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Structural damage identification by sensitivity of modal strain energy based on optimization function

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Abstract

Many methods for the assessment of structural damage based on vibration-based technique are surveyed for many decades. In this paper, an effective damage detection methods based on numerical and experimental evaluation of structural damage is presented by sensitivity of modal strain energy and using a new model updating optimization function. First, damage localization in the dynamic structures is formulated by fundamental modal strain energy (MSE) to precisely locate the eventual damage of a structure. With location determined, the corresponding damage extent can be obtained by sensitivity of modal strain energy based on changes in the stiffness components. Subsequently, using of sensitivity matrix and difference between mode shapes of healthy and damaged structures, a new optimization function is defined to accurately estimate the damage severity. For verification of the proposed methods, two illustrative examples as a numerical model of a cantilever beam and a 3-story laboratory frame as experimental model are used. In the experimental investigation, modal analysis with impact hammer is carried out to laboratory frame and vibrational modes are identified as real data. Eventually, results show that the modal strain energy indicator can be exactly detected the location of damage. Then, the estimated damage extent indicates the proposed method based on sensitivity of modal strain energy and minimization of optimization function can provide the convenient results in the numerical and experimental evaluations.

Key words: Structural damage detection, Modal strain energy indictor, Sensitivity of modal strain energy, Optimization function

1. Introduction

Vibration-based methods have been developed and applied to assessment of structural damage in many existing structures that related to civil, mechanical and aerospace engineering fields for many decades. Among the vibration-based methods, those based on modal analysis are widely used. Generally modal parameters are defined as natural frequency, mode shape and modal damping ratio. These data depend on the physical properties of a structure and not the excitation applied. Therefore, parameter identification of structures has become very important as researchers attempt to correlate changes in test data to the changes in the structural element properties. Modification of physical parameters of structures such as mass, stiffness or damping properties are lead to occur changes in the vibrational response of structures or modal data. Hence, changes in the physical properties with adversely performance of dynamic behaviour of structures are described as structural damage.

Typically, damage detection algorithms are categorized as three steps, namely detection of present of damages, detection of the structural damage locations and estimation of the damage extents [1]. Therefore, the knowledge of vibration-based methods can be used to determine the existence as well as the location and the extent of damage. For the damage detection problem, many researchers have worked in these fields for many decades. Doebling et el. [2], stubbs et al [3] and Yan et al. [4] have been provided literature reviews in damage detection process. Gudmundon [5] introduced a first order perturbation method to predict cracks, other geometrical changes and mathematically showed that the change in eigenvalue was related to the change in strain energy of the system. Lee and Chung [6] applied Gudmundson's theory to identify the location and severity of an edge crack in a cantilever beam. He et al, [7] presented a computationally attractive damage index method is proposed for structural damage detection of cylindrical shell. According this approach, the modal strain energy values computed for undamaged and damaged states were used in the corresponding damage index algorithms and two parameters as moment response power spectral density and curvature response power spectral density were divided. For all measured mode shapes, the damage index was defined by using the statistical parameter of relative root mean-square error of case before and after damage. Fan and Qiao [8] introduced a new strain-based damage detection for plate-type structures. They proposed the concepts of a damage location factor (DLF) matrix and a damage severity correction factor (DSCF) matrix, which can be derived from the elemental modal strain energy. Hence, the damage identification method using the DLF and DSCF was developed for damage localization and quantification in plate-type structures. Using finite element model updating for damage detection method, Jaishi and Ren [9] proposed a new multiobjective optimization technique for damage localization and quantification in the beam-like structures. In that methods, eigenfrequency residual and modal strain energy residual were used as two objective functions of the multiobjective optimisation. Also, Seyedpoor [10] provided a two-stage method to properly identify the site and extent of multiple damage cases in structural systems. In the first stage, a modal strain energy based index (MSEBI) was presented to precisely locate the eventual damage of a structure. The modal strain energy was calculated using the modal analysis information extracted from a finite element modelling. In the second stage, the extent of actual damage was determined via a particle swarm optimization (PSO) using the first stage results.

The objective of this article is identification of structural damage by sensitivity of modal strain energy with finite element model updating optimization function. For damage localization a modal strain energy indicator is expanded at first and then, changes of stiffness components as damage index case can be detected. Typically, the damage quantification pertains to location of damage. Therefore, the sensitivity of modal strain energy is determined to use in the model updating optimization function. After providing the optimization function, the extent of damage is estimated by minimization of proposed optimization function based on MSE sensitivity matrix and error vector containing the differences in mode shapes before and after damage. For verification of the proposed methods, two illustrative examples as a numerical model of a cantilever beam and a 3-story laboratory frame are used. In the experimental model, modal analysis with impact hammer is carried out to laboratory frame and modal parameters are identified as real data. Eventually, results show that the modal strain energy indicator can be exactly detected the location of damage. Then, the estimated damage extent indicates the proposed method based on sensitivity of modal strain energy and minimization of optimization function can provide the convenient results in the numerical and experimental evaluations.

