QUADROTOR: FULL DYNAMIC MODELING, NONLINEAR SIMULATION AND CONTROL OF ATTITUDES
SOMAYEH NOROUZI GHAZBI\textsuperscript{1a}, ALI AKBAR AKBARI \textsuperscript{1b}, MOHAMMAD REZA GHARIB\textsuperscript{1c}

\textsuperscript{1a} Ferdowsi University of Mashhad

**ABSTRACT**

In this paper, the complete nonlinear modeling of dynamical quadrotor is studied. The modeling is conducted in two parts of body modeling using Newton – Euler's method and propulsion system modeling. The propulsion system is modeled in eight phases of movement. According to the movement phase, the system dynamic switches on one of the models. Convenient access to the nonlinear model is one of the distinct advantages that, in issues related to quadrotor control, can easily utilize non-linear model, and unlike the body of research, is restricted to the body model due to the difficult access to propulsion system. After that, the complete nonlinear model is simulated in MATLAB soft. Further in the study, virtual inputs are presented to create a strong physical sense of the issue, which can also be used in designing a controller for controlling the system. Then, the accuracy of the system is evaluated by designing and implementing six dynamic performance tests on the model and three PD controllers are designed and applied to the system with the aim of controlling the attitudes.

**KEYWORDS:** Quadrotor, Dynamic Modeling, Simulation, Attitude Control, Operating Points, Movement Phase

Quadrotor is an unmanned aerial vertical take-off-landing vehicle that is classified among aerial vehicles with rotary wings. This aerial vehicle has four motors whose propulsion thrust is generated by transmission of power to propellers. This vehicle can be controlled and stabilized by altering engine rpm. Since quadrotor control, which is a vehicle with six degrees of freedom, is possible using four actuators, it is considered as an under-actuated system. Unlimited size and cost, under-actuation, high maneuverability and low sound during the movement have made this vehicle the center of attention of many researchers around the world (Hoffman et al., 2008). Quadrotor control requires an accurate model of the system. The first dynamic model of quadrotor was designed by Altug et al (2002) using Newton-Euler's method. It was a linear model with only body dynamics, which had been derived from simple hypotheses (Altug et al., 2002).

The first Lagrangian model of quadrotor was proposed and used by Castillo et al (In 2005). This model, which had been used after linearization, only included the dynamics of the body (Castillo et al., 2005). Since 2004, propulsion system was also taken into account by researchers. PID vs LQ (2008) and [5] used the propeller speed as the system input, which was more exhaustive than previous models. After that, the research aimed at more complex dynamic control of the vehicle turned to resistant methods to compensate for inaccuracy of modeling Coza et al., (2006) and Modes and Efe (2007). The engine model was added to Quadrotor model for the first time in 2006 by researchers at Aalborg University (Technical report, 2006). In this study, the engine model was defined by body equations. The use of these equations required several tests to identify each parameter. Thus, De Lellis (Modeling, 2011) presented this research in his Ph.D. thesis. Given the complexity of De Lellis’s model, the complete quadrotor model was once again removed from quadrotor system model because of the numerous tests which were required to identify parameters and mount the engine model on the body. This paper presents a model of vehicle, which is more exhaustive and accessible than De Lellis’s test. The proposed model consists of two parts:

A) Body modeling which is derived form of Newton-Euler’s equations.
B ) Propulsion system model, which is presented as a switching model. In this model, seven efficient operating points are calculated. Further, three linear models of the engine are identified by RLS method and applied to dynamic model. The use of these seven models in switching mode resolves the issue of linear approximation of the model and demonstrates the dominant feature of the nonlinear model. After modeling, to have a stronger physical sense of issue, virtual input is presented using voltage's terms combination. Similar virtual inputs were used by Balas (2007) Claudia and Luminita (2010) to link system actual inputs, i.e. voltages, and system control inputs, which were derivatives of the fourth considered modes. Here, however, these inputs are considered as control input and the benefits of each one are examined.

