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Z-interference channel with side information at the transmitters

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ABSTRACT

In this paper, channel coding strategies for the problem of communication over a two-user Z-interference channel (ZIC) in the presence of non-causal side information (NSI) at transmitters are presented. First, an achievable rate region for the general discrete memoryless ZIC-NSI is derived using rate splitting technique, superposition coding, Gel'fand–Pinsker-type binning scheme and joint typicality decoding. Next, the capacity region of a special case of the orthogonal ZIC-NSI is characterized. Then, the Gaussian version of the channel called the Gaussian Doubly Dirty ZIC (GDD-ZIC) is defined and a general outer bound to the capacity region of the GDD-ZIC is presented. After that, an inner bound for the sum-capacity of the channel using Costa's dirty paper coding is derived and thereby it is shown that Costa's strategy cannot achieve any positive rates over the GDD-ZIC for the case of strong state-interference. Finally, lattice-based achievable rates for the GDD-ZIC are obtained. These lattice-based rates are independent of the state-interference powers and thus, lattice-strategies are able to achieve positive rates for the case of strong state-interference. Moreover, it is shown that lattice-strategies are optimal for a class of the GDD-ZIC.

1. Introduction

The Z-interference channel (ZIC) is a special case of the two-user interference channel (IC) in which only one of the transmitter–receiver Q3 pairs suffers from signal-interference caused by the other transmitter–receiver pair (see Fig. 1). In other words, the ZIC has three communication links: two principal links and one interfering link. The study of the ZIC is motivated by the applications raised in some wireless communication networks such as cellular networks, wireless sensor networks and ad-hoc wireless networks. The capacity region of the general ZIC is still unknown even for the general Gaussian case. However, the capacity regions are known in some special cases, e.g., for the case that the interference is deterministic [1]. More studies on the ZIC can be found in Refs. [2–5]. It is worth mentioning that the Z channel (ZC), introduced in Ref. [6], is a three-link communication system with the same topology as the ZIC but with this difference that all three links in the ZC are principal.

Having a priori knowledge of the channel, as side information (SI) at the transmitter, has a positive impact on achieving reliable communication with high data rates. For instance, Costa in his celebrated paper [7] showed that in a single-user channel with additive white Gaussian noise (AWGN) and additive white Gaussian interference, if transmitter knows the sequence of interference symbols non-causally, then it can cancel the negative effect of the interference completely using Gaussian random binning technique and achieve the capacity of the interference-free AWGN channel. The binning schemes are the most widely used coding schemes for the problem of channel coding in the presence of non-causal SI (NSI) at the transmitter. For discrete memoryless channels with finite-alphabet inputs and outputs, the random binning techniques such as Gel'fand–Pinsker (GP) binning [8] have been used frequently for both single-user and multi-user channels (e.g., see Refs. [8–12] and references therein).

The power constrained AWGN channels with known additive interference at the transmitter are special case of channels with NSI at the transmitter which are known as dirty paper channels. Two famous coding strategies to mitigate the interference effect in dirty paper channels are (1) Costa's strategy (Gaussian random binning) [7,19], and (2) lattice strategies (structured binning) [13–17]. The question of when structured binning outperforms random binning or at least has the same performance as random binning is an open problem. However, it is shown that dirty paper channels are instances wherein lattice codes can achieve the same rates as Costa's strategy and even

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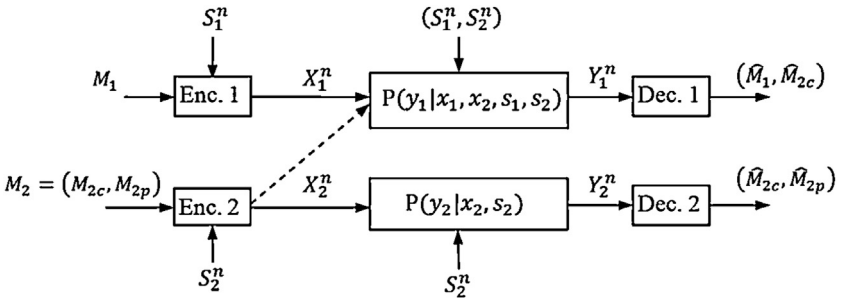

Fig. 1. Z-interference channel with side information non-causally available at the transmitters.

can beat it [13–17]. For instance, in dirty paper channels with a common interference such as the Costa's channel [7,14] or the Gaussian MAC with common interference [13,18], Costa's strategy and good lattice codes have similar capacity performance. In contrast, it is shown that in some dirty paper channels with different interferences known at different transmitters such as the Gaussian doubly dirty MAC (GDD-MAC) [13], the Gaussian triply dirty MAC (GTD-MAC) [16] and the Gaussian doubly dirty compound MAC [17], lattice strategies are better than Costa's strategy such that they are able to achieve positive rates in the strong interference conditions while Costa's strategy is not able to do this. Since we commonly consider a linear model for Gaussian channels with additive Gaussian noises and interferences and also, as lattice codes have linear structure, it seems that achieving better results for such channels by using good lattice codes is reasonable.

In this paper, we study the two-user Z-interference channel with side information S_1 and S_2 where the side information S_i, $i = 1,2$, is non-causally available only at the transmitter i (ZCI-NSI). Such setting may be motivated, for example, by a cellular communication scenario with two interfering cells as depicted in Fig. 2, where two neighboring base-stations 1 and 2, being non-causally aware of S_1 and S_2, want to communicate to users 1 and 2, respectively. Side information sequences S_1^n and S_2^n can be considered as known noises (or interferences) at base-stations 1 and 2, respectively [20]. As shown in Fig. 2, user 1 (R_{x_1}), located in the overlap area of two cells, suffers from interference caused by the signal transmitted by (non-intended) base-station 2 whereas user 2 (R_{x_2}) does not suffer from such interference because of locating far from the (non-intended) base-station 1. A similar scenario has been studied previously in Ref. [21], where authors have considered a state-dependent Z channel with a *common state* which is non-causally known to both transmitters and therefore, it can be called the *single dirty ZC*. The main contributions of Ref. [21] are (i) to derive an achievable rate based on a two-layer binning scheme for the *discrete* case of the single dirty ZC and (ii) to present a lattice-based achievable rate for a degraded version of the *Gaussian* single dirty ZC that as we will describe in the following the results presented in Ref. [21] is different from our work. Moreover, a state-dependent Gaussian ZIC with a *mismatched* receiver-side state interference and transmitter-side state cognition has been studied in Ref. [22], where a state that corrupts only the received signal at receiver 2 is known non-causally only at the transmitter 1 while the intended transmitter 2 is unaware of it, hence the problem investigated in the current work is completely different to the work done in Ref. [22].

1.1. Contributions and outline

The outline of the paper and our contributions are as follows.

(1) In Section 2, we define the general ZIC-NSI.
(2) In Section 3, we first derive a capacity inner bound for the general discrete memoryless ZIC with correlated SI known non-causally at the transmitters by making use of rate splitting technique, superposition coding, GP-type binning scheme and jointly decoding. Then, by defining a subclass of the ZIC-NSI called orthogonal ZIC-NSI we obtain the capacity region of a special case of the orthogonal ZIC-NSI.

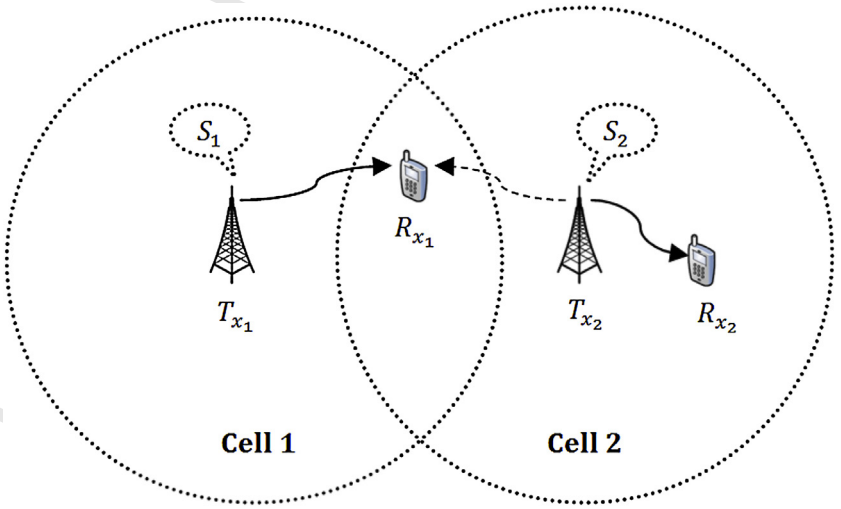

Fig. 2. A two-cell downlink communication example that can be modeled as a ZIC with non-causal side information at transmitters.

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(3) In Section 4, we first define the Gaussian ZIC with two additive state-interferences, each known non-causally at one of the transmitters only, dubbed the Gaussian Doubly Dirty ZIC (GDD-ZIC). Note that to distinguish between interference caused by transmitted signal and interference caused by state we use signal-interference to denote the former and state-interference for the latter. We then present a general outer bound to the capacity region of the GDD-ZIC and specialize it for the strong state-interference regime. Finally, we obtain an inner bound for the sum-capacity of the channel using Costa's strategy and we show that when both state-interferences are strong, the obtained achievable sum-rate vanishes like the GDD-MAC [13]. This means that in the GDD-ZIC, positive rates cannot be achieved by Costa's strategy for the case of strong state-interference.

(4) In Section 5, after giving a brief review of the required key concepts of the lattice theory, we obtain lattice-based achievable rates for the GDD-ZIC which are independent of the state-interference powers and thus, do not vanish for the case of strong state-interference. Specifically, because of the central role of the gap between signal-to-noise-ratio (SNR_1) and (signal-)interference-to-noise-ratio (INR_1) at the first receiver in the capacity analysis, we first group all possible values of the gap between SNR_1 and INR_1 into two main groups: imbalanced (large gap) and nearly balanced. Then, we derive achievable rates for these two cases and show that lattice-strategies are optimal for a class of the GDD-ZIC.

Throughout the paper, the following notations are used. Random variables and their realizations are denoted by uppercase (e.g., X) and lowercase letters (e.g., x), respectively. The set of nonnegative real numbers is denoted by $\mathbb{R}_+$ and n-sequence $(X_1, X_2, \ldots, X_n)$ is denoted by X^n or $\mathbf{X}$, interchangeably. |||| denotes the Euclidean norm, $u.c.e\{\cdot\}$ indicates the upper convex envelope and $[x]^+ \triangleq \max(0, x)$.

2. Channel model

Definition 1. A 2-user discrete memoryless Z-interference channel (DM-ZIC) with correlated SI (Fig. 1), denoted by $\{\mathcal{S}_1, \mathcal{S}_2, P(s_1, s_2), \mathcal{X}_1, \mathcal{X}_2, P(y_1, y_2|x_1, x_2, s_1, s_2), \mathcal{Y}_1, \mathcal{Y}_2\}$, consists of six finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{S}_1, \mathcal{S}_2, \mathcal{Y}_1, \mathcal{Y}_2$, a joint probability $P(s_1, s_2)$, and a transition probability function $P(y_1, y_2|x_1, x_2, s_1, s_2)$ with two conditional marginal distribution functions $P(y_1|x_1, x_2, s_1, s_2)$ and $P(y_2|x_2, s_2)$. The SI and channel are assumed to be memoryless.