2. Theory

2-1- Modal strain energy index

The equations of motion of the free vibration of a linear undamped discrete system with *N* degrees of freedom can be given by

$$K\varphi_i = \lambda_i M\varphi_i, \quad i = 1,2,...,N \tag{1}$$

Where, *M* and *K* are the mass and stiffness matrices, respectively. λ_i and φ_i are the *i*th eigenvalue (square of natural frequency, $\lambda_i=\omega_i^2$) and associated eigenvector (mode shape), respectively. Also, *N* is the total degrees of freedom of the structure. The mode shapes are usually normalized with respect to the mass matrix. Hence, in this paper considers the problem for undamped symmetric systems with distinct eigenvalues. As the mass matrix *M* is non-singular, the eigenvectors are usually normalized as

$$\varphi_i^T M\varphi_i = 1 \tag{2}$$

Since the mode shape vectors are equivalent to nodal displacements of a vibrating structure, therefore in each element of the structure strain energy is stored [10]. The strain energy of a structure due to mode shape vector are usually referred to as modal strain energy (MSE) and can be considered as a valuable parameter for damage identification. The modal strain energy in *i*th mode of the structure can be expressed as

$$MSE_i = \frac{1}{2}\varphi_i^T K\varphi_i \tag{3}$$

Assume the global stiffness matrix is assembled by *m* individual element stiffness matrices and that is

$$K = \sum_{j=1}^{e} K_e \tag{4}$$

The eth element MSE can be then given by

$$MSE_i^e = \sum_{e=1} \frac{1}{2} \varphi_i^T K_e \varphi_i, \ \ i = 1, 2, ..., N \tag{5}$$

2-1- Damage localization by modal strain energy method

It is possible to use of Eq. (5) to detection of damage location. For computational propose, it is appropriate to normalized the MSE of elements with respect to the total MSE of the structure

$$NMSE_i^e = \left| \frac{\varphi_i^T K_e \varphi_i}{\sum \varphi_i^T K_e \varphi_i} \right| \tag{6}$$

where, NMSE is the normalized MSE of *e*th element in *i*th mode of the structure. Sometimes, achievement to complete modal data is impossible. Therefore, for only *m* identified modes the MSE can be rewritten as follow

$$MNMSE_i^e = \frac{\left| \sum_{i=1}^{m} NMSE_i^e \right|}{m} \tag{7}$$

The modal strain energy index of healthy and damaged structures as form $NMSE_h$ and $NMSE_d$ must be defined to detect the damage location, respectively. Generally, placing the mode shapes of healthy and damaged structures into Eq. (6), the corresponding MSE index for m modes are obtained. On the other hand, healthy modal strain energy index is similar to typical relationship for global modal strain energy method. The modal strain energy index for damaged structure is written as follow

$$MNSE_d = \sum_{e=1} \frac{1}{2} \bar{\varphi}_i^T \alpha K_e \bar{\varphi}_i, \ \ i = 1, 2, ..., m \tag{8}$$

where, $\bar{\varphi}$ and α denote the damaged mode shape and stiffness modification factor for damaged states, respectively. The damage occurrence is led to increasing the MSE and consequently the efficient parameter NMSE for m modes. As a result, an indicator is termed as general modal strain energy index (λ_{MSE}), which can be determined as

$$\lambda_{MSE} = \frac{MNMSE_d - MNMSE_h}{\left(MNMSE\right)_h} \tag{9}$$

It should be noted that, as the damage locations are unknown for the damaged structure with respect to real data applications, therefore for this case the element stiffness matrix of the healthy structure is used for estimating the parameter $(MNMSE)_d$. According to the Eq. (9), for a healthy element the index will be equal to zero (λ_{MSE}=0) and for a damaged element the index will be greater than zero (λ_{MSE}>0).

2-3- Damage quantification by modal strain energy sensitivity analysis

Design sensitivity analysis is used to quantify the relationship between parameters used to define an optimum design and calculate outputs used to measure their performance. Design sensitivity analysis of structural and mechanical systems with respect to structural design parameters plays a critical role in inverse and identification problems in engineering applications [11]. Generally sensitivity analysis describes the rates of change of some of key properties of the dynamic model such as natural frequencies and mode shapes with small changes in some of the physical properties consist of individual mass and stiffness matrices [12]. Therefore, derivatives of dynamic response of structures toward to physical properties are usually described the sensitivity analysis in the dynamic structures.

$$\frac{\partial MSE_{ei}}{\partial p} = \varphi_i^T K_e \frac{\partial \varphi_i}{\partial p} + \frac{1}{2}\varphi_i^T \frac{\partial K_e}{\partial p}\varphi_i \qquad (10)$$

