribution by Ghosh et al. (2014).

The aim of this paper is to study the MI of DIAWs in dusty plasmas consisting of negative dust particle as well as $q$-nonextensive electrons. The layout of this article goes as follows: In Sect. 2, we present the basic equations, and using the reductive perturbation technique. Nonlinear Schrödinger equation, governing the slowly varying modulation is derived in this section too. In Sect. 3, we will discuss the numerical results of MI analysis and present the influence of nonextensive parameters and dust (or electron/ion) concentration on the growth rate of the modulational instability. Section 4 is kept for discussion and conclusions.

## 2 Basic equations and derivation of NLSE

We consider an unmagnetized dusty plasma system consisting of cold inertial ions, nonextensive distributed electrons and negatively charged immobile dust particles. The inertia of the system is provided by the ion mass and the restoring force comes from the pressure of inertialess electrons. Also the equilibrium charge neutrality condition is maintained by the stationary dust particles. The charge neutrality at equilibrium requires $n_{i0} = Z_d n_{d0} + n_{e0}$, where $n_{i0}$, $n_{e0}$ and $n_{d0}$ represent the equilibrium number densities of the ions, electrons and dust particles, respectively and $Z_d$ displays the absolute value dust charges. The usual ion fluid equations which include the continuity equation, momentum balance equation and Poisson equation, governing the DIAW as

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu n_e - n + 1 - \mu \tag{3}$$

where $n$ is the ion number density normalized by its equilibrium value $n_{i0}$, $u$ is the ion fluid speed normalized by $c_i = \sqrt{T_e/m_i}$ and $\varphi$ is the electrostatic wave potential normalized by $(T_e/e)$, where $T_e$ is electron temperature. The time $t$ and the distance $x$ are normalized by the ion plasma frequency $\omega_{pi}^{-1} = \sqrt{m_i/4\pi n_{i0}e^2}$ and the Debye length $\lambda_{Dm} = \sqrt{T_e/4\pi n_{i0}e^2}$, respectively. We have denoted $\omega_{pi}^{-1} = \sqrt{m_i/4\pi n_{i0}e^2}$.

The normalized electron density is given by Bains et al. (2011)

$$n_e = \left[1 + (q-1)\varphi\right]^{\frac{q+1}{2(q-1)}} \tag{4}$$

The parameter $q$ stands for the strength of nonextensivity. It may be useful to note that for $q < -1$, the nonextensive electron distribution (not given here) is unnormalizable. In the extensive limiting case ($q \to 1$), the electron distribution reduces to the well-known Maxwell-Boltzmann velocity distribution.

By inserting $n_e$ and $n_i$ in the Poisson's equation we get

$$\frac{\partial^2\varphi}{\partial x^2} = n_d + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 \tag{5}$$

where

$$c_1 = \frac{\mu}{2}$$
$$c_2 = \frac{\mu}{8}(q+1)(3-q) \tag{6}$$
$$c_3 = \frac{\mu}{48}(q+1)(3-q)$$

We employ the standard reductive perturbation method (Kourakis and Shukla 2005) to investigate the amplitude modulation of DIAWs in dusty plasmas with nonextensive electrons. The independent variables are stretched as $\xi = \varepsilon(x - V_0 t)$ and $\tau = \varepsilon^2 t$, where $\varepsilon$ is a small constant and $V_0$ is the wave's group velocity. It is a free parameter to be determined later by the compatibility condition. The dependent variables are expanded as follows:

$$n = 1 + \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=-\infty}^{+\infty} n_l^n(\xi,\tau)e^{il(kx-\omega t)}$$
$$u = \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=-\infty}^{+\infty} u_l^n(\xi,\tau)e^{il(kx-\omega t)} \tag{7}$$
$$\varphi = \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=-\infty}^{+\infty} \varphi_l^n(\xi,\tau)e^{il(kx-\omega t)}$$