After defining inputs, the model is simulated in Simulink environment of MATLAB software and by designing and applying six performance tests to the system, the accuracy of the model is examined.
DEGREES OF FREEDOM AND THE MOVEMENT OF VEHICLES

Quadrotor is an aerial vehicle with 6 degrees of freedom, i.e., three direct movements and three rotary movements. In this device, propellers, which are positioned opposite to each other, rotates in a direction reverse to the rotation of the coupling propellers. By changing the rpm, the size of lift force changes, and this propels the aerial vehicle. To increase or decrease the height, the rpm of all engines should change proportionally. By keeping constant the rpm of the opposite engines, and increasing the torque generated by a pair of these engines, the vehicle begins to rotate around z axis. The rotation around z-axis is called yaw. Fig. 1 is a schematic display of the engine speed during yaw movement.

If the torque of opposite motors is unequal, the vehicle begins to rotate around its vertical axis (Y and X). Rotation around x axis is called roll movement and rotation around y axis is called pitch movement. Fig. 2 and 3 present a schematic display of engine speeds in both roll and pitch movements.

If the vehicle is rotated around one of its axes, and the vertical component is equal to the total mass, the vehicle will have translational motion along horizontal components of X or Y (Fig. 4).

To simulate quadrotor, we need to initialize the parameters. To this purpose, Reference (Mahmudi et al., 2013) is used. Here, specifications of Table 2 have been given to supplement the discussion.

<table>
<thead>
<tr>
<th>Movement phase</th>
<th>Operating points</th>
<th>Equations of movement phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing height</td>
<td>1) increasing height</td>
<td>1) increasing height</td>
</tr>
<tr>
<td>Decreasing height</td>
<td>2) decreasing height</td>
<td>2) decreasing height</td>
</tr>
<tr>
<td>Rotation around y axis</td>
<td>3) rotation around y axis</td>
<td>3) rotation around y axis</td>
</tr>
<tr>
<td>Movement along the x axis</td>
<td>4) movement along the x axis</td>
<td>4) movement along the x axis</td>
</tr>
<tr>
<td>Rotation around x axis</td>
<td>5) rotation around x axis</td>
<td>5) rotation around x axis</td>
</tr>
<tr>
<td>Movement along y axis</td>
<td>6) movement along y axis</td>
<td>6) movement along y axis</td>
</tr>
<tr>
<td>Rotation around z axis</td>
<td>6) rotation around z axis</td>
<td>6) rotation around z axis</td>
</tr>
</tbody>
</table>
\[ \varphi = \theta = \psi = \phi = \dot{\psi} = x = \dot{x} = y = \dot{y} = z = \dot{z} = 0, \dot{\theta} = 0.087 \text{ rad/s} \]

\[ T_i = T_2 = T_3 = T_4, \quad \tau_i = \tau_2 = \tau_3 = \tau_4 = -\tau_4 \]

\[ \text{Increased height: } \varphi = \theta = \psi = \phi = \dot{\psi} = x = y = \dot{x} = y = z = 0, \dot{\theta} = 0.087 \text{ rad/s} \]

\[ T_i = T_2 = T_3 = T_4, \quad \tau_i = \tau_2 = \tau_3 = \tau_4 = -\tau_4 \]

\[ \text{Rotation around y axis: } \varphi = \theta = \psi = \phi = \dot{\psi} = x = y = \dot{x} = y = z = 0, \dot{\theta} = 0.087 \text{ rad} \]

\[ T_i = T_2 = T_3 = T_4, \quad \tau_i = \tau_2 = \tau_3 = \tau_4 = -\tau_4 \]

\[ \text{Movement along x axis: } \varphi = \theta = \psi = \phi = \dot{\psi} = x = y = \dot{x} = y = z = 0, \dot{\phi} = 0.087 \text{ rad} \]

\[ T_i = T_2 = T_3 = T_4, \quad \tau_i = \tau_2 = \tau_3 = \tau_4 = -\tau_4 \]

\[ \text{Rotation around x axis: } \theta = \psi = \phi = \dot{\theta} = x = \dot{x} = y = \dot{y} = z = 0, \dot{\phi} = 0.087 \text{ rad/s} \]

\[ (T_i + T_3 + T_4) \cos \theta = mg \]

\[ \text{Movement along y axis: } \theta = \psi = \phi = \dot{\theta} = x = \dot{x} = y = \dot{y} = z = 0, \dot{\phi} = 0.087 \text{ rad/s} \]