The derivatives of eigenvalues with respect to the design variable *p* can be easily obtained by differentiation of the undamped eigenvalue, but the derivatives of mode shapes cannot be found directly due to it needs overcome the singular problem [11]. For dealing with these limitation to calculation of modal strain energy sensitivity, Yan and Ren [13] derived a compact analytical expression of the element MSE sensitivity based on the algebraic method. This method computes the sensitivity of element MSE using the following as

$$\frac{\partial MSE_{ei}}{\partial p} = \varphi_i^T K^* \varphi_i \qquad (11)$$

where

$$K^* = \begin{bmatrix} K_e & 0 \end{bmatrix} . \begin{bmatrix} K - \lambda_i M & -M\varphi_i \\ -\varphi_i^T M & 0 \end{bmatrix}^{-1} . \begin{bmatrix} \lambda_i \frac{\partial M}{\partial p} - \frac{\partial K}{\partial p} \\ \frac{1}{2}\varphi_i^T \frac{\partial M}{\partial p} \end{bmatrix} + \frac{1}{2}\frac{\partial K_e}{\partial p} \qquad (12)$$

As it can be noted, this method is an accurate method, which only requires the eigenvector of interest. And it can be found the design sensitivity of element MSE in a very simple and straightforward manner. In this study, damage is assumed to be directly related to a decrease in stiffness. Therefore, damage can be located using the sensitivity of the modal strain energy with respect to the stiffness parameters. Hence, with neglecting of the mass matrix modification, the Eq. (12) is rewritten to form

$$K^* = \begin{bmatrix} K_e & 0 \end{bmatrix} . \begin{bmatrix} K - \lambda_i M & -M\varphi_i \\ -\varphi_i^T M & 0 \end{bmatrix}^{-1} . \begin{bmatrix} -\frac{\partial K}{\partial p} \\ 0 \end{bmatrix} + \frac{1}{2}\frac{\partial K_e}{\partial p} \qquad (13)$$

Once the sensitivity of modal strain energy are computed, the sensitivity matrix is build and the change in the stiffness parameters is estimated by minimizing of the optimization function,

$$J = \left(S_{MSE}\Delta k - \Delta\varphi\right)^T W_{\varepsilon\varepsilon}\left(S_{MSE}\Delta k - \Delta\varphi\right) + \Delta k^T W_{kk}\Delta k \qquad (14)$$

S_{MSE} is the modal strain energy sensitivity matrix, which can be described as follow

$$S_{MSE} = \varphi_i^T\left(\begin{bmatrix}K_e & 0\end{bmatrix}.\begin{bmatrix}K-\lambda_i M & -M\varphi_i \\ -\varphi_i^T M & 0\end{bmatrix}^{-1}.\begin{bmatrix}-\dfrac{\partial K}{\partial p} \\ 0\end{bmatrix} + \frac{1}{2}\frac{\partial K_e}{\partial p}\right)\varphi_i \qquad (15)$$

$\Delta\varphi$ is the error vector containing the differences in mode shapes before and after damage. $W_{\varepsilon\varepsilon}$, W_{kk} are positive definite weighting matrices. $W_{\varepsilon\varepsilon}$ is a diagonal matrix whose elements are given by the reciprocals of the variance of the corresponding measurements. $W_{kk}=\alpha I$ is a diagonal matrix whose elements are equal to the regularization parameter α (Tikonov regularization). A detailed explanation of the derivation of this equation is found in the book of Friswell and Mottershead [14]. The solution of equation (14) is obtained through least squares as follow,

$$\{\Delta k\} = \left[S_{MSE}^T W_{\varepsilon\varepsilon} S_{MSE} + W_{kk}\right]^{-1} S_{MSE}^T W_{\varepsilon\varepsilon}\Delta\varphi \qquad (16)$$

where, Δk is the damage quantification based on modal strain energy sensitivity method. In the numerical evaluation, the damage detection process is usually carried out to induce the damage index and according to proposed method, predicated damage index can be estimated as stiffness reduction.

3. Application

3-1- A cantilever beam

In this section, the damage detection and damage severity models describes in the preceding section used to identify the location and determine the magnitude of reduction of stiffness on a cantilever beam. The beam has been shown in Fig. 1. The length, thickness and width of the beam are 1.20, 0.05 and 0.1 m, respectively. The mass density is 7850 kg/m3 and the elasticity modulus is 210 GPa. In this example, the first 5 vibrating modes are used for identifying the damage. Therefore, consider the incomplete modal data are available.