In the ZIC with two correlated SI non-causally available at the encoders, transmitter i, $i = 1, 2$, being non-causally aware of SI sequence s_i^n, wishes to send a message $M_i \in \mathcal{M}_i = \{1, \cdots, 2^{nR_i}\}$ to receiver i over the channel (Fig. 1). The messages M_1 and M_2 are assumed to be independently and uniformly generated over $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively.

A $(2^{nR_1}, 2^{nR_2}, n, \varepsilon)$ code for the ZIC-NSI consists of two message sets $\mathcal{M}_1, \mathcal{M}_2$, two encoding functions $f_i : \mathcal{M}_i \times \mathcal{S}_i^n \to \mathcal{X}_i^n$, $i = 1, 2$, and two decoding functions $g_i : \mathcal{Y}_i^n \to \mathcal{M}_i$, $i = 1, 2$, such that the probability of decoding error is not larger than ε, where the probability of decoding error is defined as

$$P_e^{(n)} = \{g_1(y_1^n) \neq M_1 \text{ or } g_2(y_2^n) \neq M_2 | (M_1, M_2) \text{ is sent}\}$$

A pair (R_1, R_2) of nonnegative real values is called an achievable rate for the ZIC-NSI if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n, \varepsilon)$ codes. The capacity region of the ZIC-NSI is the closure of the set of all achievable rates.

3. Discrete memoryless Z-interference channel with non-causal side information

In this section, we first derive an achievable rate region for the general DM-ZIC-NSI. Then, by defining a subclass of the ZIC-NSI called orthogonal ZIC-NSI we obtain the capacity region of a special case of the orthogonal ZIC-NSI.

In the ZIC-NSI shown in Fig. 1, due to the interfering link from transmitter 2 to receiver 1, the first receiver may be able to partially decode the message sent by the second transmitter and thereby it can reduce the effective interference and improve its performance. Therefore, we use rate splitting technique to split the message at transmitter 2 into two sub-messages: (i) the crossly observable (or common) sub-message (M_{2c}) which is decodable by both decoders and (ii) the crossly non-observable (or private) sub-message (M_{2p}) which is decodable only by the second decoder. We use auxiliary random variables U_{2c} and U_{2p} to encode sub-messages M_{2c} and M_{2p}, respectively, and also random variable U_1 to encode message M_1. It is worth noting that, by using rate splitting technique the second transmitter sees a two-receiver broadcast channel (BC) with degraded message sets.

For the DM-ZIC-NSI (Fig. 1), let $\mathcal{P}_{\text{ZIC-NSI}}$ be the set of all distributions of the form (1).

$$p(s_1, s_2, u_1, u_{2c}, u_{2p}, x_1, x_2, y_1, y_2) = p(s_1, s_2)p(x_1, u_1|s_1)p(x_2, u_{2p}, u_{2c}|s_2)p(y_1|x_1, x_2, s_1, s_2)\, p(y_2|x_2, s_2). \quad (1)$$

By (1), the following Markov chains hold:

$$(U_1, X_1) \leftrightarrow S_1 \leftrightarrow S_2 \leftrightarrow (U_{2c}, U_{2p}, X_2) \quad (2)$$

$$(U_1, U_{2c}, U_{2p}, Y_2) \leftrightarrow (X_1, X_2, S_1, S_2) \leftrightarrow Y_1 \quad (3)$$

$$(S_1, U_1, U_{2c}, U_{2p}, X_1, Y_1) \leftrightarrow (X_2, S_2) \leftrightarrow Y_2 \quad (4)$$

Define $\mathcal{R}_{\text{ZIC-NSI}}(p)$ as the set of all non-negative triples (R_1, R_{2c}, R_{2p}) satisfying

$$R_1 \leq I(Y_1; U_1|U_{2c}) - I(S_1; U_1|U_{2c}) = A \quad (5)$$

$$R_{2c} \leq I(Y_1; U_{2c}|U_1) - I(S_2; U_{2c}|U_1) = B \quad (6)$$

$$R_1 + R_{2c} \leq I(Y_1; U_1, U_{2c}) - I(S_1, S_2; U_1, U_{2c}) = C \quad (7)$$

$$R_{2p} \leq I(Y_2; U_{2p}|U_{2c}) - I(S_2; U_{2p}|U_{2c}) = D \quad (8)$$

$$R_{2c} + R_{2p} \leq I(Y_2; U_{2c}, U_{2p}) - I(S_2; U_{2c}, U_{2p}) = E \quad (9)$$

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for some $p \in \mathcal{P}_{\text{ZIC-NSI}}$.

Theorem 1. *The closure of the convex hull of the set $\mathcal{R}_{\text{ZIC-NSI}}$ is achievable for the discrete memoryless Z-interference channel with two correlated SI, each known non-causally to one of the encoders (Fig. 1), where*

$$\mathcal{R}_{\text{ZIC-NSI}} = \bigcup_{p \in \wp_{\text{ZIC-NSI}}} \mathcal{R}_{\text{ZIC-NSI}}(p) \tag{10}$$

Proof. For brevity, we here give only an outline of the proof. Encoder 1 applies a GP-like binning scheme against s_1^n to generate u_1^n for encoding message M_1 coordinated with s_1^n. As we mentioned above, encoder 2 first splits the message M_2 to two sub-messages M_{2c} and M_{2p} with rates R_{2c} and R_{2p}, respectively, such that $M_2 = (M_{2c}, M_{2p})$ and $R_2 = R_{2c} + R_{2p}$. Next, encoder 2 encodes the sub-message M_{2c} into codeword u_{2c}^n by applying GP-like binning scheme against s_2^n. Then, it encodes the sub-message M_{2p} into codeword u_{2p}^n superimposed onto the u_{2c}^n by applying GP-like binning scheme against s_2^n. Note that by splitting the message M_2 to two sub-messages and letting M_{2c} be decoded at the first decoder, receiver 1 is able to reduce the effective interference and therefore, the rate R_1 is boosted. Finally, the rate region $\mathcal{R}_{\text{ZIC-NSI}}$ is achieved by joint typicality decoding [11].

Remark 1. In the absence of receiver 2 and setting $U_{2p} = \emptyset$, the achievable rate region $\mathcal{R}_{\text{ZIC-NSI}}$ reduces to the achievable rate region of the MAC with (non-causal) correlated SI (MAC-NSI).

By applying the Fourier–Motzkin elimination [23] to the implicit achievable rate region $\mathcal{R}_{\text{ZIC-NSI}}$, we now obtain an explicit description of it in terms of rate pair (R_1, R_2), denoted by $\mathcal{R}^*_{\text{ZIC-NSI}}$.

Define $\mathcal{R}^*_{\text{ZIC-NSI}}(p)$ as the set of all non-negative rate pairs (R_1, R_2) satisfying

$$R_1 \le \min\{A, C\} \tag{11}$$

$$R_2 \le \min\{B + D, E\} \tag{12}$$

$$R_1 + R_2 \le C + D \tag{13}$$

for some $p \in \mathcal{P}_{\text{ZIC-NSI}}$, where the bound constants A–E are the same as (5)–(9)

Theorem 2. *The closure of the convex hull of the set $\mathcal{R}^*_{\text{ZIC-NSI}}$ is achievable for the discrete memoryless ZIC-NSI where*

$$\mathcal{R}^*_{\text{ZIC-NSI}} = \bigcup_{p \in \mathcal{P}_{\text{ZIC-NSI}}} \mathcal{R}^*_{\text{ZIC-NSI}}(p) \tag{14}$$

Proof. Considering (5)–(9), $R_{2c} \ge 0$, $R_{2p} \ge 0$, $R_2 = R_{2c} + R_{2p}$ and applying the Fourier–Motzkin elimination technique [23] to them, Theorem 2 is proved.

Since the capacity region of the MAC with two SI, each available non-causally at one of the encoders, is unknown in general, the capacity region of the ZIC-NSI is unknown in general. Due to this, we focus here on a special case of the ZIC-NSI called *orthogonal ZIC-NSI*.

Definition 2. The two-user ZIC-NSI is said to be orthogonal if the first receiver alphabet $\mathcal{Y}_1 = \mathcal{Y}_{11} \times \mathcal{Y}_{12}$ and $p(y_1|x_1, x_2, s_1, s_2) = p(y_{11}|x_1, s_1)p(y_{12}|x_2, s_2)$.

Definition 3. For a two-receiver BC $(X \to (Y_1, Y_2))$ with degraded message sets and without SI

(i) if $I(Y_1; U_c) \ge I(Y_2; U_c)$ for all $U_c \to X \to (Y_1, Y_2)$, then it is said that the receiver Y_1 is less noisy than the receiver Y_2 [24],
(ii) the conditions $I(Y_1; U_c) \ge I(Y_2; U_c)$ and $I(Y_1; X) \le I(Y_2; X)$ can be held simultaneously [25].

Theorem 3. *The capacity region of the discrete memoryless orthogonal Z-interference channel with two independent SI, each known non-causally to one of the encoders, is given by*

$$C^{*\text{-ortho}}_{\text{ZIC-NSI}} = \bigcup_{\substack{p(x_1,u_1,s_1)p(x_2,u_{2p},u_{2c},s_2) \\ \text{subjectto}: I(Y_{12};U_{2c}) \ge I(Y_2;U_{2c})}} \left\{ \begin{array}{l} (R_1, R_2) \in \mathbb{R}_+^2 : \\ R_1 \le I(Y_{11}; U_1) - I(S_1; U_1) \\ R_2 \le I(Y_2; U_{2c}, U_{2p}) - I(S_2; U_{2c}, U_{2p}) \end{array} \right\} \tag{15}$$

Proof. The converse part uses the converse proof of the Gel'fand–Pinsker Theorem [8] twice. In addition, for the orthogonal ZIC-NSI when S_1 and S_2 are independent, it is easy to verify that the inequalities (11)–(13) boil down to

$$R_1 \le I(Y_{11}; U_1) - I(S_1; U_1) \tag{16}$$

$$R_2 \le \min\{I(Y_{12}; U_{2c}), I(Y_2; U_{2c})\} + I(Y_2; U_{2p}|U_{2c}) - I(S_2; U_{2c}, U_{2p}) \tag{17}$$

Therefore, the achievability proof is completed by applying the constraint $I(Y_{12}; U_{2c}) \ge I(Y_2; U_{2c})$.

As seen in Theorem 3, when S_1 and S_2 are independent the channel is indeed decomposed into two separate sub-channels. This is no longer true for the case where S_1 and S_2 are correlated and therefore, determining the capacity region is still difficult for the general case.

4. Gaussian Z-interference channel with additive interference

In this section, we first define the Gaussian Doubly Dirty ZIC and then present an outer bound for it. Finally, we obtain an inner bound for the sum-capacity of the channel using Costa's strategy and thereby we show that, similar to the GDD-MAC [13], when both state-interferences are strong, the obtained inner bound vanishes.