where $n$, $u$ and $\varphi$ satisfy the reality condition $A_{-l}^{(n)} = (A_l^{(n)})^*$ and the asterisk denotes complex conjugate. Substituting these expressions along with stretching coordinates into Eqs. (1) and (6) and collecting the terms in the different powers of $\varepsilon$, we can obtain nth-order reduced equations. We obtain the first-order ($n = 1$) equation quantities with $l = 1$ as follows

$$-i\omega n_1^{(1)} + iku_1^{(1)} = 0$$
$$-i\omega u_1^{(1)} - ik\varphi_1^{(1)} = 0 \tag{8}$$
$$n_1^{(1)} + (k^2 + c_1)\varphi_1^{(1)} = 0$$

The solution for the first harmonics is

$$n_1^{(1)} = -(k^2 + c_1)\varphi_1^{(1)}, \qquad u_1^{(1)} = -\frac{\omega}{k}(k^2 + c_1)\varphi_1^{(1)} \tag{9}$$

that give rise to the following dispersion relation for the DIAWs

$$\frac{\omega^2}{k^2} = \frac{1}{(k^2 + c_1)} \tag{10}$$

At second order of $\varepsilon$, we expect to calculate expressions for the group velocity $V_0$, and for the zeroth and second harmonics. For $n = 2$ and $l = 1$, we need to impose a compatibility condition in the form

$$\frac{\partial\varphi_1^{(1)}}{\partial T} + V_0\frac{\partial\varphi_1^{(1)}}{\partial x} = 0 \tag{11}$$

where the group velocity $V_0(k) = \frac{\partial\omega}{\partial k} = \omega'(k)$ is given by

$$V_0 = \frac{\omega}{k}(1 - \omega^2) = c_1\frac{\omega^3}{k^3} = \frac{c_1}{(k^2 + c_1)^{3/2}} \tag{12}$$

The group speed (12) describes the propagation velocity of the wave pocket in the media. No wave packet propagation while group velocity is zero. Figure 1 presents the wave group velocity as functions of wave number $k$ for

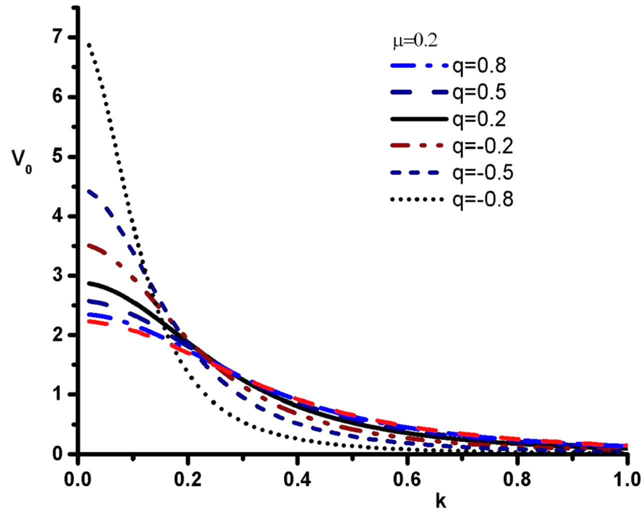


**Fig. 1** Group velocity $V_0$ as functions of wave number with different values of nonextensive parameter $q$ and fixed value for $\mu = 0.2$

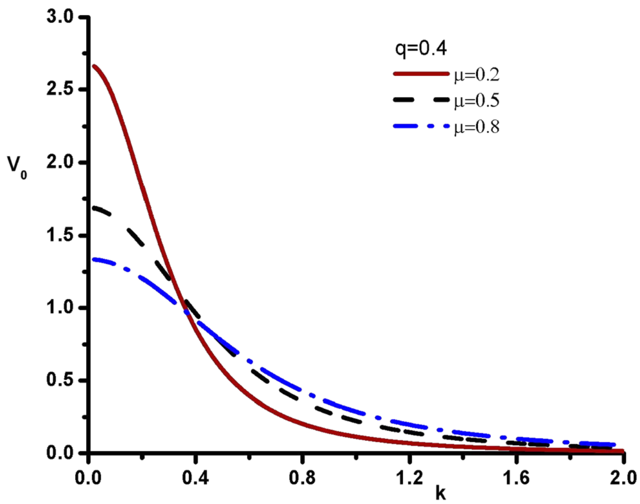


**Fig. 2** Group velocity $V_0$ as functions of wave number with different values of $\mu$ and fixed value for nonextensive parameter $q = 0.4$