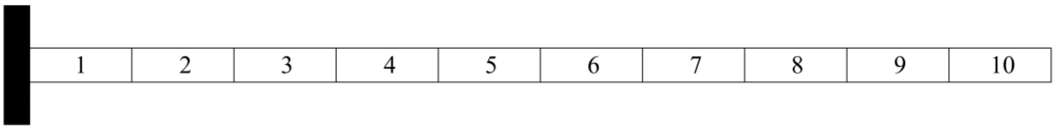


Figure 1. A cantilever beam

The finite element analysis is carried out to simulate the modal data, using two-node beam elements [15]. Here, four damage cases are assumed to investigate the capabilities of the proposed methods in detection of the occurred damage of a flexural structure. In the first damage case, the stiffness of element 2 was decreased by 30%. In damage case number two, the stiffness of elements 5 reduced by 40%. In the third damage case, the stiffness of element

2 and 5 decreased via 20% and 30%, respectively. Finally, in the damage case number four, the stiffness of element 8 reduced by 25%.

Based on the proposed damage assessment algorithms, location of induced damages is detected by modal strain energy index from Eq. (9). Corresponding to damage quantification, sensitivity of MSE is firstly determined to specify the changes of dynamic behaviour. Subsequently, difference of eigenvectors (mode shapes) between healthy and damaged structures are computed. Eventually, using of Eq. (16) the vectors of damage parameters will be estimated.

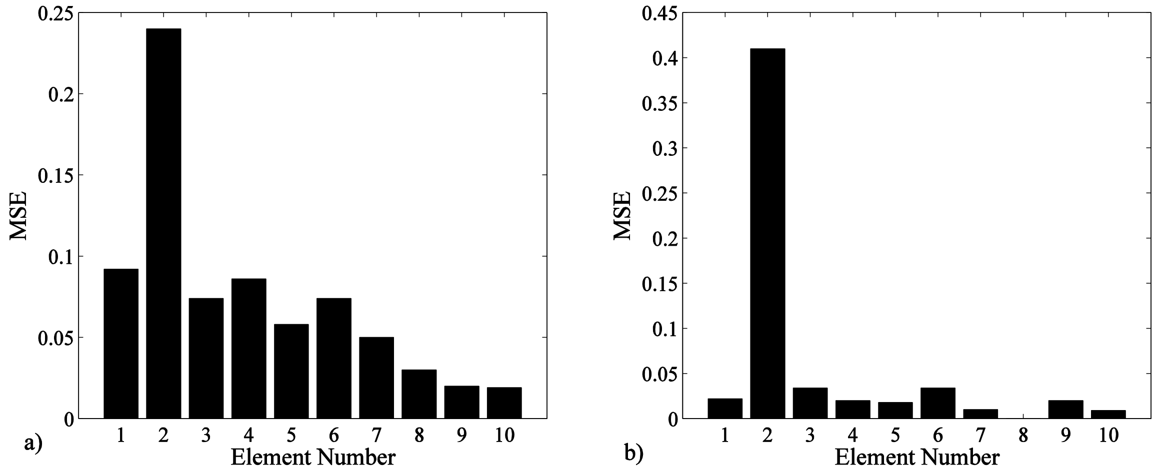


Figure 2. Damage localization of the cantilever beam in scenario 1, a) For 3 identified modes, b) For 10 identified modes

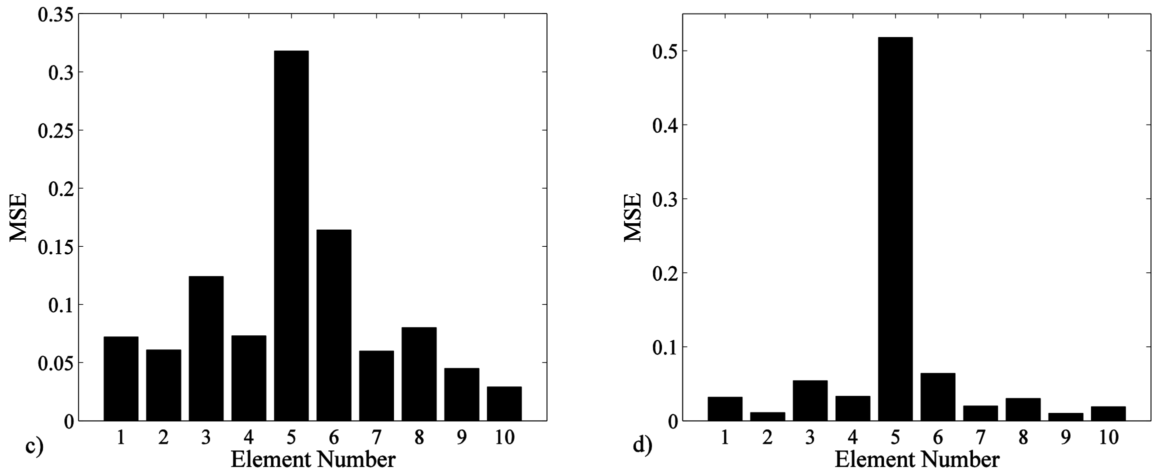


Figure 3. Damage localization of the cantilever beam in scenario 2, c) For 3 identified modes, d) For 10 identified modes