\[ (T_i + T_3 + T_4) \cos \phi = mg \]

\[ \text{Rotation around the z axis: } \varphi = \theta = \psi = \phi = \dot{\psi} = x = \dot{x} = y = \dot{y} = z = 0, \dot{\phi} = 0.087 \text{ rad/s} \]

\[ (T_i + T_3 + T_4) \cos \phi = mg \]

\[ \text{Buoyancy: } \varphi = \theta = \psi = \phi = \dot{\psi} = x = \dot{x} = y = \dot{y} = z = 0 \]

\[ (T_i + T_3 + T_4) \cos \phi = mg \]

**Table 2. Parameter initialization used in the dynamical equations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>Quadrotor arm length</td>
<td>0.232m</td>
</tr>
<tr>
<td>b</td>
<td>Propeller lift coefficient</td>
<td>3.13 \times 10^2 \text{ Ns/m}</td>
</tr>
<tr>
<td>d</td>
<td>Propeller resistance coefficient</td>
<td>7.5 \times 10^{-3} \text{ m.s}</td>
</tr>
<tr>
<td>m</td>
<td>Total quadrotor mass</td>
<td>0.52 kg</td>
</tr>
<tr>
<td>Ix</td>
<td>Moment of inertia about X axis</td>
<td>6.228 \times 10^{-3} \text{ kg.m}</td>
</tr>
<tr>
<td>Iy</td>
<td>Moment of inertia about Y axis</td>
<td>6.228 \times 10^{-3} \text{ kg.m}</td>
</tr>
<tr>
<td>Iz</td>
<td>Moment of inertia about Z axis</td>
<td>1.121 \times 10^{-2} \text{ kg.m}</td>
</tr>
<tr>
<td>Jr</td>
<td>Moment of propeller inertia about Z axis</td>
<td>6 \times 10^{-3} \text{ kg.m}</td>
</tr>
<tr>
<td>tad</td>
<td>Coefficient of drag force exerted on the body in translational motion</td>
<td>0.5 kg.m.s</td>
</tr>
<tr>
<td>rad</td>
<td>Coefficient of drag force exerted on the body in rotational motion</td>
<td>0.02 kg.m.s</td>
</tr>
</tbody>
</table>

The engine used in this study was a DC brushless model (T. 09/2215 E, M, A, X 6). Technical characteristic of this model are shown in Table 3:

**Table 3. Technical characteristics of the engine T. 09/2215 E, M, A, X 6**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (v)</td>
<td>12</td>
</tr>
<tr>
<td>Maximum power (W)</td>
<td>130</td>
</tr>
<tr>
<td>Weight (gr)</td>
<td>55</td>
</tr>
<tr>
<td>relative propulsion force (g / g)</td>
<td>13.64</td>
</tr>
</tbody>
</table>

Using the equations of the system as well technical characteristics of quadrotor and engines, we will need engine model around operating points given in Table 4:

**Table 4. operating speed of engines**

<table>
<thead>
<tr>
<th>Operating point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular speed (rpm)</td>
<td>1895</td>
<td>1900</td>
<td>1905</td>
<td>1910</td>
<td>1915</td>
<td>1920</td>
<td>2090</td>
</tr>
</tbody>
</table>

According to Table 4, linear model of engine around seven operating point is required. In what follows, these models are derived.
ENGINE MODELING

As noted earlier, DC Brushless motor was used in this study. According to reference (Dvorak, 2012) and based on recursive least squares model, this engine was evaluated and the results are shown in Table 5 (Eq.1 to 7).

Table 5. Functions identified for engine model about operating points

<table>
<thead>
<tr>
<th>Function identified around operating points in point ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>$G(s) = \frac{625}{s+19.02}$</td>
</tr>
</tbody>
</table>

MODELING THE BODY

This section deals with nonlinear modeling of quadrotor. This model is used in the following sections to be simulated in the Simulink environment of MATLAB software and design controller. The assumptions used in developing the models include assumptions concerning the dynamic modeling of the body and the modeling of propulsion system. Assumptions concerning the dynamic modeling of the body are as follows:

A) Quadrotor is a rigid vehicle.
B) Quadrotor has the intended symmetrical structure (the inertia matrix is diagonal).
C) Estimating the aerodynamic resistance forces, which are generated by translational and rotational movements of the vehicle, exerted on the body of the quadrotor with a linear function of rotational and translational speed (Madani and Benallegue, 2006).

