Indian J.Sci.Res.1(2) : 759-769, 2014

ISSN:2250-0138(Online)
ISSN : 0976-2876 (Print)

QUADROTOR: FULL DYNAMIC MODELING, NONLINEAR SIMULATION AND CONTROL OF ATTITUDES

SOMAYEH NOROUZI GHAZBI[1a], ALI AKBAR AKBARI [b], MOHAMMAD REZA GHARIB[c]

[abc] Ferdowsi University of Mashhad

ABSTRACT

In this paper, the complete nonlinear modeling of dynamical quadrotor is studied. The modeling is conducted in two parts of body modeling using Newton – Euler's method and propulsion system modeling. The propulsion system is modeled in eight phases of movement. According to the movement phase, the system dynamic switches on one of the models. Convenient access to the nonlinear model is one of the distinct advantages that, in issues related to quadrotor control, can easily utilize non- linear model, and unlike the body of research, is restricted to the body model due to the difficult access to propulsion system. After that, the complete nonlinear model is simulated in MATLAB soft. Further in the study, virtual inputs are presented to create a strong physical sense of the issue, which can also be used in designing a controller for controlling the system. Then, the accuracy of the system is evaluated by designing and implementing six dynamic performance tests on the model and three PD controllers are designed and applied to the system with the aim of controlling the attitudes.

KEYWORDS: Quadrotor, Dynamic Modeling, Simulation, Attitude Control, Operating Points, Movement Phase

Quadrotor is an unmanned aerial vertical take-off-landing vehicle that is classified among aerial vehicles with rotary wings. This aerial vehicle has four motors whose propulsion thrust is generated by transmission of power to propellers. This vehicle can be controlled and stabilized by altering engine rpm.

Since quadrotor control, which is a vehicle with six degrees of freedom, is possible using four actuators, it is considered as an under-actuated system. Unlimited size and cost, under-actuation, high maneuverability and low sound during the movement have made this vehicle the center of attention of many researchers around the world (Hoffman et al., 2008). Quadrotor control requires an accurate model of the system. The first dynamic model of quadrotor was designed by Altug et al (2002) using Newton-Euler's method.It was a linear model with only body dynamics, which had been derived from simple hypotheses (Altug et al., 2002).

The first Lagrangian model of quadrotor was proposed and used by Castillo et al (In 2005). This model, which had been used after linearization, only included the dynamics of the body (Castillo et al., 2005).

Since 2004, propulsion system was also taken into account by researchers. PID vs LQ (2008) and [5] used the propeller speed as the system input, which was more exhaustive than previous models. After that, the research aimed at more complex dynamic control of the vehicle turned to resistant methods to compensate for inaccuracy of modeling Coza et al., (2006) and Modes and Efe (2007). The engine model was added to Quadrotor model for the first time in 2006 by researchers at Aalborg University (Technical report, 2006). In this study, the engine model was defined by body equations. The use of these equations required several tests to identify each parameter. Thus, De Lellis (Modeling, 2011) presented this research in his Ph.D. thesis. Given the complexity of De Lellis's model, the complete quadrotor model was once again removed from quadrotor system model because of the numerous tests which were required to identify parameters and mount the engine model on the body. This paper presents a model of vehicle, which is more exhaustive and accessible than De Lellis's test. The proposed model consists of two parts:

A) Body modeling which is derived form of Newton-Euler's equations.

B) Propulsion system model, which is presented as a switching model. In this model, seven efficient operating points are calculated. Further, three linear models of the engine are identified by RLS method and applied to dynamic model. The use of these seven models in switching mode resolves the issue of linear approximation of the model and demonstrates the dominant feature of the nonlinear model.

After modeling, to have a stronger physical sense of issue, virtual input is presented using voltage's terms combination. Similar virtual inputs were used by Balas (2007) Claudia and Luminita (2010) to link system actual inputs, i.e. voltages, and system control inputs, which were derivatives of the fourth considered modes. Here, however, these inputs are considered as control input and the benefits of each one are examined.

After defining inputs, the model is simulated in Simulink environment of MATLAB software and by designing and applying six performance tests to the system, the accuracy of the model is examined.

[1]**Corresponding author**

DEGREES OF FREEDOM AND THE MOVEMENT OF VEHICLES

Quadrotor is an aerial vehicle with 6 degrees of freedom, i.e., three direct movements and three rotary movements. In this device, propellers, which are positioned opposite to each other, rotates in a direction reverse to the rotation of the coupling propellers. By changing the rpm, the size of lift force changes, and this propels the aerial vehicle. To increase or decrease the height, the rmp of all engines should change proportionally. By keeping constant the rmp of the opposite engines, and increasing the torque generated by a pair of these engines, the vehicle begins to rotate around z axis. The rotation around z-axis is called yaw. Fig. 1 is a schematic display of the engine speed during yaw movement.

If the torque of opposite motors is unequal, the vehicle begins to rotate around its vertical axis (Y and X). Rotation around x axis is called roll movement and rotation around y axis is called pitch movement. Fig. 2 and 3 present a schematic display of engine speeds in both roll and pitch movements.