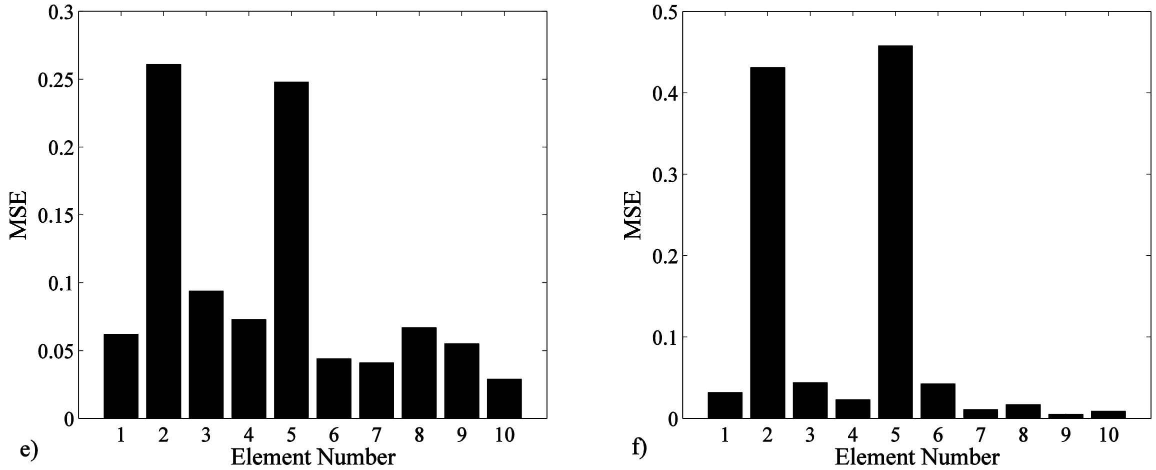


Figure 4. Damage localization of the cantilever beam in scenario 3, e) For 3 identified modes, f) For 10 identified modes

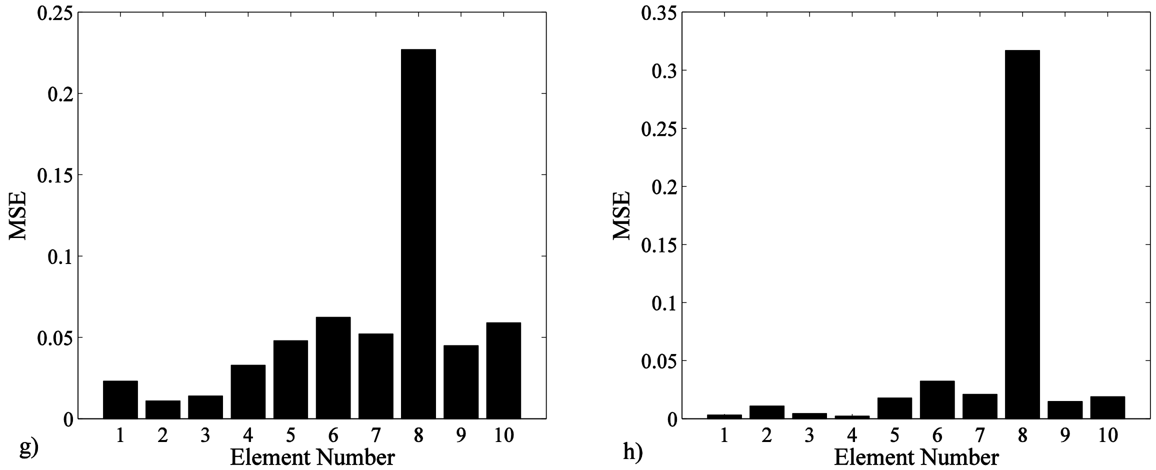


Figure 5. Damage localization of the cantilever beam in scenario 4, g) For 3 identified modes, h) For 10 identified modes

It can be observed that the modal strain energy index achieves to true location of induced damage cases, even for multiple damage case and limitation of identified modes. According to Figs. 2-5, the location of induced damage is precisely detected, while 10 identified modes have better results than 3 identified modes. The error function for undamaged element in each damage cases is inconsiderable values and according to all Figs. 2-5, the peaks of damaged element are clearly demonstrated to damage localization. Also, Figs. 6-9, illustrate the extent of damage cases based on sensitivity modal strain energy and optimization function.