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4.1. Gaussian channel model

The two-user Gaussian ZIC with additive state-interference is modeled as

$$Y_1 = h_{11}X_1 + h_{12}X_2 + g_{11}S_1 + g_{12}S_2 + Z_1 \tag{18}$$

$$Y_2 = h_{22}X_2 + g_{22}S_2 + Z_2 \tag{19}$$

where coefficients h_r, g_r, $r \in \{11, 12, 22\}$, are assumed to be static real numbers, the channel input X_i, $i = 1, 2$, is subjected to the power constraint $1/n\sum_{t=1}^{n} X_{i,t}^2 \le P_i$, the channel noise Z_i is an AWGN as $Z_i \sim \mathcal{N}(0, N_i)$ and is independent of $Z_{3-i}, S_1, S_2, X_1, X_2$, the state-interference sequence $\mathbf{S}_i$ which is known non-causally only at the transmitter i is assumed to be i.i.d. Gaussian as $S_i \sim \mathcal{N}(0, Q_i)$. We also assume that state-interferences S_1 and S_2 are independent. We call this channel the *Gaussian Doubly Dirty ZIC (GDD-ZIC)*.

In a two-user GDD-ZIC, the signal-to-noise and (signal-)interference-to-noise ratios are defined as: $\mathrm{SNR}_1 = h_{11}^2 P_1/N_1$, $\mathrm{INR}_1 = h_{12}^2 P_2/N_1$ and $\mathrm{SNR}_2 = h_{22}^2 P_2/N_2$. Depending on the values of INR_1 and SNR_2, the two-user GDD-ZIC can be classified into the following two cases:

(i) if $\mathrm{INR}_1 \ge \mathrm{SNR}_2$ or equivalently $h_{12}^2/h_{22}^2 \ge N_1/N_2$, the channel is a GDD-ZIC with strong signal-interference,
(ii) if $\mathrm{INR}_1 < \mathrm{SNR}_2$ or equivalently $h_{12}^2/h_{22}^2 < N_1/N_2$, the channel is a GDD-ZIC with weak signal-interference.

4.2. Outer bound

Theorem 4. *An outer bound on the capacity region of the Gaussian Doubly Dirty ZIC defined by (18) and (19) consists of rate pairs (R_1, R_2) satisfying*

$$R_2 \le \frac{1}{2}\log\left(1 + \frac{h_{22}^2 P_2}{N_2}\right) \tag{20}$$

$$R_1 + R_2 \le \min\left\{\frac{1}{2}\log\left(\frac{\left(h_{11}\sqrt{P_1} + h_{12}\sqrt{P_2} + g_{11}\sqrt{Q_1}\right)^2 + N_1}{Q_1}\cdot\frac{N_1 + h_{11}^2 P_1}{g_{11}^2 N_1}\right),\right.$$
$$\left.\frac{1}{2}\log\left(\frac{\left(h_{11}\sqrt{P_1} + h_{12}\sqrt{P_2} + g_{12}\sqrt{Q_2}\right)^2 + N_1}{Q_2}\cdot\frac{N_1 + h_{12}^2 P_2}{g_{12}^2 N_1}\right)\right\} \tag{21}$$

Proof. The bound (20) is proved by considering the point-to-point channel between the second transmitter–receiver pair. Hence, we only prove (21). Considering the receiver 1 and applying the Fano's inequality we can write:

$$\begin{aligned} n(R_1 + R_2 - \varepsilon_n) &\le I(M_1, M_2; \mathbf{Y}_1) \\ &= I(M_1, M_2, \mathbf{S}_1, \mathbf{S}_2; \mathbf{Y}_1) - I(\mathbf{S}_1, \mathbf{S}_2; \mathbf{Y}_1 | M_1, M_2) \qquad (22) \\ &= h(\mathbf{Y}_1) - h(\mathbf{Y}_1|M_1, M_2, \mathbf{S}_1, \mathbf{S}_2) - h(\mathbf{S}_1, \mathbf{S}_2|M_1, M_2) + h(\mathbf{S}_1, \mathbf{S}_2|M_1, M_2, Y_1) \\ &\le h(\mathbf{Y}_1) + h(\mathbf{S}_2|\mathbf{Y}_1) + h(\mathbf{S}_1|M_1, M_2, \mathbf{Y}_1, \mathbf{S}_2) - h(\mathbf{Z}_1) - h(\mathbf{S}_1) - h(\mathbf{S}_2) \qquad (23) \\ &\le h(\mathbf{Y}_1|\mathbf{S}_2) + h\left(\frac{1}{g_{11}}(h_{11}\mathbf{X}_1 + \mathbf{Z}_1)\right) - h(\mathbf{Z}_1) - h(\mathbf{S}_1) \qquad (24) \\ &\le \frac{n}{2}\log\left(\frac{\left(h_{11}\sqrt{P_1} + h_{12}\sqrt{P_2} + g_{11}\sqrt{Q_1}\right)^2 + N_1}{Q_1}\cdot\frac{N_1 + h_{11}^2 P_1}{g_{11}^2 N_1}\right) \qquad (25) \end{aligned}$$

where $\varepsilon_n \to 0$ as $n \to \infty$. Here, (22) comes from (18) and the fact that $\mathbf{X}_i$, $i = 1, 2$, is a function of $(\mathbf{S}_i, M_i)$ as well as the fact that $\{\mathbf{S}_1, \mathbf{S}_2, M_1, M_2\}$ are mutually independent, (23) is due to the fact that the conditioning reduces entropy, (24) follows from (18) and these facts that the conditioning reduces entropy and $\mathbf{X}_2 = f_2(\mathbf{S}_2, M_2)$, and finally (25) follows from the Cauchy–Schwarz inequality as well as the fact that the differential entropy is maximized by a Gaussian distribution for a fixed second moment. In addition, we can rewrite (22)–(25) as

$$\begin{aligned} n(R_1 + R_2 - \varepsilon_n) &\le h(\mathbf{Y}_1) - h(\mathbf{Z}_1) - h(\mathbf{S}_1) - h(\mathbf{S}_2) + h(\mathbf{S}_1|M_1, M_2, \mathbf{Y}_1) + h(\mathbf{S}_2|M_1, M_2, \mathbf{Y}_1, \mathbf{S}_1) \qquad (26) \\ &\le h(\mathbf{Y}_1) + h(\mathbf{S}_1|\mathbf{Y}_1) + h(\mathbf{S}_2|M_1, M_2, \mathbf{Y}_1, \mathbf{S}_1) - h(\mathbf{Z}_1) - h(\mathbf{S}_1) - h(\mathbf{S}_2) \qquad (27) \\ &\le h(\mathbf{Y}_1|\mathbf{S}_1) + h\left(\frac{1}{g_{12}}(h_{12}\mathbf{X}_2 + \mathbf{Z}_1)\right) - h(\mathbf{Z}_1) - h(\mathbf{S}_2) \qquad (28) \\ &\le \frac{n}{2}\log\left(\frac{\left(h_{11}\sqrt{P_1} + h_{12}\sqrt{P_2} + g_{12}\sqrt{Q_2}\right)^2 + N_1}{Q_2}\cdot\frac{N_1 + h_{12}^2 P_2}{g_{12}^2 N_1}\right) \qquad (29) \end{aligned}$$

Corollary 1. *For the strong state-interference regime, i.e., when $Q_i \to \infty$, $i = 1, 2$, the capacity outer bound given in Theorem 4 is reduced to the set of all rate pairs (R_1, R_2) satisfying*

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$$R_2 \le \frac{1}{2}\log\left(1+\frac{h_{22}^2P_2}{N_2}\right) \tag{30}$$

$$R_1+R_2 \le \frac{1}{2}\log\left(1+\frac{\min(h_{11}^2P_1, h_{12}^2P_2)}{N_1}\right) \tag{31}$$

In this case, if signal-interference is weak, i.e., $h_{12}^2/h_{22}^2 < N_1/N_2$, then the capacity region of the GDD-ZIC is contained in the following region.

$$R_1+R_2 \le \frac{1}{2}\log\left(1+\frac{\min\left(h_{11}^2P_1, h_{12}^2P_2\right)}{N_1}\right) \tag{32}$$

4.3. An inner bound using Costa's strategy

In this subsection, we exploit Costa's strategy to obtain an achievable sum-rate for a simple case of the GDD-ZIC and thereby, we show that this achievable sum-rate vanishes when both state-interferences are strong.

Assume that in the ZIC-NSI studied in Section 3 there is no private sub-message as well as assume that state-interferences S_1 and S_2 are independent. Considering Theorem 2 and letting $U_{2p}=\emptyset$, one can easily show that the following sum-rate is achievable for this special case of the ZIC-NSI.

$$R_1+R_2 \le \min\left\{I(Y_1;U_{2c}), I(Y_2;U_{2c})\right\} + I(Y_1;U_1|U_{2c}) - I(S_1;U_1) - I(S_2;U_{2c}) \tag{33}$$

We now use Costa's strategy [7] to define the auxiliary random variables U_1 and U_{1c} as

$$U_1 = X_1 + \alpha_1 S_1 \tag{34}$$

$$U_{2c} = X_2 + \alpha_2 S_2 \tag{35}$$

where $X_1 \sim \mathcal{N}(0, P_1)$, $X_2 \sim \mathcal{N}(0, P_2)$, and $\{X_1, X_2, S_1, S_2\}$ are mutually independent. Substituting U_1 and U_{2c} in the sum-rate (33), and considering Y_1 and Y_2 as

$$Y_1 = X_1 + X_2 + S_1 + S_2 + Z_1 \tag{36}$$

$$Y_2 = X_2 + S_2 + Z_2 \tag{37}$$

we obtain the following Costa-based sum-rate for this simple case of the GDD-ZIC.

Theorem 5. *If we use Costa's strategy, i.e., define the auxiliary random variables U_1 and U_{2c} as (34) and (35), then the sum-rate (33) is bounded from above by*

$$R_1+R_2 \le [\min\{L_1^{\text{Costa}}, L_2^{\text{Costa}}\}]^+ \tag{38}$$

where

$$L_1^{\text{Costa}} = \frac{1}{2}\log\left(\frac{(P_1+P_2+Q_1+Q_2+N_1)P_1P_2}{(1-\alpha_1)^2(P_2+\alpha_2^2Q_2)P_1Q_1+(1-\alpha_2)^2(P_1+\alpha_1^2Q_1)P_2Q_2+(P_1+\alpha_1^2Q_1)(P_2+\alpha_2^2Q_2)N_1}\right) \tag{39}$$

$$L_2^{\text{Costa}} = \frac{1}{2}\log\left(\frac{\left\{(P_1+Q_1+N_1)(P_2+\alpha_2^2Q_2)+(1-\alpha_2)^2P_2Q_2\right\}(P_2+Q_2+N_2)P_1P_2}{\left\{(1-\alpha_1)^2(P_2+\alpha_2^2Q_2)P_1Q_1+(1-\alpha_2)^2(P_1+\alpha_1^2Q_1)P_2Q_2+(P_1+\alpha_1^2Q_1)(P_2+\alpha_2^2Q_2)N_1\right\}\left\{N_2(P_2+\alpha_2^2Q_2)+(1-\alpha_2)^2P_2Q_2\right\}}\right) \tag{40}$$

Proof. The proof of Theorem 5 follows by substituting U_1, U_{2c}, Y_1 and Y_2 defined by (34)–(37) in the sum-rate (33) and doing some calculations.