And assumptions used in modeling the propulsion system include:

A) Estimating the resistance torque acting on the propeller as a proportional function of the square of the propeller’s velocity (Meriam and Kraige, 2007).
B) Estimating the thrust force as a proportional function of the square of the propellers’ velocity (Dvorak, 2012).

COORDINATE SYSTEM

Quadrotor modeling requires defining various coordinate systems, each of which have different applications. For example, to determine the movement equations, a coordinate system fixed on the quadrotor is required; forces and torques are measured in body coordinate system and GPS finds positions in the fixed land system. Therefore, the knowledge of coordinate systems and their applications is a perquisite. The necessary coordinate systems have been defined in Table 6. Fig.6 show the body and inertial coordinate system.

Using definitions presented in Table 6, the transfer equation from body coordinate system to inertial coordinate system was achieved according to Eq. 8:

$$ R_i^b = R_c^b \times R_i^c \times R_i^b $$ (8)

Body Modeling Using Newton-Euler’s Method

In mathematical modeling of the body, the famous Newton – Euler’s formula is used. For the translational and rotational motion, Newton – Euler’s formula is defined according to Eq. 9 and Eq. 10:

$$ \vec{F} = \frac{d(\vec{v})}{dt} $$ (9)
$$ \vec{M} = \frac{d(\vec{H})}{dt} $$ (10)

Where $\vec{F}$ is the sum of external forces and $\vec{M}$ is the sum of external torques acting on the vehicle.

Linear velocity of the vehicle $\vec{V}$ and $\vec{H}$ is the angular momentum of the vehicle. The main forces acting on quadrotor are mass and lift forces. These equations can be written in the body coordinate system and inertia coordinate systems. To derive equations form body coordinate system, forces need to be displayed in body coordinate system. In Eq. 11, the gravity force is displayed in body coordinate system and in Eq. 12, the total force acting on the body in the body coordinate system are measured.

Table 6. Coordinate systems

<table>
<thead>
<tr>
<th>Coordinate system</th>
<th>Position</th>
<th>Transition matrix</th>
</tr>
</thead>
</table>
Fixed system and origin point, based on the desired point on the ground

Axis is along system \( F^r \) and origin point, based on the gravity center of the vehicle

Rotated \( F^r \) system with angle \( \psi \) around \( \hat{k} \) axis

\[
\mathbf{K}_{\psi} = \begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Rotated \( F^v \) system with angle \( \theta \) around \( \hat{j} \) axis

\[
\mathbf{K}_{\theta} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Rotated \( F^z \) system with angle \( \phi \) around \( \hat{i} \) axis

\[
\mathbf{K}_{\phi} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\]

\[
\bar{g}_{\text{body}} = R_{\psi,\theta,\phi} \begin{bmatrix}
0 \\
0 \\
-g
\end{bmatrix} + \begin{bmatrix}
g s \rho \\
-g c_s c_\phi \\
-g c_s c_\phi
\end{bmatrix}
\]

\[
\bar{F}_{\text{Body}} = \begin{bmatrix}
0 \\
0 \\
T
\end{bmatrix} + m \begin{bmatrix}
+g s \rho \\
-g c_s c_\phi \\
-g c_s c_\phi
\end{bmatrix} = m \begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix}
\]

In the above formula, components of linear acceleration are in the direction of body coordinate system and \( T \) is the total "lift" force. Lift force and total "lift" force are calculated according to Eq. 13 and Eq. 14:

\[
T_i = b \omega_i^3, \quad i = 1, 2, 4
\]

\[
T = \sum_{i=1}^{4} T_i
\]

The Eq. 13, \( b \) is the aerodynamic coefficient of the thrust force. To obtain transitional movement equations in inertia coordinate system, the total lift force vector needs to be transferred from body coordinate system to inertial coordinate system. As such, we will have (Eq.15):

\[
\bar{F}_{\text{inertial}} = \begin{bmatrix}
(c_\psi s_\phi + s_\psi s_\phi), T \\
(s_\psi s_\phi - c_\psi s_\phi), T \\
-mg + (c_\phi c_\psi), T
\end{bmatrix} = \begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix}
\]

Where \( a_x, a_y \) and \( a_z \) are the components of linear momentum in the direction of inertia coordinate system.