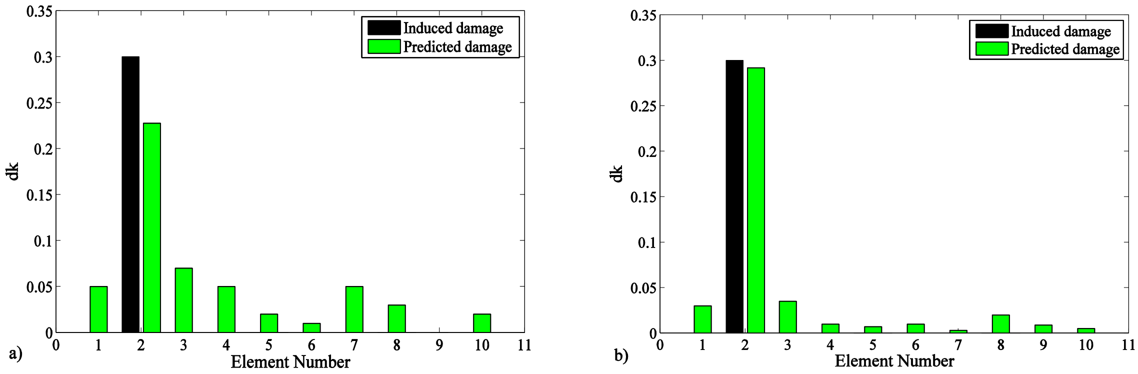


Figure 6. Damage quantification of the cantilever beam in scenario 1, a) For 3 identified modes, b) For 10 identified modes

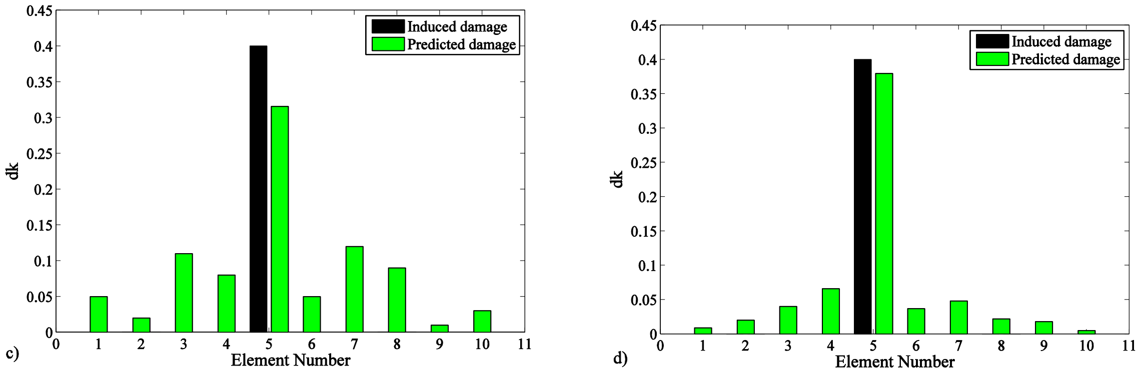


Figure 7. Damage quantification of the cantilever beam in scenario 2, a) For 3 identified modes, b) For 10 identified modes

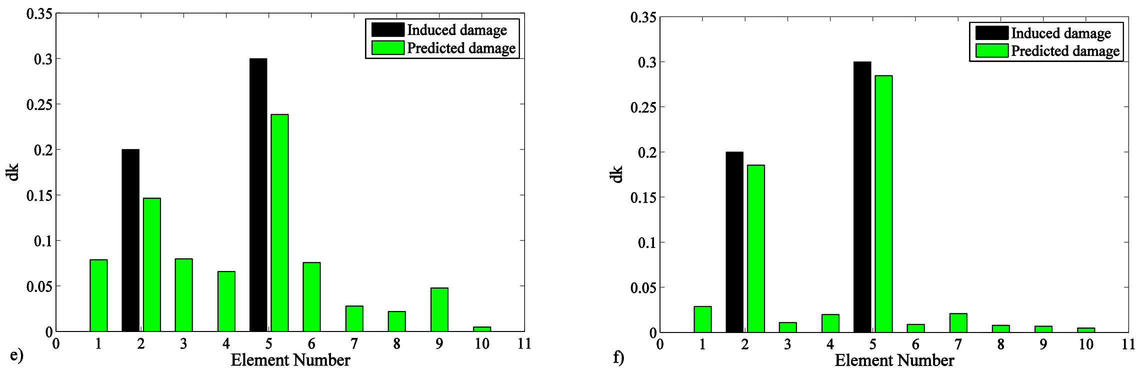
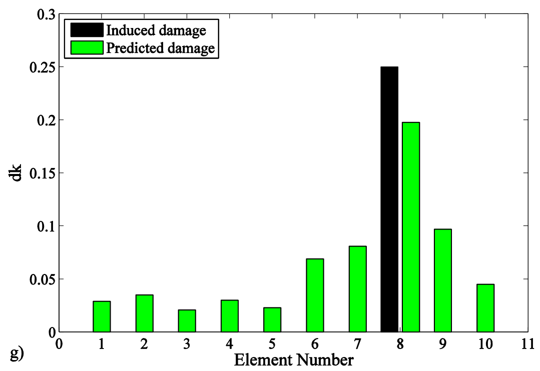