Corollary 2. *From (39) and (40), we observe that*

(1) when $Q_1, Q_2 \to \infty$, L_1^{Costa} acts as $\frac{1}{2}\log\left(\frac{(Q_1+Q_2)P_1P_2}{Q_1Q_2\left\{(1-\alpha_1)^2\alpha_2^2P_1+(1-\alpha_2)^2\alpha_1^2P_2+\alpha_1^2\alpha_2^2N_1\right\}}\right)$ and hence $L_1^{\text{Costa}} \to -\infty$ as $Q_1, Q_2 \to \infty$. Note that L_1^{Costa} is the sum-rate bound corresponding to the MAC seen by receiver 1 and as shown in Ref. [13, Proposition 1] we have $[L_1^{\text{Costa}}]^+ \underset{Q_1,Q_2\to\infty}{\to} [-\infty]^+ = 0$.

(2) When $Q_1, Q_2 \to \infty$, L_2^{Costa} acts as $\frac{1}{2}\log\left(\frac{\alpha_2^2P_1P_2Q_1Q_2}{Q_1Q_2\left\{(1-\alpha_1)^2\alpha_2^2P_1+(1-\alpha_2)^2\alpha_1^2P_2+\alpha_1^2\alpha_2^2N_1\right\}\left\{N_2\alpha_2^2+(1-\alpha_2)^2P_2\right\}}\right)$ and hence $L_2^{\text{Costa}} \to \frac{1}{2}\log\left(\frac{\alpha_2^2P_1P_2}{((1-\alpha_1)^2\alpha_2^2P_1+(1-\alpha_2)^2\alpha_1^2P_2+\alpha_1^2\alpha_2^2N_1)(N_2\alpha_2^2+(1-\alpha_2)^2P_2)}\right)$ as $Q_1, Q_2 \to \infty$. Note that L_2^{Costa} is the sum-rate bound corresponding to two separate point-to-point channels between transmitter 1 and receiver 1 and between transmitter 2 and receiver 2. In this case, if $P_1 \gg N_1$ and $P_2 \gg N_2$, then the proper Costa's factors are $\alpha_1=\alpha_2=1$ and $L_2^{\text{Costa}} \to \frac{1}{2}\log\left(\frac{P_1P_2}{N_1N_2}\right) = \frac{1}{2}\log\left(\frac{P_1}{N_1}\right)+\frac{1}{2}\log\left(\frac{P_2}{N_2}\right)$ which is the summation of the capacities of two separate point-to-point channels for high signal-to-noise ratio, as we expected.

Therefore, the achievable sum-rate (38) in Theorem 5 is

$$R_1+R_2 \le \left[\min\{L_1^{\text{Costa}}, L_2^{\text{Costa}}\}\right]^+ \underset{Q_1,Q_2\to\infty}{\to} [L_1^{\text{Costa}}]^+ \underset{Q_1,Q_2\to\infty}{\to} 0 \tag{41}$$

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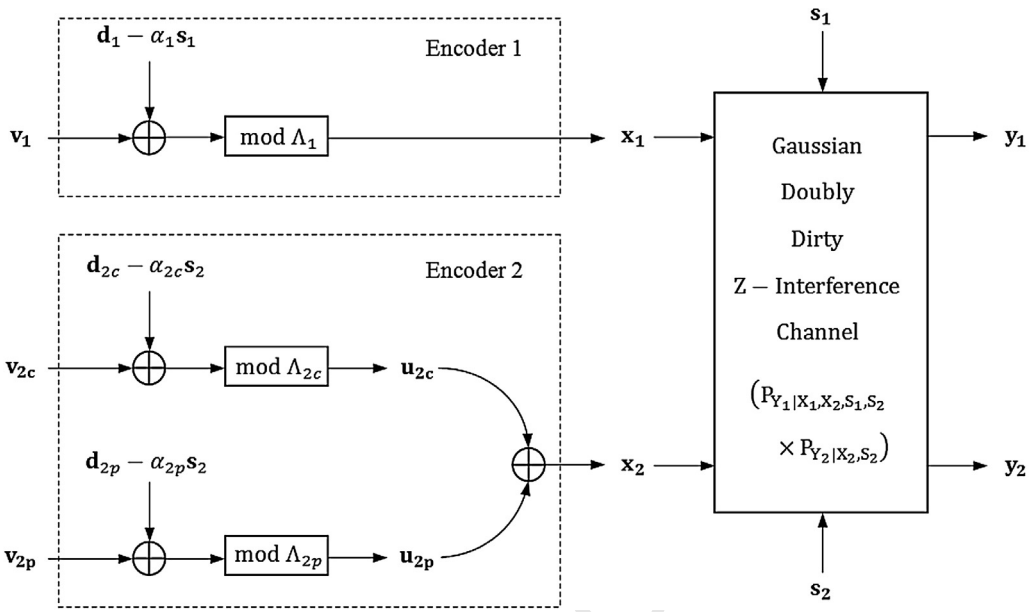

Fig. 3. Encoding structure for the Gaussian doubly dirty Z-interference channel.

This shows that in the GDD-ZIC and in the limit of strong state-interferences S_1 and S_2, Costa's strategy in which the auxiliary random variables U_1 and U_{2c} are selected as (34) and (35) is unable to achieve positive rate.

5. Lattice-based inner bounds to the capacity region of the GDD-ZIC

In this section, after giving a brief review of the required key concepts of the lattice theory, we obtain lattice-based achievable rates for the GDD-ZIC which are independent of the state-interference powers and thus, do not vanish for the case of strong state-interference. Specifically, because of the central role of the gap between SNR_1 and INR_1 at the first receiver in the capacity analysis, we first group all possible values of the gap between SNR_1 and INR_1 into two main imbalanced (large gap) and nearly balanced groups. Then, we derive achievable rates for these two cases and show that lattice-strategies are optimal for a class of the GDD-ZIC.

5.1. Lattice preliminaries

In this subsection, for the sake of brevity, we review just those necessary definitions and properties of the lattice theory that will be used in the following. More detailed studies about lattices can be found in Refs. [9,13–17].

Let Λ be an n-dimensional lattice in the Euclidian space $\mathbb{R}^n$.

- The nearest neighbor quantizer $Q_\Lambda : \mathbb{R}^n \to \Lambda$ is defined by $Q_\Lambda(\mathbf{x}) \triangleq \text{argmin}_{\boldsymbol{l}\in\Lambda}||\mathbf{x} - \boldsymbol{l}||$.
- The fundamental Voronoi region of Λ is defined as the set of all points in $\mathbb{R}^n$ closer to the origin than to any other lattice point, i.e., $\mathcal{V} \triangleq \{\mathbf{x} \in \mathbb{R}^n : Q_\Lambda(\mathbf{x}) = 0\}$. The volume of $\mathcal{V}$ is $V = \text{vol}(\mathcal{V}) = \int_\mathcal{V} d\mathbf{x}$.
- The modulo-Λ operation which gives the quantization error of $\mathbf{x}$ with respect to the lattice Λ is defined as

$$\mathbf{x} \bmod \Lambda = \mathbf{x} - Q_\Lambda(\mathbf{x}) \tag{42}$$

- The following properties are satisfied by the modulo-Λ operation:

$$[\mathbf{x} \bmod \Lambda + \mathbf{y}] \bmod \Lambda = [\mathbf{x} + \mathbf{y}] \bmod \Lambda \tag{43}$$

$$[\alpha\mathbf{x}] \bmod \Lambda = \alpha \left[\mathbf{x} \bmod \frac{\Lambda}{\alpha}\right] \tag{44}$$

- If the average (per dimension) second moment of Λ is denoted by σ^2_Λ, then the normalized second moment (NSM) is given by $G(\Lambda) = \frac{\sigma^2_\Lambda}{V^{2/n}}$. The NSM is commonly used as a figure of merit for showing the quantization efficiency of a lattice quantizer with respect to the mean-squared error (MSE) measure.
- It is known that an n-dimensional sphere has the smallest NSM among all n-dimensional shapes of equal volume V. As the NSM of an infinite-dimensional sphere is $(1/2\pi e)$ therefore, for all n, we have $G(\Lambda) \geq (1/2\pi e)$. Moreover, it is known that for large enough n, there exist a sequence of good lattice quantizers Λ_n with spherical Voronoi region and $G(\Lambda_n) = (1/2\pi e)$ (good lattices for quantization). In other words, for lattices that are good for quantization, we can say that an n-dimensional random vector $\mathbf{D}$ which is distributed uniformly over $\mathcal{V}$ ($\mathbf{D} \sim \text{Unif}(\mathcal{V})$) acts as a white Gaussian noise with normalized differential entropy $(1/n)h(\mathbf{D}) = (1/2)\log(2\pi e\sigma^2_\Lambda)$.

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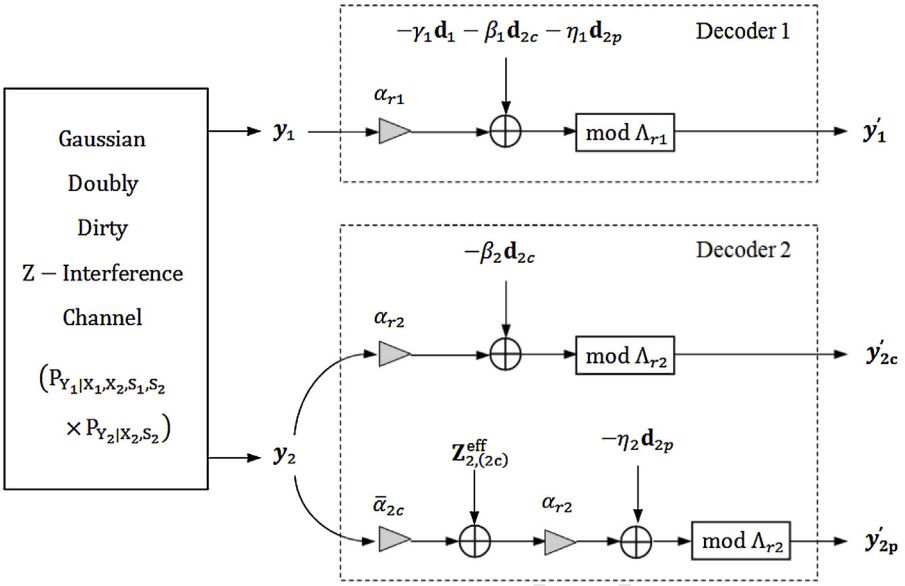

Fig. 4. Decoding structure for the Gaussian doubly dirty Z-interference channel.