different values of nonextensive parameter $q$. This figure shows that plasmas with lower nonextensive parameter have greater group velocities. This means that wave packets travel with greater velocities in plasmas out of their thermal equilibrium.

Figure 2 demonstrates variation of group velocity with wave number $k$ for different values of $\mu$. The group velocity decreases with increasing values of $\mu$. Therefore one can concluded that DIAWs group velocity have higher values in plasmas with greater population of ions in comparison with electron density.

The expressions for the amplitudes corresponding to the first harmonics in order $\varepsilon^2$ are given by

$$\begin{aligned}
&-i\omega n_1^{(2)} + iku_1^{(2)} = V_0\frac{\partial n_1^{(1)}}{\partial \xi} - \frac{\partial u_1^{(1)}}{\partial \xi}\\
&-i\omega u_1^{(2)} + ik\varphi_1^{(2)} = V_0\frac{\partial u_1^{(1)}}{\partial \xi} + \frac{\partial \varphi_1^{(1)}}{\partial \xi}\\
&-n_1^{(2)} - \left(k^2 + c_1\right)\varphi_1^{(2)} = 2ik\frac{\partial \varphi_1^{(1)}}{\partial \xi}
\end{aligned}\tag{13}$$

The second-harmonic modes $n = 2$, $l = 2$ arising from the nonlinear self-interaction of the carrier waves are obtained in terms of $(\varphi_1^{(1)})^2$ as

$$\begin{aligned}
n_2^{(2)} &= A_n\left(\phi_1^{(1)}\right)^2\\
u_2^{(2)} &= A_u\left(\phi_1^{(1)}\right)^2\\
\phi_2^{(2)} &= A_\phi\left(\phi_1^{(1)}\right)^2
\end{aligned}\tag{14}$$

where

$$\begin{aligned}
A_\varphi &= -\frac{c_2}{3k^2} - \frac{k^2}{2\omega^4}\\
A_n &= -c_2 - \left(\frac{k^2 + 3k^2\omega^2}{\omega^2}\right)A_\varphi\\
A_u &= \frac{\omega}{k}\left(A_n - \frac{k^4}{\omega^4}\right)
\end{aligned}\tag{15}$$

The zeroth-harmonic mode (in terms of $|\varphi_l^{(1)}|^2 = (\varphi_l^{(1)})^*\varphi_l^{(1)}$) also appears due to the self interaction of the modulated carrier wave. An expression for $\varphi_l^{(1)}$ cannot be determined completely within the second order and we will have to consider the third-order equations. Thus, the $l = 0$ components of the third-order part of the reduced equations determine the following second-order quantities in the zeroth-harmonic

$$\begin{aligned}
n_0^{(2)} &= B_n\left|\varphi_1^{(1)}\right|^2\\
u_0^{(2)} &= B_u\left|\varphi_1^{(1)}\right|^2\\
\varphi_0^{(2)} &= B_\varphi\left|\varphi_1^{(1)}\right|^2
\end{aligned}\tag{16}$$

where

$$\begin{aligned}
B_\varphi &= \frac{2c_2V_0^2 + (3c_1 + k^2)}{1 - c_1V_0^2}\\
B_n &= -c_1B_\varphi - 2c_2\\
B_u &= -\frac{2k^3}{\omega^3} + V_0B_n
\end{aligned}\tag{17}$$