Figure 8. Damage quantification of the cantilever beam in scenario 3, a) For 3 identified modes, b) For 10 identified modes



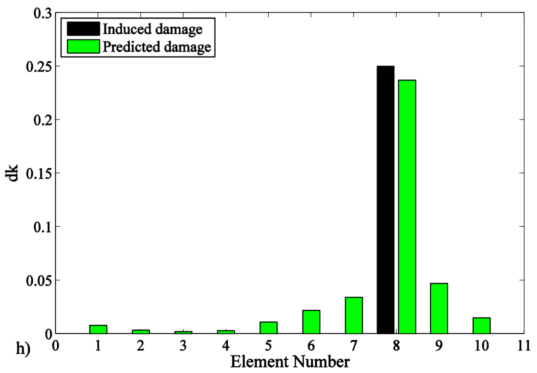


Figure 9. Damage quantification of the cantilever beam in scenario 4, a) For 3 identified modes, b) For 10 identified modes

As can be seen, extent of damage cases were estimated for 3 and 10 identified modes, respectively. The figures belong to three identified modes, error function of predicted damage than induce damage has considerable values, and hence the damage quantifications do not validate based on numerical results. Whereas, for ten identified modes, error function of predicted have better results and close to induced damage. As a result, whatever the number of modes increases, more accurate results can be achieved.

3-2- A 3-story experimental frame

For experimental evaluation, the 3-story laboratory frame used in this study was approximately 2.1 m high and constructed from equal angel aluminum column sections and steel floor plates bolted together with aluminum brackets as shown in Fig. 10. The steel floor plates were 4 mm thick and 650 mm×650 mm square. Hence, each story has 1.3532 kg weight. The column sections at each story were 30 mm×30 mm equal angles. Two section thicknesses were used for the columns, either 4.5 mm or 3 mm, for the undamaged and damaged states, respectively. Each column was made of 3×0.7 m high segments, rather than one long angle, in order to make them easily replaceable for simulation of localized damage at difference stories. Based on actual properties of components of each story, stuffiness values including of 40099 N/m, 9891 N/m and 9078 N/m for the first, second and third stories, respectively. The proposed damage localization approach was accomplished on three-story laboratory frame by experimental modal analysis. A modal analysis with impact hammer is performed in the undamaged state. Table (1) and (2) indicate the extracted modal parameters of healthy frame.

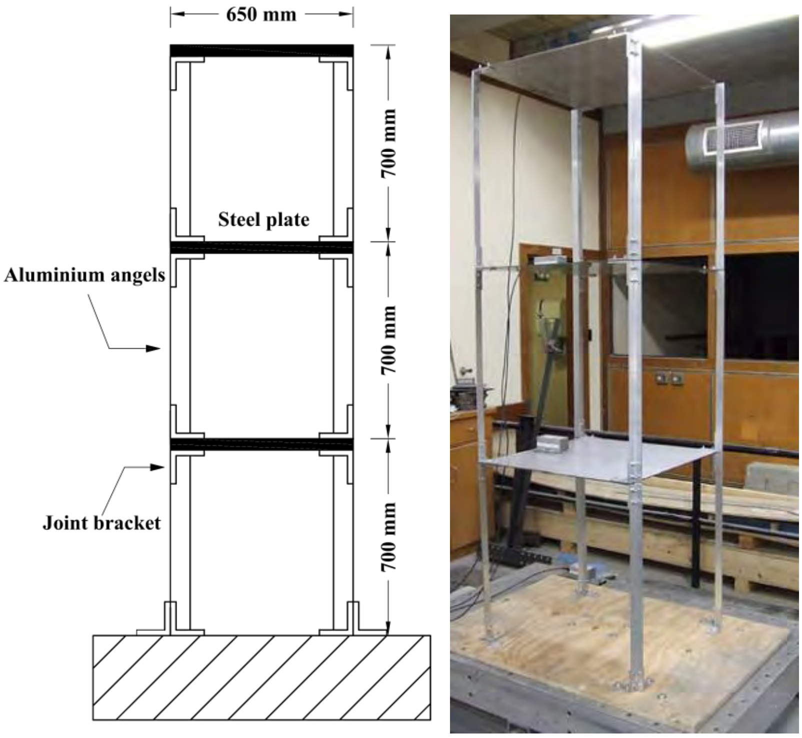


Figure 10. The 3-story laboratory frame

Table 1. Experimental mode shapes of 3-story laboratory frame

[φ]	Mode 1	Mode 2	Mode 3
Story 1	0.0896	0.2131	0.6338
Story 2	0.4215	0.5664	-0.2320
Story 3	0.6309	-0.4463	0.0566

Table 2. Experimental natural frequency of 3-story laboratory frame (Hz)

Degree of freedoms	Mode 1	Mode 2	Mode 3
Natural frequency	1.93	5.52	8.55

According to identifying of modal parameters of healthy structures, two damage cases were introduced and experimental modal analysis was separately carried out for all cases of damage in the three-story laboratory frame. In the first damage case, the aluminum angels with 30 mm×30 mm and thickness of 3mm were replaced rather than the columns of first story. In third damage case, the columns of third story as well as the first story were replaced with columns similar to damage case number one. Therefore, changes of dynamic behaviour of laboratory frame can be illustrated based on Table 3.