To derive the equations of rotational motion, Newton-Euler's equation is used for rotational movement. In Eq. 10, total

\[
\bar{H}_{\text{Body}} = \begin{bmatrix}
I_x & 0 & 0 \\
0 & I_y & 0 \\
0 & 0 & I_z
\end{bmatrix} \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix}
\]

\[
\bar{H}_{\text{Blade}} = \begin{bmatrix}
0 \\
0 \\
J \cdot \Omega_{\text{blades}}
\end{bmatrix}
\]

Where \( \bar{H}_{\text{body}} \) is angular momentum of the body, \( \bar{H}_{\text{blade}} \) is the angular momentum caused by rotation of the propeller in body coordinate system and \( I_x, I_y \) and \( I_z \) are the moments of inertia of the vehicles. Because of body symmetry, the products of inertia became zero. \( \Omega_{\text{blades}} \) is the difference between clockwise and counterclockwise propeller's rotation. As a result, the angular momentum in body coordinate system is calculated according to Eq. 18.

\[
\bar{H} = \begin{bmatrix}
I_x \omega_x \\
I_y \omega_y \\
I_z \omega_z + J \cdot \Omega_{\text{blades}}
\end{bmatrix}
\]
Now, the total drag force in inertia coordinate system needs to be computed, which requires general Eq. 19 \[17\]. This equation is used to calculate torque in inertial coordinate system based on angular moments in body coordinate system as well as the rotation velocity of body coordinate system.

\[
\dot{M} = \frac{d(\dot{H})}{dt} = \frac{d(\dot{H})}{dt} + \Omega \times \dot{H}
\]

\[19\]

\[
\frac{d(\dot{H})}{dt} = \begin{bmatrix}
I_x \omega_x + \omega_y \omega_z (1_z - 1_y) + j_r \omega_y \\
I_y \omega_y + \omega_x \omega_z (1_z - 1_x) + j_r \omega_x \\
I_z \omega_z + \omega_x \omega_y (1_y - 1_z)
\end{bmatrix}
\]

\[20\]

In Eq. 20, the angular velocity of the body can be calculated using Euler’s angular velocity as follows \[21\] :

\[
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} = \begin{bmatrix}
\phi \\
0 + R^T_{\psi}(\varphi) \dot{\psi} \\
0
\end{bmatrix}
\]

\[21\]

Which results in \[22\] :

\[
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} = \begin{bmatrix}
1 & \sin \varphi \tan \theta & \cos \varphi \tan \theta \\
0 & \cos \varphi & -\sin \varphi \\
0 & \sin \varphi \sec \theta & \cos \varphi \sec \theta
\end{bmatrix}^{-1} \begin{bmatrix}
\dot{\varphi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\]

\[22\]

To find the left side of Eq. 19, the external torques acting on the system should be calculated. Such torques is the result of lift or moments differences caused by rotations of the propeller. External torques in the body coordinate system is calculated according to Eq. 23.

\[
\dot{M}_b = \begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} = \begin{bmatrix}
(T_1 - T_3)l \\
(T_4 - T_2)l \\
(-\tau_1 + \tau_2 - \tau_3 + \tau_4)
\end{bmatrix}
\]

\[23\]

In the above equation, \( \tau_x \), \( \tau_y \) and \( \tau_z \) are respectively roll, pitch and yaw torques; \( l \) is the distance between the center of body mass and the axis of propeller rotation and \( \tau_i \) is the resistant torque, which is generated in the opposite direction of propeller rotation and is exerted on the propeller by air. This torque is defined as follows \[24\] :

\[
\tau_i = d \omega^2 i = 1 \text{ to } 4
\]

\[24\]