Finally, substitution of all the previous derived expressions into the components for $n = 3$, $l = 1$ of the reduced equations lead to the following NLS equation:

$$i\frac{\partial \Phi}{\partial \tau} + P\frac{\partial^2 \Phi}{\partial \xi^2} + Q|\Phi|^2\Phi = 0 \tag{18}$$

for the slow evolution of the first-order amplitude of the plasma perturbation potential $\varphi_l^{(1)} \equiv \Phi$. In the above equation, the coefficients $P$ and $Q$ are given by

$$\begin{aligned}
P &= -\frac{3}{2}c_1\frac{\omega^5}{k^4}\\
Q &= \frac{\omega^3}{2k^2}\bigg[3c_3 + 2c_2(A_\varphi + B_\varphi)\\
&\qquad - 2\frac{k}{\omega}\left(k^2 + c_1\right)(A_u + B_u) - \left(k^2 + c_1\right)(A_n + B_n)\bigg]
\end{aligned}\tag{19}$$

## 3 Modulational instability (MI)

The MI of DIAWs can be studied by adding a small perturbation $\delta\phi$ to the solution. Therefore, we set $\Phi = (\Phi_0 + \delta\varphi)e^{i\Delta\tau}$, where $\Phi_0$ is the amplitude of the pump carrier which is much larger than the perturbation, i.e., $\Phi_0 \gg |\delta\phi|$ and $\Phi = \varphi_1^{(1)}$, also here $\Delta$ is a nonlinear frequency shift produced by the nonlinear interaction. After substituting $\Phi = (\Phi_0 + \delta\varphi)e^{i\Delta\tau}$ into Eq. (18) and collecting terms of the same order, we find (Fedele and Schamel 2002)

$$\Delta = -Q|\Phi_0|^2 \tag{20}$$

and

$$i\frac{\partial\delta\varphi}{\partial t} + P\frac{\partial^2\delta\varphi}{\partial Z^2} + Q|\Phi_0|^2\left(\delta\varphi + \delta\varphi^*\right) = 0 \tag{21}$$

where $\delta\varphi^*$ is the conjugate of $\delta\varphi$. Upon assuming the amplitude perturbation varying as $e^{(ikZ - i\Omega t)}$, we obtain the following nonlinear dispersion relation

$$\Omega^2 = P^2k^2\left(k^2 - 2Q|\Phi_0|^2/P\right) \tag{22}$$

For all values of $k$, the DIAW is stable in the presence of small perturbation if $PQ < 0$. In this situation $\Omega$ is always a real number. On the other hand, the modulation instability (MI) would set in as $\Omega$ becomes imaginary when $PQ > 0$. This happens when the modulation wave number $k$ of external perturbation is less than the critical value $k_{cr} = \sqrt{2Q|\Phi_0|^2/P}$. In the region $PQ > 0$ and $k < k_{cr}$, the MI growth rate becomes

$$\Gamma = lm(\Omega) = \sqrt{P^2k^2\left(k_{cr}^2 - k^2\right)} \tag{23}$$

Obviously, the growth rate reaches its maximum value, $\Gamma_{\max} = Q|\Phi_0|^2$, for $k = \sqrt{k_{cr}^2}/2$.

It would be illuminating to numerically analyze different parameter regimes where stable or unstable waves are created. $P$ and $Q$, the coefficients of dispersive and nonlinear terms, respectively, are related to relative density of electron $\omega_{pi}^{-1} = \sqrt{m_i/4\pi n_{i0}e^2}$, and spectral index $q$, and these parameters would definitely affect the stability characteristics of DIAW s in our plasma model. Our primary aim is to demonstrate the effects of nonextensive electrons on the instability of DIAWs. For $PQ > 0$, we expect MI is occurred. It is clear from (10) that the coefficient $P$ is always negative. But numerical analysis shows that $Q$ can be negative or positive. Apparently, the coefficients of dispersion term $P$ and nonlinear term $Q$ are related to nonextensive parameter $q$ and the rate of electron to ion concentrations $\mu$. Therefore, we expect that these parameters affect the instable characteristics, which may develop in the plasma model. To investigate these effects in more detail, we plot the ratio of $P/Q$ versus the carrier wave number $k$ for different values