5.2. Lattice-based transmission scheme

The proposed lattice-based encoding scheme for the GDD-ZIC is as follows (Fig. 3). Encoder 1 by utilizing the lattice Λ_1 with second moment $\sigma^2_{\Lambda_1} = P_1$ and fundamental Voronoi region $\mathcal{V}_1$ generates $\mathbf{X}_1$ as (45) and transmits it.

$$\mathbf{X}_1 = [\mathbf{V}_1 - \alpha_1\mathbf{S}_1 + \mathbf{D}_1] \bmod \Lambda_1 \tag{45}$$

In (45) vector $\mathbf{V}_1$ and dither signal $\mathbf{D}_1$ are uniformly distributed over $\mathcal{V}_1$, i.e., $\mathbf{D}_1 \sim \text{Unif}(\mathcal{V}_1)$, $\mathbf{V}_1 \sim \text{Unif}(\mathcal{V}_1)$, and message M_1 is carried by vector $\mathbf{V}_1$. Encoder 2 uses the lattices Λ_{2c} and Λ_{2p}, respectively, with second moments $\sigma^2_{\Lambda_{2c}} = \rho P_2$ and $\sigma^2_{\Lambda_{2p}} = \bar{\rho} P_2$ and Voronoi regions $\mathcal{V}_{2c}$ and $\mathcal{V}_{2p}$. This encoder transmits $\mathbf{X}_2 = \mathbf{U}_{2c} + \mathbf{U}_{2p}$ where $\mathbf{U}_{2c}$ and $\mathbf{U}_{2p}$ are generated as

$$\mathbf{U}_{2c} = [\mathbf{V}_{2c} - \alpha_{2c}\mathbf{S}_2 + \mathbf{D}_{2c}] \bmod \Lambda_{2c} \tag{46}$$

$$\mathbf{U}_{2p} = [\mathbf{V}_{2p} - \alpha_{2p}\mathbf{S}_2 + \mathbf{D}_{2p}] \bmod \Lambda_{2p} \tag{47}$$

where $\mathbf{D}_{2c} \sim \text{Unif}(\mathcal{V}_{2c})$, $\mathbf{V}_{2c} \sim \text{Unif}(\mathcal{V}_{2c})$, $\mathbf{D}_{2p} \sim \text{Unif}(\mathcal{V}_{2p})$, $\mathbf{V}_{2p} \sim \text{Unif}(\mathcal{V}_{2p})$, and vectors $\mathbf{V}_{2c}$ and $\mathbf{V}_{2p}$ carry messages M_{2c} and M_{2p}, respectively. Note that $\mathbf{V}_1, \mathbf{V}_{2c}$ and $\mathbf{V}_{2p}$ are mutually independent. In addition, $\mathbf{D}_1, \mathbf{D}_{2c}$ and $\mathbf{D}_{2p}$ are pair-wise independent dither signals and are known to both decoders. It is worth mentioning that $\mathbf{X}_1$ and $\mathbf{X}_2$ satisfy the power constraints P_1 and P_2, respectively. Decoders also use lattice-based decoding schemes as shown in Fig. 4 that will be described in detail at the time of proving Theorem 6. In order to derive the desired results, the lattices Λ_1, Λ_{2c}, Λ_{2p}, Λ_{r1}, Λ_{r2} and also the scalars $\alpha_1, \alpha_{2c}, \alpha_{2p}, \alpha_{r1}, \alpha_{r2}, \gamma_1, \gamma_2, \beta_1, \beta_2, \eta_1, \eta_2$ should be determined properly.

Notice that the performance of the proposed lattice-based encoding scheme is dependent on proper selecting of the factors $\alpha_1, \alpha_{2c}, \alpha_{2p}$ and their uniqueness. It is shown that the optimal α^* in general is as $\alpha^* = \text{SNR}/(1 + \text{SNR})$ where SNR is the signal-to-noise ratio at receiver such that for high SNR, $\alpha^* \to 1$. Since in the GDD-ZIC the first receiver receives two signals $\mathbf{X}_1$ and $\mathbf{X}_2$ transmitted by transmitters 1 and 2, respectively, there are two SNRs and therefore two factors α at the first receiver: (1) SNR caused by $\mathbf{X}_1$ (denoted by SNR_1) and related α_1; (2) SNR caused by $\mathbf{X}_2$ (denoted by INR_1) and related α_{2c}. The gap between SNR_1 and INR_1 can be large or small. If this SNR gap is large (the *imbalanced* SNRs) then one of two α_1 or α_{2c} is approximately 1 (the factor related to the greater SNR) and the other optimal factor should be determined. Also, if the SNR gap is small (the *nearly-balanced* SNRs) then both of factors α_1 and α_{2c} should be determined properly. Based on our analysis (Appendices A and B) the criterion of being large is the distance between geometric mean and smaller SNR, i.e., $\sqrt{\text{SNR}_1\text{INR}_1} - \min(\text{SNR}_1, \text{INR}_1) \geq 1$. Therefore, because of the central role of the gap between SNR_1 and INR_1 at the first receiver in the capacity analysis, we group all possible values of the gap into following two main groups:

1. *Imbalanced Case (Im):* When the gap between SNR_1 and INR_1 at the first receiver is large, i.e., $\sqrt{\text{SNR}_1\text{INR}_1} - \min(\text{SNR}_1, \text{INR}_1) \geq 1$ or equivalently $N_1 \leq \sqrt{h^2_{11}h^2_{12}P_1P_2} - \min(h^2_{11}P_1, h^2_{12}P_2)$.
2. *Nearly Balanced Case (NB):* When SNR_1 and INR_1 at the first receiver are nearly balanced, i.e., $\sqrt{\text{SNR}_1\text{INR}_1} - \min(\text{SNR}_1, \text{INR}_1) < 1$ or equivalently $N_1 > \sqrt{h^2_{11}h^2_{12}P_1P_2} - \min(h^2_{11}P_1, h^2_{12}P_2)$.

5.2.1. Capacity inner bounds for the imbalanced case

Theorem 6. *(Imbalanced Case): When $N_1 \leq \sqrt{h^2_{11}h^2_{12}P_1P_2} - \min(h^2_{11}P_1, h^2_{12}P_2)$ and $h^2_{11}P_1 \neq h^2_{12}P_2$, an achievable rate region for the GDD-ZIC is given by*

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$$\mathcal{R}^{lm}_{\text{GDD-ZIC}} = \bigcup_{\rho\in[0,1],\alpha_{2c}\in[0,1]} \left\{ \begin{array}{l} R_1 \geq 0, R_2 \geq 0 \\ R_2 \leq \frac{1}{2}\log\left(\left(\frac{h_{22}^2\rho P_2}{(\alpha_{2c}-1)^2 h_{22}^2\rho P_2 + \alpha_{2c}^2(h_{22}^2\bar{\rho}P_2+N_2)}\right)\left(1+\frac{h_{22}^2\bar{\rho}P_2}{N_2}\right)\right) \\ R_1+R_2 \leq \min\left\{\frac{1}{2}\log\left(1+\frac{h_{11}^2P_1}{N_1}\right), \frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right)\right\} \end{array} \right\} \tag{48}$$

Proof. The proof is given in Appendix A.

Remark 2. As seen in (48), the achievable rate region obtained by lattice strategies is independent of the state-interference power and therefore, contrary to the Costa-based rate region, the lattice-based rate region $R^{lm}_{\text{GDD-ZIC}}$ does not vanish under the strong state-interference regime. Actually, as shown in Appendix A, the proposed lattice alignment scheme achieves positive rates over the GDD-ZIC by employing interference alignment (via using the same lattices (up to scaling)) and interference concentration (via using proper scaling factors).

Remark 3. As seen in (48), rate-terms R_1 and R_2 depend on α_{2c} only. The optimal minimum mean-square error (MMSE) factor from the first receiver standpoint is $\alpha^*_{2c} = (h_{12}^2P_2)/(h_{12}^2P_2+N_1)$, and from the second receiver standpoint is $\alpha^{**}_{2c} = (h_{22}^2\rho P_2)/(h_{22}^2P_2+N_2)$. As $\alpha^*_{2c} \neq \alpha^{**}_{2c}$ in general, the MAC (seen by the first receiver) and the point-to-point channel (seen by the second receiver) in the GDD-ZIC are not capacity-achieving concurrently. In the following Corollary, we present two achievable rate regions for the GDD-ZIC, each of which is obtained by considering the optimal MMSE factor from the viewpoint of one of the receivers. We can combine these two achievable rate regions using the time-sharing technique and derive an enlarged achievable rate region.

Corollary 3. *When* $N_1 \leq \sqrt{h_{11}^2h_{12}^2P_1P_2} - \min(h_{11}^2P_1, h_{12}^2P_2)$ *and* $h_{11}^2P_1 \neq h_{12}^2P_2$, *by considering* $\alpha^*_{2c} = (h_{12}^2P_2)/(h_{12}^2P_2+N_1)$ *we obtain an achievable rate region for the GDD-ZIC which is given by*

$$R^{lm-1}_{\text{GDD-ZIC}} = \bigcup_{\rho\in[0,1]} \left(\begin{array}{l} R_1 \geq 0, R_2 \geq 0 \\ R_2 \leq \frac{1}{2}\log\left(\left(\frac{h_{22}^2\rho(h_{12}^2P_2+N_1)^2}{N_1^2h_{22}^2\rho + h_{12}^4P_2(h_{22}^2\bar{\rho}P_2+N_2)}\right)\left(1+\frac{h_{22}^2\bar{\rho}P_2}{N_2}\right)\right) \\ R_1+R_2 \leq \min\left\{\frac{1}{2}\log\left(1+\frac{h_{11}^2P_1}{N_1}\right), \frac{1}{2}\log\left(1+\frac{h_{12}^2P_2}{N_1}\right)\right\} \end{array} \right) \tag{49}$$

and by considering $\alpha^{**}_{2c} = (h_{22}^2\rho P_2)/(h_{22}^2P_2+N_2)$ we obtain another achievable rate region for the GDD-ZIC which is given by

$$R^{lm-2}_{\text{GDD-ZIC}} = \bigcup_{\rho\in[0,1]} \left\{ \begin{array}{l} R_1 \geq 0, R_2 \geq 0 \\ R_2 \leq \frac{1}{2}\log\left(1+\frac{h_{22}^2P_2}{N_2}\right) \\ R_1+R_2 \leq \min\left\{\frac{1}{2}\log\left(1+\frac{h_{11}^2P_1}{N_1}\right), \frac{1}{2}\log\left(\frac{h_{12}^2(h_{22}^2P_2+N_2)^2}{N_1h_{22}^4\rho^2P_2 + h_{12}^2(h_{22}^2\bar{\rho}P_2+N_2)^2}\right)\right\} \end{array} \right\} \tag{50}$$

In Theorem 7 we show that lattice-strategies are optimal for a class of the GDD-ZIC.

Theorem 7. *Consider the case where* $M_2 = M_{2c}$ *and* $M_{2p} = \emptyset$, *and suppose that* $N_1 \leq \sqrt{h_{11}^2h_{12}^2P_1P_2} - \min(h_{11}^2P_1, h_{12}^2P_2)$ *and* $h_{11}^2P_1 \neq h_{12}^2P_2$. *The capacity region of the GDD-ZIC when signal-interference is weak (i.e.,* $h_{12}^2/h_{22}^2 < N_1/N_2$*) is given by the set of all rate pairs* (R_1, R_2) *satisfying*

$$R_1+R_2 \leq \min\left\{\frac{1}{2}\log\left(1+\frac{h_{11}^2P_1}{N_1}\right), \frac{1}{2}\log\left(1+\frac{h_{12}^2P_2}{N_1}\right)\right\} \tag{51}$$

Proof. The converse follows from (32). Moreover, one can easily verify that by setting $\rho = 1$ in (49) and considering $\frac{h_{12}^2}{h_{22}^2} < \frac{N_1}{N_2}$ the inequality $\frac{h_{12}^2P_2+N_1}{N_1} < \frac{h_{22}^2(h_{12}^2P_2+N_1)^2}{N_1^2h_{22}^2+h_{12}^4P_2N_2}$ is held and thus, the direct part is proved.