In Eq. 24, \( d \) is the aerodynamic coefficient of resistant torque. Now, the external torques acting on inertia coordinate system can be achieved \[25\] :

\[
\dot{M} = R_{\psi \omega \theta} \dot{M}_b
\]

\[25\]

AERODYNAMIC RESISTANCE

What is meant by aerodynamic resistances, which are added to the equations in this section, are forces and torques exerted on the vehicle by the air. Given the small cross section of the quadrotor and its low velocity (1 to 2 meters per second), the effect of resistant forces will be negligible. However, the following two equations are used to add these two to the set of equations (Eq. 26 and Eq. 27).

\[
\dot{F}_{ad} = -t_{ad} \dot{V}
\]

\[26\]

In the above equation, \( \dot{M} \) is external torque vector acting on inertia coordinate system and \( H \) is angular momentum in body coordinate system that rotates with angular velocity of \( \Omega = [\omega_x, \omega_y, \omega_z] \). Drawing on Eq. 18 and Eq. 19, the following equation is derived: \[20\].

\[
\dot{M}_a = r_{ad} \Omega
\]

\[27\]

In the above equations, \( r_{ad} \) and \( t_{ad} \) are the coefficients of total drag force acting on a body in translational movement and total drag force acting on the body in rotational movement. These two equations yield force in inertia coordinate system \[13\].

DEFINING NEW CONTROL INPUTS

In determining the system inputs, i.e., control inputs, it is necessary to understand the correct structure of the system and control targets. Actual system inputs are the type of inputs that are fed to the system during operation. As to the quadrotor, actual inputs are voltage systems. In many cases, control targets need to define other inputs, which are known as virtual inputs.

In this study, the following categories of inputs are introduced as input virtual to the system.

\( U_1 = V_1 - V_3 \) : Rotation around X horizontal axis

\( U_2 = V_4 - V_2 \) : Rotation around Y horizontal axis

\( U_3 = V_1 - V_2 + V_3 - V_4 \) : Rotation around Z axis
Movement along Z axis

As it can be seen, the new inputs are a mixture of system actual inputs. If system actual inputs are used to control system, they cannot provide a detailed understanding of the system behavior. For example, it is not possible to determine the kind of movement generated by an increase or decrease in engine voltage (which is one of the system actual inputs), without observing other system inputs. However, since the voltage corresponding to the external force and torque are generated by the engine and the propeller, an increase or decrease in the first input of the new control inputs system is equal to an increases or decreases in the difference of two opposite engines. Given the description provided in Section 2, zero, positive or negative values of the difference between opposite engines would lead to the stability, clockwise and counterclockwise rotation around the axis perpendicular to the connecting axis of two engines. Thus, the new set of inputs provides a better description of subsequent behavior of the system.

Dynamic Simulation Of The System In Simulink Environment Of MATLAB Software

The propulsion system has been simulated in the Simulink environment of MATLAB software.

In Fig. 6m it has been shown as a sub-system of propulsion system-total.
Figure 8. Rigid Body Subsystem: simulator of dynamic equations - the relationship between forces and the location of the vehicle

Figure 9.

Figure 10. Dynamic model subsystem

MODEL EVALUATION

This section examines the reliability of the dynamic model and the designed simulator. Verification is conducted on the using the following five tests:

TEST ONE

In this test, all inputs are equal to zero. Since all voltages are zero and only weight force is exerted, vehicle is expected to stay on the ground. Given the fact that ground proviso has been taken into account, this is equivalent to a continuous decrease of the height. Fig. 9 shows the system outputs in terms of four zero inputs. As it can be seen, the system performance is as expected.

Figure 11. System response in the test in which all inputs are zero


**TEST TWO**

In this test, all system inputs except for the lift force are estimated to be zero. The total force is equal to mg given to the system. The lack of difference between opposite engines halts the rotary movement around the horizontal axes and vertical axes. Further, quadrotor is expected to stay in place without any variation in its height. The output torque of engines should also halt any movement. The system response to the test is shown in Fig. 10.