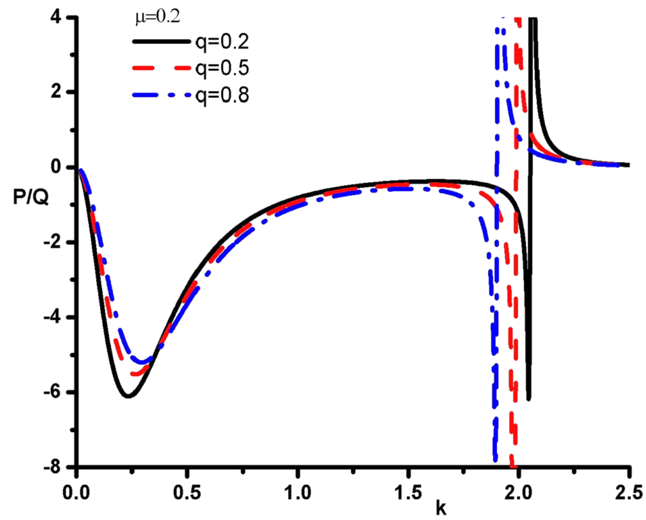


**Fig. 3** Variation of the NLSE coefficients ratio $P/Q$ with the carrier wave number $k$ for different values of $q > 0$. All the plots have been sketched with $\mu = 0.2$

of plasma parameters. $Q = 0$ corresponds to zero dispersion point leading to $P/Q \to \pm\infty$. The corresponding value of $k(= k_c)$ is called critical or threshold wave number for the onset of MI.

The results of $P/Q$ as functions of $k$ with $\mu = 0.2$ for three different values of nonextensive parameter $q$ have been displayed in Fig. 3. It is observed that both stable and unstable regions are obtained in this situation. The wave remains stable at small $k < k_c$ and the MI sets in for $k > k_c$. Dark solitons occur in the former case, i.e. for large wavelengths, while bright envelope solitons occurs in the latter region. In the absence of nonextensive distributed electrons ($q = 1$) corresponding to the Maxwellian case for electrons, we have found $k_c = 1.86$ (not plotted in Fig. 1). The critical wave number shits to higher values i.e. smaller wave lengths as nonextensive parameter decreases. According to our calculations which have been plotted in Fig. 1, for $q = 0.8$ the critical wave number is $k_c = 1.91$, with $q = 0.5$ it becomes $k_c = 1.98$ and the critical wave number is $k_c = 2.06$ for $q = 0.2$. Therefore it is obvious from the graph, the presence of nonextensive distributed electrons markedly increases the threshold $k_c$. It may be noted that the existence of nonextensive particles reduces the index $q$. It is not a general behavior and in some cases the critical velocity increases as the nonextensive parameter $q$ increases (electron distribution goes toward usual Maxwellian case). This situation will be discussed latter.

It is found from Fig. 3 that the DIAW remains stable for $k < k_c$ and MI sets in when $k > k_c$. The dark solitons occur in the former case, i.e. for large wavelength, while bright envelope solitons occur in the latter region. Figure 3 shows that, when the instability sets in, the critical value $k_c$ decreases with an increase in the value of the nonextensive $q$.