5.2.2. Capacity inner bounds for the nearly balanced case

Theorem 8. *(Nearly Balanced Case): When* $N_1 > \sqrt{h_{11}^2h_{12}^2P_1P_2} - \min(h_{11}^2P_1, h_{12}^2P_2)$, *an achievable rate region for the GDD-ZIC is given by*

$$R^{NB}_{\text{GDD-ZIC}} = \bigcup_{\rho\in[0,1],\alpha_1,\alpha_{2c}\in[0,1]} \left\{ \begin{array}{l} R_1 \geq 0, R_2 \geq 0 \\ R_2 \leq \frac{1}{2}\log\left(\left(\frac{h_{22}^2\rho P_2}{(\alpha_{2c}-1)^2h_{22}^2\rho P_2 + \alpha_{2c}^2(h_{22}^2\bar{\rho}P_2+N_2)}\right)\left(1+\frac{h_{22}^2\bar{\rho}P_2}{N_2}\right)\right) \\ R_1+R_2 \leq \min\left\{u.c.e.\{[L_1^{NB}]^+\}, u.c.e\{[L_2^{NB}]^+\}\right\} \end{array} \right\} \tag{52}$$

where

$$L_1^{NB} = \frac{1}{2}\log\left(\frac{h_{11}^2P_1}{(\alpha_{2c}-1)^2\left(\frac{\alpha_1}{\alpha_{2c}}\right)^2h_{12}^2P_2 + (\alpha_1-1)^2h_{11}^2P_1 + \alpha_1^2N_1}\right), L_2^{NB}$$

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$$= \frac{1}{2}\log\left(\frac{h_{12}^2 P_2}{(\alpha_1-1)^2\left(\frac{\alpha_{2c}}{\alpha_1}\right)^2 h_{11}^2 P_1 + (\alpha_{2c}-1)^2 h_{12}^2 P_2 + \alpha_{2c}^2 N_1}\right) \tag{53}$$

Proof. The proof is given in Appendix B.

Remark 4. As seen in (52), rate-terms R_1 and R_2 (i) are independent of the state-interference power, and (ii) depend on α_1 and α_{2c}. The optimal MMSE factors from the viewpoint of the first receiver are $\alpha_1^* = (h_{11}^2 P_1 + h_{11}h_{12}\sqrt{P_1P_2})/(h_{11}^2 P_1 + h_{12}^2 P_2 + N_1)$ and $\alpha_{2c}^* = (h_{12}^2 P_2 + h_{11}h_{12}\sqrt{P_1P_2})/(h_{11}^2 P_1 + h_{12}^2 P_2 + N_1)$, and the optimal MMSE factor from the viewpoint of the second receiver is $\alpha_{2c}^{**} = (h_{22}^2\rho P_2)/(h_{22}^2 P_2 + N_2)$. In the following, we present two achievable rate regions for the GDD-ZIC, each of which is obtained by considering the optimal MMSE factors from the viewpoint of one of the receivers. We can combine these two achievable rate regions using the time-sharing technique and derive an enlarged achievable rate region.

Corollary 4. *When* $N_1 > \sqrt{h_{11}^2 h_{12}^2 P_1 P_2} - \min(h_{11}^2 P_1, h_{12}^2 P_2)$, *by considering* $\alpha_1^* = (h_{11}^2 P_1 + h_{11}h_{12}\sqrt{P_1P_2})/(h_{11}^2 P_1 + h_{12}^2 P_2 + N_1)$ *and* $\alpha_{2c}^* = (h_{12}^2 P_2 + h_{11}h_{12}\sqrt{P_1P_2})/(h_{11}^2 P_1 + h_{12}^2 P_2 + N_1)$ *we obtain an achievable rate region for the GDD-ZIC which is given by*

$$R_{\text{GDD-ZIC}}^{NB-1} = \bigcup_{\rho\in[0,1]} \left\{ \begin{array}{l} R_1 \geq 0, R_2 \geq 0 \\ R_2 \leq \frac{1}{2}\log\left(\left(\frac{h_{22}^2\rho P_2}{(\alpha_{2c}^*-1)^2 h_{22}^2\rho P_2 + (\alpha_{2c}^*)^2(h_{22}^2\bar{\rho}P_2+N_2)}\right)\left(1+\frac{h_{22}^2\bar{\rho}P_2}{N_2}\right)\right) \\ R_1 + R_2 \leq u.c.e\left\{\left[\frac{1}{2}\log\left(\frac{h_{11}^2P_1+h_{12}^2P_2+N_1}{2N_1+\left(\sqrt{h_{12}^2P_2}-\sqrt{h_{11}^2P_1}\right)^2}\right)\right]^+\right\} \end{array} \right\} \tag{54}$$

and by considering $\alpha_{2c}^{**} = (h_{22}^2\rho P_2)/(h_{22}^2 P_2 + N_2)$ we obtain another achievable rate region for the GDD-ZIC which is given by

$$R_{\text{GDD-ZIC}}^{NB-2} = \bigcup_{\rho\in[0,1],\alpha_1\in[0,1]} \left\{ \begin{array}{l} R_1 \geq 0, R_2 \geq 0 \\ R_2 \leq \frac{1}{2}\log\left(1+\frac{h_{22}^2P_2}{N_2}\right) \\ R_1 + R_2 \leq \min\left\{u.c.e\left\{[L_1^{NB}(\alpha_1,\alpha_{2c}^{**})]^+\right\}, u.c.e\left\{[L_2^{NB}(\alpha_1,\alpha_{2c}^{**})]^+\right\}\right\} \end{array} \right\} \tag{55}$$

6. Conclusion

We considered the ZIC with NSI at transmitters. First, an achievable rate region was established for the general discrete memoryless case and was shown that the rate region is tight for a special case of the orthogonal ZIC-NSI. We then defined the corresponding Gaussian case and presented a capacity outer bound for the GDD-ZIC. Finally, by obtaining achievable rate regions based on Costa's strategy and lattice-strategies, we showed that unlike Costa's strategy, lattice-strategies succeed to achieve positive rates under the strong state-interference regime.

Proof of Theorem 6.

We first show the achievability of $R_2 \leq \frac{1}{2}\log\left(\left(\frac{h_{22}^2\rho P_2}{(\alpha_{2c}-1)^2 h_{22}^2\rho P_2+\alpha_{2c}^2(h_{22}^2\bar{\rho}P_2+N_2)}\right)\left(1+\frac{h_{22}^2\bar{\rho}P_2}{N_2}\right)\right)$. In accordance with the lattice encoding scheme described in Section 5 and depicted in Fig. 3, encoder 2 uses the lattices Λ_{2c} and Λ_{2p}, respectively, with second moments $\sigma_{\Lambda_{2c}}^2 = \rho P_2$ and $\sigma_{\Lambda_{2p}}^2 = \bar{\rho}P_2$ and generates $\mathbf{U}_{2c}$ and $\mathbf{U}_{2p}$ as (A-1) and (A-2) and transmits $\mathbf{X}_2 = \mathbf{U}_{2c} + \mathbf{U}_{2p}$.

$$\mathbf{U}_{2c} = \left[\mathbf{V}_{2c} - \frac{g_{22}\alpha_{2c}}{h_{22}}\mathbf{S}_2 + \mathbf{D}_{2c}\right] \bmod \Lambda_{2c} \tag{A-1}$$

$$\mathbf{U}_{2p} = \left[\mathbf{V}_{2p} - \frac{g_{22}\bar{\alpha}_{2c}\alpha_{2p}}{h_{22}}\mathbf{S}_2 + \mathbf{D}_{2p}\right] \bmod \Lambda_{2p} \tag{A-2}$$

We now set $\Lambda_{r2} = h_{22}\Lambda_{2c}$. As shown in Fig. 4, decoder 2 upon receiving $\mathbf{Y}_2$ calculates $\mathbf{Y}'_{2c}$ as

$$\mathbf{Y}'_{2c} = [\alpha_{r2}\mathbf{Y}_2 - \beta_2\mathbf{D}_{2c}] \bmod \Lambda_{r2} \tag{A-3}$$

$$= [\alpha_{2c}\mathbf{Y}_2 - h_{22}\mathbf{D}_{2c}] \bmod \Lambda_{r2} \tag{A-4}$$

$$= [\alpha_{2c}(h_{22}(\mathbf{U}_{2c}+\mathbf{U}_{2p}) + g_{22}\mathbf{S}_2 + \mathbf{Z}_2) - h_{22}D_{2c}] \bmod \Lambda_{r2} \tag{A-5}$$

$$= \left[h_{22}\mathbf{V}_{2c} + (\alpha_{2c}-1)h_{22}\mathbf{U}_{2c} + h_{22}\left(\mathbf{U}_{2c} - \left(\mathbf{V}_{2c} - \frac{g_{22}\alpha_{2c}}{h_{22}}\mathbf{S}_2 + \mathbf{D}_{2c}\right)\right) + \alpha_{2c}(h_{22}\mathbf{U}_{2p} + \mathbf{Z}_2)\right] \bmod \Lambda_{r2} \tag{A-6}$$

$$= [h_{22}\mathbf{V}_{2c} + (\alpha_{2c}-1)h_{22}\mathbf{U}_{2c} + \alpha_{2c}(h_{22}\mathbf{U}_{2p} + \mathbf{Z}_2)] \bmod \Lambda_{r2} \tag{A-7}$$

$$= [h_{22}\mathbf{V}_{2c} + \mathbf{Z}_{2,(2c)}^{\text{eff}}] \bmod \Lambda_{r2} \tag{A-8}$$