Table 3. Natural frequencies evaluation in the damage cases for 3-story laboratory frame (Hz)

Damage cases	Mode 1	Mode 2	Mode 3

Case 1	1.82	4.93	7.78
Case 2	1.70	4.72	7.51

In the first stage of identifying the damage induced, the damage localization for complete modal data by indicator modal strain energy index is detected. Figs. 11a-b show the values of MSE for damage localization. In can be seen, in the experimental evaluation, the location of damage is also predicted based on identified modal data. In the other words, the highest columns of charts show the damage location.

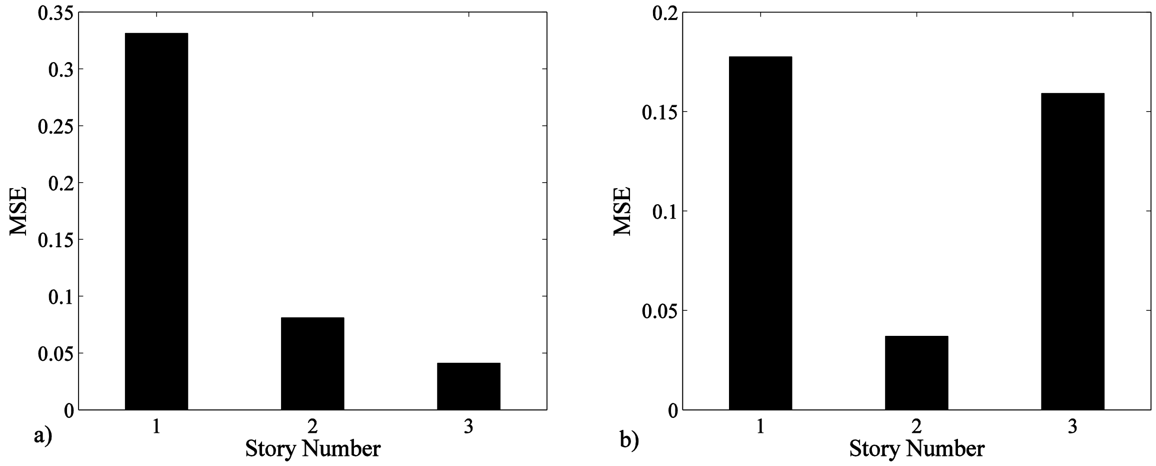


Figure 11. Damage localization of experimental frame, a) Damage case 1, b) Damage case 2

The damage parameter Δk, based on Eq. (16) and identified experimental modal data has been estimated. As mentioned before, the induced damage in the experimental evaluation, imposed to laboratory frame by replacing the columns of selected stories. Therefore, for representation of damage severity the changed stiffness matrix of damaged structures will be compared to initial stiffness quantities of healthy frame. Generally, stiffness matrix of damaged frame is calculated by summation of stiffness damage parameters as well as stiffness matrix of healthy frame as $K_d=K_h- \Delta k$.

Table 4. Damage severity assessment by comparison of stiffness components

Damage cases	Stiffness of the 3–story laboratory frame (N/m)		
	1st	2nd	3rd
Healthy	40099	9891	9087
Damage case 1	33089	9459	8764
Damage case 2	31646	9267	7663

As can be seen, using of degraded material and sections in the laboratory frame is led to reduction of stiffness values as well as adversely dynamic behaviour of structure. Also, the effect of reduction in damage case 2 is larger than case 1. Furthermore, in the all damage cases, stiffness of second story (healthy story) has been slowly decreased. As a result, connection in each story has great influence to distribute the damage effects.

4. Conclusion

A method to detect and locate damage based on sensitivity of modal strain energy has been implemented. The proposed method for damage localization utilizes the modal strain energy indictor (MSE) of healthy and damaged structures. For damage quantification, the sensitivity of modal strain energy was determined at first and then, the model updating optimization function as well as the error vectors containing of differences of mode shapes were used to estimate changes of stiffness components as damage index. The approaches were verified with two examples as numerical investigation on the cantilever beam and experimental evaluation on the 3-story laboratory frame. In the numerical model, results were compared with those obtained with the numerical induced damage index, when incomplete modal data were present. In contrast, in the experimental model, replacing of column of laboratory frame was introduced the damage index. Therefore, in this state, results were determined based on comparison of stiffness components of damaged frame with corresponding healthy frame. Consequently, numerical and experimental results show that the proposed methods can accurately detect the damage location and severity.

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