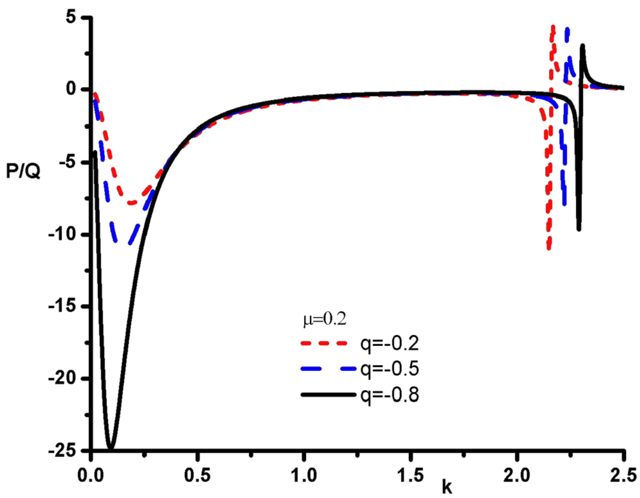


**Fig. 4** Variation of the NLSE coefficients ratio $P/Q$ with the carrier wave number $k$ for different values of $q < 0$

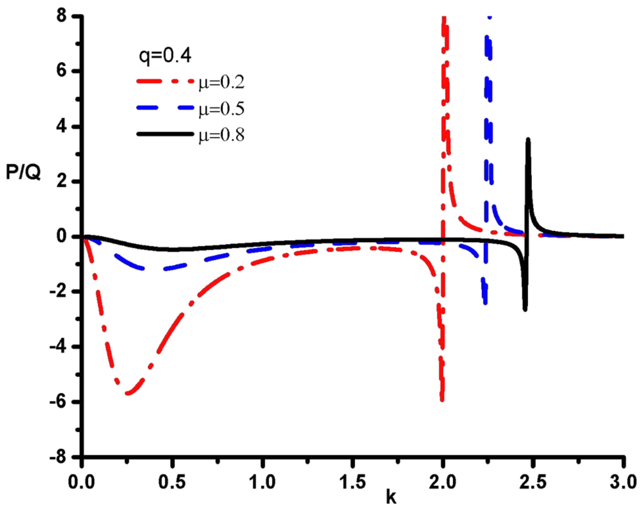


**Fig. 5** Variation of the NLSE coefficients ratio $P/Q$ with the carrier wave number $k$ for different values of $\mu$ and fixed value of $q > 0$

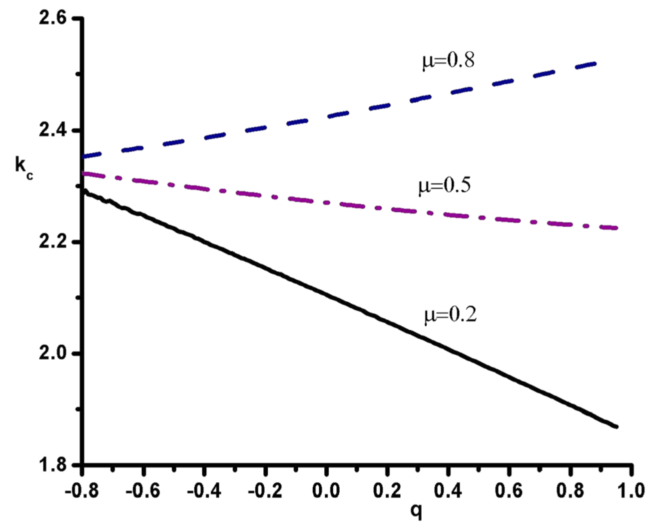


**Fig. 6** Threshold wave number $k_c$ as functions of nonextensivity index $q$ with different values of $\mu$

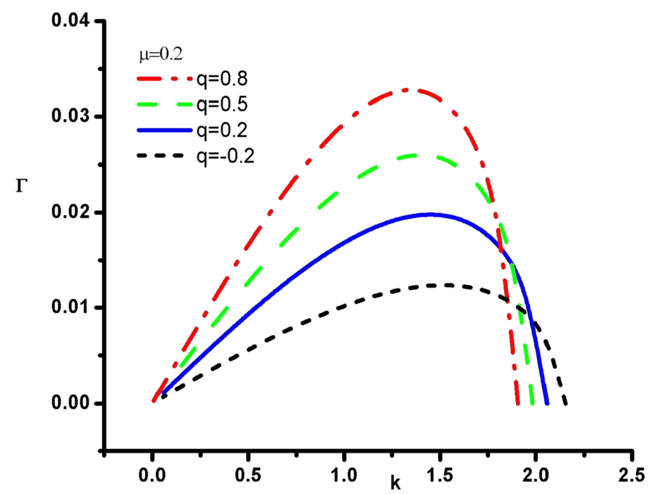


**Fig. 7** Grow rate $\Gamma$ as functions of $k$ with different values of nonextensive parameter $q$ and fixed value $\mu = 0.2$