![Figure 12](image)

**Figure 12.** System response in a test in which all inputs except for lift force are equal to zero and total lift force equals the mass

**TEST THREE**

In this test, total lift force equals the mass, the difference between the two opposite engines (the first and the third engine) equals zero, the difference between the first and the fourth engines is a positive value and the sum of resisting torques equals zero. It is expected that the vehicle begins rotating about one of its horizontal axes, and moves along another horizontal axis. Further, since the rotation of vehicle around horizontal axis has a reducing effect on the vertical component of total lift, the height of vehicle is decreased. The results of these tests are shown in Fig.11, which indicate the compatibility of the system performance with the expectations of the tests.

![Figure 13](image)

**Figure 13.** Fig. 11: System response in the test in which total lift force equals mass, the difference between the first and the third engines equal a positive value and other inputs are equal to zero

**TEST FOUR**

In this test, total lift force is equal to the mass; the sum of resistant torques is equal to a positive value in the system and other inputs are zero. In this test, it is expected that the system begins rotating about z axis. The system response to the test is shown in Fig. 12. As it can be seen, test four also was able to fulfill the intended expectations.

![Figure 14](image)

**Figure 14.** System response to the test in which total lift force equals the mass, resistant torques is equal to a positive value and other inputs are zero

**TEST FIVE**

In this test, by feeding positive input into the total velocity of engines, and feeding positive input into the force difference between the first and the third engines, it is expected that
propeller rotation about z axis and the body rotation around its horizontal axis generate rotation around the other axis. In fact, in this test, the gyroscopic movement of the system is investigated. Fig. 13 shows the system output in this test.

Figure 15. gyroscopic effect of the propeller

As it can be observed, gyroscopic phenomenon has not affected outputs. The torque generated by the gyroscope of propeller is equal to $3.005 \times 10^{-3}$. Since the inertia of propeller is insignificant, the gyroscopic thrust force would be paltry, making it unable to generate a significant movement.

ANGLES CONTROL

In this section, angles are controlled in the buoyancy phase and PD controller is used. Control laws used in Eq. 28 to 30 are shown below.

\[
U_1 = Kp_1(\phi_d - \phi) + Kd_1(\dot{\phi}_d - \dot{\phi}) \quad (28)
\]

\[
U_2 = Kp_2(\theta_d - \theta) + Kd_2(\dot{\theta}_d - \dot{\theta}) \quad (29)
\]

\[
U_3 = Kp_3(\psi_d - \psi) + Kd_3(\dot{\psi}_d - \dot{\psi}) \quad (30)
\]

Controller’s coefficients were obtained by trial and error. These coefficients are shown in Table 7:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Kp_1$</td>
<td>0.4</td>
<td>$Kd_1$</td>
<td>0.055</td>
</tr>
<tr>
<td>$Kp_2$</td>
<td>0.25</td>
<td>$Kd_2$</td>
<td>0.06</td>
</tr>
<tr>
<td>$Kp_3$</td>
<td>-4</td>
<td>$Kd_3$</td>
<td>-2</td>
</tr>
</tbody>
</table>

To evaluate the efficacy and accuracy of the presented control laws, they were used to control the stability of quadrotor in Hover movement condition. In this case, quadrotor, which is floated 1 meter above the ground, is released with initial conditions. Quadrotor is expected to reach equilibrium state ($\phi = \theta = \psi = 0$ rad.) in an optimal time.

Fig. 14 shows the variation of quadrotor’s external angles after applying the above rules and conditions. Further, the findings of Mian and Wang (Mian and Wang, 2007), which is shown by dashed line, have been shown for the comparison of the results. In their study, Mian and Wang examined a quadrotor control (0.6 m and 0.6 kg) using feedback linearization method.

Euler’s method was used. Moreover, the propulsion system in each phase was modeled separately using recursive least squares method. To do so, first the movement phase was defined and operating points were identified. The model switches on propulsion system in each movement phase. An advantage of this model was convenience of obtaining nonlinear model. Such modeling helps researchers include propulsion system in their study model. After that, it was simulated in the Simulink environment of MATLAB software. Finally, the design and implementation of six tests on the model confirmed the accuracy of the obtained model. In the final section, three angle controllers for controlling design angels were designed and applied to the system. A comparison of this controller with the one proposed by Mian and Wang
shows that PD controllers were able to detect system inputs 85 % faster.

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