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where $\mathbf{Z}_{2,(2c)}^{\text{eff}} = \left[(\alpha_{2c}-1)h_{22}\mathbf{U}_{2c} + \alpha_{2c}(h_{22}\mathbf{U}_{2p}+\mathbf{Z}_2)\right] \bmod \Lambda_{r2}$, (A-5) follows from (19); (A-7) comes from (42)–(44) and the fact that $h_{22}Q_{\Lambda_{2c}}(\mathbf{V}_{2c} - (g_{22}\alpha_{2c}/h_{22})\mathbf{S}_2 + \mathbf{D}_{2c}) \in \Lambda_{r2}$ because $\Lambda_{r2} = h_{22}\Lambda_{2c}$. Actually, after the modulo-Λ_{r2} operation, the terms $h_{22}Q_{\Lambda_{2c}}(\mathbf{V}_{2c} - (g_{22}\alpha_{2c}/h_{22})\mathbf{S}_2 + \mathbf{D}_{2c})$ disappears because of aligning the interference sequence $\mathbf{S}_2$ with Λ_{r2}. Therefore, we can write

$$R_{2c} = \frac{1}{n}I(\mathbf{V}_{2c};\mathbf{Y}'_{2c}) = \frac{1}{n}\{h(\mathbf{Y}'_{2c}) - h(\mathbf{Y}'_{2c}|\mathbf{V}_{2c})\} \tag{A-9}$$

$$= \frac{1}{n}\left\{h(\mathbf{Y}'_{2c}) - h([(\alpha_{2c}-1)h_{22}\mathbf{U}_{2c} + \alpha_{2c}(h_{22}\mathbf{U}_{2p}+\mathbf{Z}_2)] \bmod \Lambda_{r2})\right\} \tag{A-10}$$

$$\geq \frac{1}{2}\log\left(\frac{h_{22}^2\rho P_2}{G(\Lambda_{r2})}\right) - \frac{1}{2}\log\left(2\pi e((\alpha_{2c}-1)^2h_{22}^2\rho P_2 + \alpha_{2c}^2(h_{22}^2\bar{\rho}P_2+N_2))\right) \tag{A-11}$$

$$= \frac{1}{2}\log\left(\frac{h_{22}^2\rho P_2}{(\alpha_{2c}-1)^2h_{22}^2\rho P_2 + \alpha_{2c}^2(h_{22}^2\bar{\rho}P_2+N_2)}\right) - \frac{1}{2}\log(2\pi eG(\Lambda_{r2})) \tag{A-12}$$

Note that $\mathbf{Y}'_{2c}$ is uniform over $\mathcal{V}_{r2} = h_{22}\mathcal{V}_{2c}$. Here, (A-11) follows from the fact for a fixed second moment, Gaussian distribution maximizes the entropy. Accordingly, if Λ_{r2} is a good lattice for quantization, then

$$R_{2c} \leq \frac{1}{2}\log\left(\frac{h_{22}^2\rho P_2}{(\alpha_{2c}-1)^2h_{22}^2\rho P_2 + \alpha_{2c}^2(h_{22}^2\bar{\rho}P_2+N_2)}\right) \tag{A-13}$$

As shown in Fig. 4, decoder 2 after calculating $\mathbf{Y}'_{2c}$ and obtaining $\mathbf{Z}_{2,(2c)}^{\text{eff}}$ and also setting $\Lambda_{r2} = h_{22}\Lambda_{2p}$ computes $\mathbf{Y}'_{2p}$ as

$$\mathbf{Y}'_{2p} = [\alpha_{r2}(\bar{\alpha}_{2c}\mathbf{Y}_2 + \mathbf{Z}_{2,(2c)}^{\text{eff}}) - \eta_2\mathbf{D}_{2p}] \bmod \Lambda_{r2} \tag{A-14}$$

$$= [\alpha_{2p}(\bar{\alpha}_{2c}\mathbf{Y}_2 + \mathbf{Z}_{2,(2c)}^{\text{eff}}) - h_{22}\mathbf{D}_{2p}] \bmod \Lambda_{r2} \tag{A-15}$$

$$= [\alpha_{2p}\bar{\alpha}_{2c}(h_{22}(\mathbf{U}_{2c}+\mathbf{U}_{2p}) + g_{22}\mathbf{S}_2 + \mathbf{Z}_2) + \alpha_{2p}(\alpha_{2c}-1)h_{22}\mathbf{U}_{2c} + \alpha_{2p}\alpha_{2c}(h_{22}\mathbf{U}_{2p}+\mathbf{Z}_2) - h_{22}\mathbf{D}_{2p}] \bmod \Lambda_{r2}$$

$$= [\alpha_{2p}\bar{\alpha}_{2c}g_{22}\mathbf{S}_2 + \alpha_{2p}(h_{22}\mathbf{U}_{2p}+\mathbf{Z}_2) - h_{22}\mathbf{D}_{2p}] \bmod \Lambda_{r2} \tag{A-16}$$

$$= \left[h_{22}\mathbf{V}_{2p} + (\alpha_{2p}-1)h_{22}\mathbf{U}_{2p} + h_{22}\left(\mathbf{U}_{2p} - \left(\mathbf{V}_{2p} - \frac{g_{22}\alpha_{2p}\bar{\alpha}_{2c}}{h_{22}}\mathbf{S}_2 + \mathbf{D}_{2p}\right)\right) + \alpha_{2p}Z_2\right] \bmod \Lambda_{r2} \tag{A-17}$$

$$= [h_{22}\mathbf{V}_{2p} + (\alpha_{2p}-1)h_{22}\mathbf{U}_{2p} + \alpha_{2p}\mathbf{Z}_2] \bmod \Lambda_{r2} \tag{A-18}$$

$$= [h_{22}\mathbf{V}_{2p} + \mathbf{Z}_{2,(2p)}^{\text{eff}}] \bmod \Lambda_{r2} \tag{A-19}$$

where $\mathbf{Z}_{2,(2p)}^{\text{eff}} = [(\alpha_{2p}-1)h_{22}\mathbf{U}_{2p} + \alpha_{2p}\mathbf{Z}_2] \bmod \Lambda_{r2}$. Here, (A-18) follows from the fact that $h_{22}Q_{\Lambda_{2p}}\left(\mathbf{V}_{2p} - \frac{g_{22}\alpha_{2p}\bar{\alpha}_{2c}}{h_{22}}\mathbf{S}_2 + \mathbf{D}_{2p}\right) \in \Lambda_{r2}$, because $\Lambda_{r2} = h_{22}\Lambda_{2p}$. Therefore, we can write

$$R_{2p} = \frac{1}{n}I(\mathbf{V}_{2p};\mathbf{Y}'_{2p}) = \frac{1}{n}\{h(\mathbf{Y}'_{2p}) - h(\mathbf{Y}'_{2p}|\mathbf{V}_{2p})\} \tag{A-20}$$

$$= \frac{1}{n}\{h(\mathbf{Y}'_{2p}) - h([(\alpha_{2p}-1)h_{22}\mathbf{U}_{2p} + \alpha_{2p}\mathbf{Z}_2] \bmod \Lambda_{r2})\} \tag{A-21}$$

$$\geq \frac{1}{2}\log\left(\frac{h_{22}^2\bar{\rho}P_2}{G(\Lambda_{r2})}\right) - \frac{1}{2}\log\left(2\pi e((\alpha_{2p}-1)^2h_{22}^2\bar{\rho}P_2 + \alpha_{2p}^2N_2)\right) \tag{A-22}$$

$$= \frac{1}{2}\log\left(\frac{h_{22}^2\bar{\rho}P_2}{(\alpha_{2p}-1)^2h_{22}^2\bar{\rho}P_2 + \alpha_{2p}^2N_2}\right) - \frac{1}{2}\log(2\pi eG(\Lambda_{r2})) \tag{A-23}$$

Consequently, if Λ_{r2} is a good lattice for quantization, then

$$R_{2p} \leq \frac{1}{2}\log\left(\frac{h_{22}^2\bar{\rho}P_2}{(\alpha_{2p}-1)^2h_{22}^2\bar{\rho}P_2 + \alpha_{2p}^2N_2}\right) \tag{A-24}$$

By solving $\partial R_{2p}/\partial\alpha_{2p} = 0$ we obtain the optimal MMSE factor $\alpha_{2p}^{**} = h_{22}^2\bar{\rho}P_2/(h_{22}^2\bar{\rho}P_2 + N_2)$. Substituting α_{2p}^{**} into (A-24) yields

$$R_{2p} \leq \frac{1}{2}\log\left(1 + \frac{h_{22}^2\bar{\rho}P_2}{N_2}\right) \tag{A-25}$$

We now show that the sum-rate R_1+R_2 presented in (48) is achievable for the imbalanced case. First, notice that the imbalanced case $N_1 \leq \sqrt{h_{11}^2h_{12}^2P_1P_2} - \min(h_{11}^2P_1, h_{12}^2P_2)$ for $h_{12}^2P_2 \neq h_{11}^2P_1$, in turn, includes two sub-cases: $h_{12}^2P_2 \geq h_{11}^2P_1\left(\frac{h_{11}^2P_1+N_1}{h_{11}^2P_1}\right)^2$ and $h_{11}^2P_1 \geq h_{12}^2P_2\left(\frac{h_{12}^2P_2+N_1}{h_{12}^2P_2}\right)^2$. Therefore, the sum-rate R_1+R_2 in the rate region is the minimum value of *two* terms. Second, if we prove that two

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following points are achievable, then the achievability of the sum-rate R_1+R_2 in (48) will be proved by using time sharing between (A-26) and (A-27).

$$(R_1,R_2)\le\left(\min\left\{\frac{1}{2}\log\left(1+\frac{h_{11}^2P_1}{N_1}\right),\quad\frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right)\right\},\quad 0\right)\tag{A-26}$$

$$(R_1,R_2)\le\left(0,\min\left\{\frac{1}{2}\log\left(1+\frac{h_{11}^2P_1}{N_1}\right),\quad\frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right)\right\}\right)\tag{A-27}$$

Here, for the sake of brevity, we only prove the achievability of (A-26) for the case where $h_{11}^2P_1\ge h_{12}^2P_2\left(\frac{h_{12}^2P_2+N_1}{h_{12}^2P_2}\right)^2$ or equivalently, the achievability of $(R_1,R_2)\le\left(\frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right),0\right)$. In the same way, one can show that other terms in (A-26) and (A-27) are also achievable. As $R_2=0$ in the point $(R_1,R_2)\le\left(\frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right),0\right)$, we set $\mathbf{V}_2=0$ and consider the case where the second transmitter acts as a helper for the first transmitter. Encoder 1 utilizes the lattice Λ_1 with second moment $\sigma_{\Lambda_1}^2=P_1$ and generates $\mathbf{X}_1$ as (A-28) and transmits it. Encoder 2 sets $\rho=1$ and by using the lattices Λ_{2c} with second moments $\sigma_{\Lambda_{2c}}^2=\rho P_2=P_2$ generates $\mathbf{X}_2$ as (A-29) and transmits it.

$$\mathbf{X}_1=\left[\mathbf{V}_1-\frac{g_{11}}{h_{11}}\mathbf{S}_1+\mathbf{D}_1\right]\bmod\Lambda_1\tag{A-28}$$

$$\mathbf{X}_2=\mathbf{U}_{2c}=\left[-\frac{g_{12}\alpha_{2c}}{h_{12}}\mathbf{S}_2+\mathbf{D}_{2c}\right]\bmod\Lambda_{2c}\tag{A-29}$$