It means that the MI occurs at smaller wavelengths as electrons evolve toward their thermodynamic equilibrium. The effect of the nonextensive parameter in the range of values smaller than zero has been presented in Fig. 4. The figure indicates that the critical wave number $k_c$ shifts toward higher wave lengths as index $q$ decreases, and this treatment of $k_c$ is in agreement to the earlier observations ($q > 0$). The results for $q < 0$ are in agreement with results of Bains et al. (2011) for ion acoustic waves in plasmas with $q$-nonextensive distributed electrons. Nevertheless similar behavior of the critical value $k_c$ is observed in both types of positive and negative values of nonextensive parameter. If we increase the value of $q$, the critical value $k_c$ decreases as one can understand from Figs. 3 and 4.

The effects of electron relative density on the DIAW characters can be explained using Fig. 5. This figure indicates that the threshold wave number $k_c$ increases with increasing values of $\mu$. Dark solitons occur in the former case, i.e., for large wavelengths, while bright envelope solitons occurs in the latter region. Therefore it can be concluded that stable solitary waves with larger wave numbers are propagated in plasmas with greater electron relative population. Such this situation also is occurred for negative index $q$ as one can find from Fig. 6.

To glean better information about the nature of MI, the growth rate $\Gamma$ has been plotted as a function of $k$ for three different values of nonextensive parameter $q$ with electron relative density $\mu = 0.2$, as shown in Fig. 7. The corresponding critical wave numbers $k_c$, below which the MI growth rate reaches its maximum (as written before) are $k_c = 1.91, 1.98, 2.06, 2.15$ for nonextensive values $q = 0.8$, 0.5, 0.2 and $-0.2$ respectively. One can find that the elec-

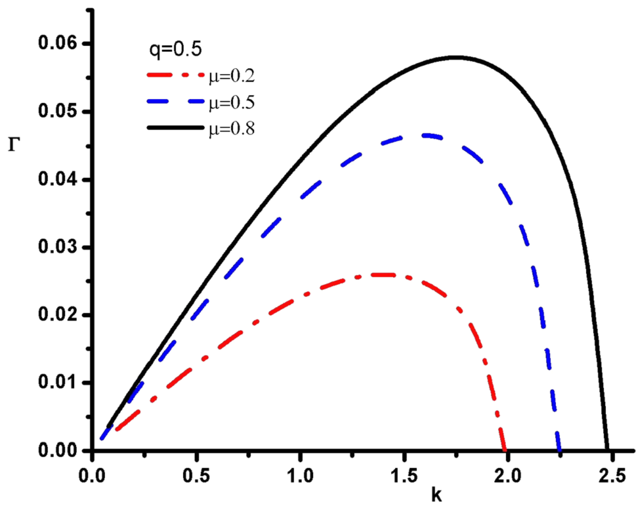


**Fig. 8** Grow rate $\Gamma$ as functions of $k$ with different values of relative density of electrons and fixed value $q = 0.5$

tron nonextensivity have crucial role on the growing rate of the MI, which increases as the electrons evolve toward their thermodynamic equilibrium.

Although the general trend is of a similar type in all the plots, there is a substantial difference in the magnitude of the growth rate $\Gamma$ with different values of $q$. The growths exhibit an increase with $k$, attaining maximum values followed by a sharp decrease in all three cases. However the threshold wave number $k_c$ increases as $q$ decreases for all values of this parameter, but the maximum grow rate $\Gamma$ decreases with lower values of index $q$. It may be noted that the nonextensivity effects are more dominant in lower values of $q$.

Similar behavior is observed in the $\Gamma$ versus $k$ graphs plotted for different values of electron relative density, $\mu$ in Fig. 8. Increasing the population of electrons compared with ions leads to a significant increase in the growth rate, as shown in Fig. 8. Both the maximum grow rate and threshold wave number $k_c$ increases as $\mu$ increases.

Presented results can be compared with the results of recent investigations. The modulational instability of ion-acoustic waves in two-component plasmas containing nonextensive electrons has been investigated in Bains et al. (2011). Our derived relations with $\mu = 1$ are fully compatible with the calculations presented in Bains et al. (2011). According to what has been reported in Bains et al. (2011), the critical value $k_c$ increases with an increase in the value of the nonextensive index for $q > 0$. This situation is different from our results where such increase in $q$ would lead to a decrease in the critical wave number as Fig. 3 presents. But for $q < 0$ our results are in agreement with results of Bains et al. (2011). Such problem also exist in variation of grow rate too. Our results also show the similar behavior as explained before. Bains et al. have investigated the dynamics of low-phase velocity nonlinear dust acoustic waves in three component, homogeneous, unmagnetized dusty plasmas whose constituents are negatively charged inertial dust particles, $q$-distributed electrons and ions (Bains et al. 2013). Our derived Eqs. (6)–(17) are in agreement with the relations calculated in Bains et al. (2013). The general behavior of nonlinear waves is the same and nonextensivity plays similar role. According to Bains et al. (2013) existence of nonextensive electrons causes a shift to lower wave numbers in the critical wave number $k_c$ while in our plasma system such shift is occurred toward the greater wave number.

Eslami et al. have investigated Modulational instability of ion acoustic waves in unmagnetized plasmas composed of cold ions and nonextensive positrons and electrons (Eslami et al. 2011). Our results also qualitatively are in agreement with the results of their work, with $\mu = 1$ in our calculations and neglecting the effects of positrons in the results of Eslami et al. (2011).

## 4 Conclusions

A nonlinear Schrodinger equation was derived for the one-dimensional nonlinear propagation of DIAWs in homogeneous unmagnetized dusty plasmas consisting of cold inertial ions, nonextensive distributed electrons and negatively charged immobile dust particles by applying the standard reductive perturbation method. The amplitude modulation was considered to study the modulational instability of the DIAWs. A nonlinear dispersion relation and group velocity for the wave pocket propagation were derived too. The influence of nonextensive index $q$ and relative density of electrons $\mu$ on the modulation instability is studied. It was shown that the DIAW remains stable for $k < k_c$ and MI sets in when $k > k_c$ where $k_c$ is a critical wave number in which the nonlinear coefficient $Q$ becomes zero. The critical wave number $k_c$ shifts toward larger wave numbers as index $q$ decreases. Also the threshold wave number $k_c$ increases with increasing values of $\mu$. To find better information about the nature of MI, the growth rate $\Gamma$ also was studied. The cut-off wave number of the grow rate increases as $q$ decreases but the maximum grow rate $\Gamma$ decreases with lower values of nonextensive parameter $q$. It was shown that increasing the population of electrons compared with ions leads to a significant increase in the growth rate.

Outcomes were compared with the results of related published paper and one would find a very good agreement between the presented results and the results of recent works.

The present study could be helpful to understand some features of the nonlinear waves in stellar polytropes (Plastino and Plastino 1993) hadronic matter and quark-gluon plasmas (Gervino et al. 2012) proto-neutron stars (Lavagno and Pigato 2011) and dark-matter halos (Feron and Hjorth 2008) where electron follows the nonextensive distribution.

The interesting nature of nonextensive electrons should be verified by further laboratory experiments. We propose to perform a laboratory experiment which will be able to identify the special new features of the DIASWs in nonextensive dusty plasmas that have been predicted in this investigation.

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