Now, let us set $\Lambda_{r1}=\alpha_{2c}h_{11}\Lambda_1=h_{12}\Lambda_{2c}$. As shown in Fig. 4, decoder 1 upon receiving $\mathbf{Y}_1$ calculates $\mathbf{Y}_1'$ as

$$\mathbf{Y}_1'=[\alpha_{r1}\mathbf{Y}_1-\gamma_1\mathbf{D}_1-\beta_1\mathbf{D}_{2c}-\eta_1\mathbf{D}_{2p}]\bmod\Lambda_{r1}\tag{A-30}$$

$$=[\alpha_{2c}\mathbf{Y}_1-\alpha_{2c}h_{11}\mathbf{D}_1-h_{12}\mathbf{D}_{2c}]\bmod\Lambda_{r1}\tag{A-31}$$

$$=[\alpha_{2c}(h_{11}\mathbf{X}_1+h_{12}\mathbf{X}_2+g_{11}\mathbf{S}_1+g_{12}\mathbf{S}_2+\mathbf{Z}_1-h_{11}\mathbf{D}_1)-h_{12}\mathbf{D}_{2c}]\bmod\Lambda_{r1}\tag{A-32}$$

$$=\left[\alpha_{2c}h_{11}\mathbf{V}_1+\alpha_{2c}h_{11}\left(\mathbf{X}_1-\left(\mathbf{V}_1-\frac{g_{11}}{h_{11}}\mathbf{S}_1+\mathbf{D}_1\right)\right)+\alpha_{2c}(h_{12}\mathbf{X}_2+\mathbf{Z}_1)-h_{12}\left(-\frac{g_{12}\alpha_{2c}}{h_{12}}\mathbf{S}_2+\mathbf{D}_{2c}\right)\right]\bmod\Lambda_{r1}\tag{A-33}$$

$$=[\alpha_{2c}h_{11}\mathbf{V}_1+h_{12}(\alpha_{2c}-1)\mathbf{X}_2+\alpha_{2c}\mathbf{X}_1]\bmod\Lambda_{r1}\tag{A-34}$$

$$=[\alpha_{2c}h_{11}\mathbf{V}_1+\mathbf{Z}_{1,(2c)}^{\text{eff}}]\bmod\Lambda_{r1}\tag{A-35}$$

where $\mathbf{Z}_{1,(2c)}^{\text{eff}}=[h_{12}(\alpha_{2c}-1)\mathbf{X}_2+\alpha_{2c}\mathbf{Z}_1]\bmod\Lambda_{r_1}$. As seen, the effective noise $\mathbf{Z}_{1,(2c)}^{\text{eff}}$ does not include any self-noise caused by the first transmitter (strong transmitter with higher power constraint) because this transmitter performs interference concentration by using scaling factor $\alpha_1=g_{11}/h_{11}$. Notice that (A-32) follows from (18); (A-34) comes from (42)–(44) and the fact that $\alpha_{2c}h_{11}Q_{\Lambda_1}\left(\mathbf{V}_1-\frac{g_{11}}{h_{11}}\mathbf{S}_1+\mathbf{D}_1\right)\in\Lambda_{r1}$ because $\Lambda_{r1}=\alpha_{2c}h_{11}\Lambda_1$. Actually, the interference alignment is also performed by utilizing the same lattices (up to scaling, i.e., $\Lambda_{r1}=\alpha_{2c}h_{11}\Lambda_1=h_{12}\Lambda_{2c}$). Therefore, we can write

$$R_1=\frac{1}{n}I(V_1;\mathbf{Y}_1')=\frac{1}{n}\left\{h(\mathbf{Y}_1')-h(\mathbf{Y}_1'|\mathbf{V}_1)\right\}\tag{A-36}$$

$$=\frac{1}{n}\left\{h(\mathbf{Y}_1')-h([h_{12}(\alpha_{2c}-1)\mathbf{X}_2+\alpha_{2c}\mathbf{Z}_1]\bmod\Lambda_{r1})\right\}\tag{A-37}$$

$$\ge\frac{1}{2}\log\left(\frac{\alpha_{2c}^2h_{11}^2P_1}{G(\Lambda_{r1})}\right)-\frac{1}{2}\log\left(2\pi e((\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1)\right)\tag{A-38}$$

$$=\frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right)-\frac{1}{2}\log(2\pi eG(\Lambda_{r1}))\tag{A-39}$$

Note that $\mathbf{Y}_1'$ is uniform over $\mathcal{V}_{r1}=\alpha_{2c}h_{11}\mathcal{V}_1$. Here, (A-38) follows from the fact for a fixed second moment, Gaussian distribution maximizes the entropy. Accordingly, if Λ_{r1} is a good lattice for quantization, then

$$R_1\le\frac{1}{2}\log\left(\frac{h_{12}^2P_2}{(\alpha_{2c}-1)^2h_{12}^2P_2+\alpha_{2c}^2N_1}\right)\tag{A-40}$$

Proof of Theorem 8.

The achievability proof of the rate R_2 given in (52) is as above. Here, we prove the achievability of the sum-rate R_1+R_2 given in (52). First, note that the nearly balanced case $N_1>\sqrt{h_{11}^2h_{12}^2P_1P_2}-\min(h_{11}^2P_1,h_{12}^2P_2)$, in turn, includes two sub-cases: $h_{11}^2P_1\le h_{12}^2P_2\le h_{11}^2P_1\left(\frac{h_{11}^2P_1+N_1}{h_{11}^2P_1}\right)^2$ and $h_{12}^2P_2\le h_{11}^2P_1\le h_{12}^2P_2\left(\frac{h_{12}^2P_2+N_1}{h_{12}^2P_2}\right)^2$. Hence, the sum-rate R_1+R_2 in the rate region is the minimum value of *two*

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terms. Second, if we prove that two following points are achievable, then the achievability of the sum-rate R_1+R_2 in (52) will be proved using time sharing between (B-1) and (B-2).

$$(R_1, R_2) \le \left(\min\left\{u.c.e\{[L_1^{NB}]^+\}, u.c.e\{[L_2^{NB}]^+\}\right\}, 0\right) \tag{B-1}$$

$$(R_1, R_2) \le \left(0, \min\left\{u.c.e\{[L_1^{NB}]^+\}, u.c.e\{[L_2^{NB}]^+\}\right\}\right) \tag{B-2}$$

Here, for the sake of brevity, we only prove the achievability of (B-2) for the case where $h_{11}^2P_1 \le h_{12}^2P_2 \le h_{11}^2P_1\left(\frac{h_{11}^2P_1+N_1}{h_{11}^2P_1}\right)^2$ or equivalently, the achievability of the point $\left(0, \frac{1}{2}\log\left(\frac{h_{11}^2P_1}{(\alpha_{2c}-1)^2\left(\frac{\alpha_1}{\alpha_{2c}}\right)^2h_{12}^2P_2+(\alpha_1-1)^2h_{11}^2P_1+\alpha_1^2N_1}\right)\right)$. In the same way, one can show that other terms in (B-1) and (B-2) are achievable. As in this point $R_1=0$ therefore, we set $\mathbf{V}_1=0$. Encoders 1 and 2 generate $\mathbf{X}_1$ and $\mathbf{X}_2$ as follows and send them.

$$\mathbf{X}_1 = \left[\frac{g_{11}\alpha_1}{h_{11}}\mathbf{S}_1 + \mathbf{D}_1\right] \bmod \Lambda_1 \tag{B-3}$$

$$\mathbf{X}_2 = \mathbf{U}_{2c} = \left[\mathbf{V}_{2c} - \frac{\alpha_{2c}g_{12}}{h_{12}}\mathbf{S}_2\right] \bmod \Lambda_{2c} \tag{B-4}$$

After receiving $\mathbf{Y}_1$ and setting $\Lambda_{r1}=h_{11}\Lambda_1=(\alpha_1/\alpha_{2c})h_{12}\Lambda_{2c}$, decoder 1 calculates $\mathbf{Y}_1'$ as

$$\mathbf{Y}_1' = [\alpha_{r1}\mathbf{Y}_1 - \gamma_1\mathbf{D}_1 - \beta_1\mathbf{D}_{2c} - \eta_1\mathbf{D}_{2p}] \bmod \Lambda_{r1} \tag{B-5}$$

$$= [\alpha_1\mathbf{Y}_1 - h_{11}\mathbf{D}_1] \bmod \Lambda_{r1} \tag{B-6}$$

$$= [\alpha_1(h_{11}\mathbf{X}_1 + h_{12}\mathbf{X}_2 + g_{11}\mathbf{S}_1 + g_{12}\mathbf{S}_2 + \mathbf{Z}_1) - h_{11}\mathbf{D}_1] \bmod \Lambda_{r1} \tag{B-7}$$

$$= \left[\frac{\alpha_1}{\alpha_{2c}}h_{12}V_{2c} + (\alpha_{2c}-1)\frac{\alpha_1}{\alpha_{2c}}h_{12}X_2 + \frac{\alpha_1}{\alpha_{2c}}h_{12}\left(X_2 - \left(V_{2c} - \frac{\alpha_{2c}g_{12}}{h_{12}}S_2\right)\right) + \alpha_1Z_1 + \alpha_1h_{11}X_1 - h_{11}\left(-\frac{g_{11}\alpha_1}{h_{11}}S_1 + D_1\right)\right] \bmod \Lambda_{r1} \tag{B-8}$$

$$= \left[\frac{\alpha_1}{\alpha_{2c}}h_{12}\mathbf{V}_{2c} + (\alpha_{2c}-1)\frac{\alpha_1}{\alpha_{2c}}h_{12}\mathbf{X}_2 + (\alpha_1-1)h_{11}\mathbf{X}_1 + \alpha_1\mathbf{Z}_1\right] \bmod \Lambda_{r1} \tag{B-9}$$

Therefore, we can write

$$R_{2c} = \frac{1}{n}I(\mathbf{V}_{2c};\mathbf{Y}_1') = \frac{1}{n}\{h(\mathbf{Y}_1') - h(\mathbf{Y}_1'|\mathbf{V}_{2c})\} \tag{B-10}$$

$$= \frac{1}{n}\left\{h(\mathbf{Y}_1') - h\left(\left[(\alpha_{2c}-1)\frac{\alpha_1}{\alpha_{2c}}h_{12}\mathbf{X}_2 + (\alpha_1-1)h_{11}\mathbf{X}_1 + \alpha_1\mathbf{Z}_1\right] \bmod \Lambda_{r1}\right)\right\} \tag{B-11}$$

$$\ge \frac{1}{2}\log\left(\frac{(\alpha_1/\alpha_{2c})^2h_{12}^2P_2}{G(\Lambda_{r1})}\right) - \frac{1}{2}\log\left(2\pi e\left((\alpha_{2c}-1)^2\left(\frac{\alpha_1}{\alpha_{2c}}\right)^2h_{12}^2P_2 + (\alpha_1-1)^2h_{11}^2P_1 + \alpha_1^2N_1\right)\right) \tag{B-12}$$

$$= \frac{1}{2}\log\left(\frac{h_{11}^2P_1}{(\alpha_{2c}-1)^2(\alpha_1/\alpha_{2c})^2h_{12}^2P_2 + (\alpha_1-1)^2h_{11}^2P_1 + \alpha_1^2N_1}\right) - \frac{1}{2}\log(2\pi eG(\Lambda_{r1})) \tag{B-13}$$

Accordingly, if Λ_{r1} is a good lattice for quantization, then

$$R_{2c} \le \frac{1}{2}\log\left(\frac{h_{11}^2P_1}{(\alpha_{2c}-1)^2(\alpha_1/\alpha_{2c})^2h_{12}^2P_2 + (\alpha_1-1)^2h_{11}^2P_1 + \alpha_1^2N_1}\right) \tag{B-14}$$

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