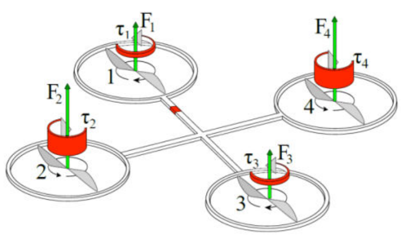


Figure 1. Schematic display of the engine speed during yaw movement (Huang, 2009)

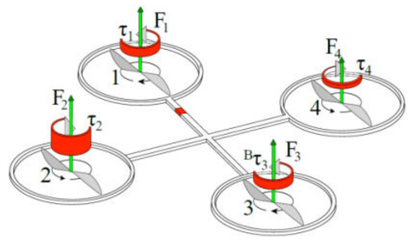


Figure 2. Roll movement (Huang, 2009)

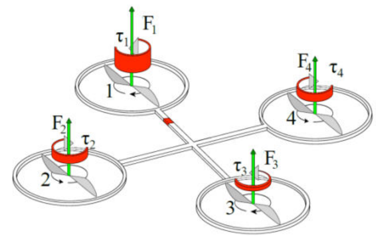


Figure 3. Pitch movement (Huang, 2009)

If the vehicle is rotated around one of its axes, and the vertical component is equal to the total mass, the vehicle will have translational motion along horizontal components of X or Y (Fig. 4).

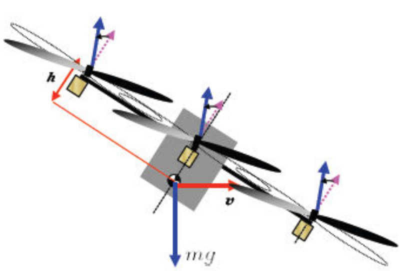


Figure 4. The position of quadrotor during translational movement (Pipatpaibul and Ouyang, 2011)

Figure 5.

Given that the number of degree of freedom is larger than operators (6 degrees of freedom and 4 engines) in the vehicle, quadrotor is considered as an under-actuator system. In recent years, under-actuator systems have attracted the attention of designers for reasons such as a) low energy consumption, b) lower number of actuators and c) low cost. Obviously, the most important advantage of using under-actuator systems is that under certain conditions, system control with less operators (or control input) is possible. Using fewer operators makes the control design more difficult, especially in the presence of uncertainty.

OPERATING POINTS OF THE SYSTEM

In this section, the operating points of the system are determined, which can be used to measure the operating velocity of propulsion system in each operating point of quadrotor. These operating speeds will be used to identify the engine model around each point. The quadrotor movement system can be simulated by combining seven movements 1) increasing height; 2) decreasing height; 3) rotation around y axis; 4) movement along the x axis; 5) rotation around x axis 6) movement along y axis and 6) rotation around z axis.

To determine the speed of engine, first the desired operating characteristics of the quadrotor need to be determined for each movement. These characteristic include angular and linear position as well as angular and linear velocity. Table 1 shows the assumptions and the equations of the system in each movement.

To simulate quadrotor, we need to initialize the parameters. To this purpose, Reference (Mahmudi et al., 2013) is used. Here, specifications of Table 2 have been given to supplement the discussion.

Table 1. operating points and equations for each movement phase

Movement phase	Operating points	Equations of movement phase

Increased height	$\varphi=\theta=\psi=\dot{\varphi}=\dot{\theta}=\dot{\psi}=x=y=\dot{x}=\dot{y}=0,\dot{z}=0.45(m/s)$	$T_1=T_2=T_3=T_4,\tau_1=\tau_3=-\tau_2=-\tau_4$
Decreased height	$\varphi=\theta=\psi=\dot{\varphi}=\dot{\theta}=\dot{\psi}=x=y=\dot{x}=\dot{y}=0,\dot{z}=-0.45(m/s)$	$T_1=T_2=T_3=T_4,\tau_1=\tau_3=-\tau_2=-\tau_4$
Rotation around y axis	$\varphi=\theta=\psi=\dot{\varphi}=\dot{\psi}=x=y=\dot{x}=\dot{y}=\dot{z}=0,\dot{\theta}=0.087(rad/s)$	$T_1=T_3,\quad T_1+T_3+T_2+T_4=mg$ $\tau_1=\tau_3=-\tau_2=-\tau_4$
Movement along x axis	$\varphi=\psi=\dot{\varphi}=\dot{\theta}=\dot{\psi}=x=y=\dot{y}=z=\dot{z}=0,\dot{x}=0.45(m/s)$ $\theta=0.087(rad)$	$T_1=T_2=T_3=T_4,\quad \tau_1=\tau_3=-\tau_2=-\tau_4$ $(T_1+T_3+T_2+T_4)\cos\theta=mg$
Rotation around x axis	$\varphi=\theta=\psi=\dot{\theta}=\dot{\psi}=x=y=\dot{x}=\dot{y}=\dot{z}=0,\dot{\varphi}=0.087(rad/s)$	$T_2=T_4,\quad T_1+T_3+T_2+T_4=mg,$ $\tau_1=\tau_3=-\tau_2=-\tau_4$
Movement along y axis	$\theta=\psi=\dot{\varphi}=\dot{\theta}=\dot{\psi}=x=\dot{x}=y=z=\dot{z}=0,\dot{y}=0.45(m/s)$ $\varphi=0.087(rad)$	$T_1=T_2=T_3=T_4,\quad \tau_1=\tau_3=-\tau_2=-\tau_4$ $(T_1+T_3+T_2+T_4)\cos\varphi=mg$
Rotation around the z axis	$\varphi=\theta=\psi=\dot{\varphi}=\dot{\theta}=x=\dot{x}=y=\dot{y}=z=\dot{z}=0,$ $\dot{\psi}=0.087(rad/s)$	$T_1=T_3,\quad T_2=T_4\quad T_1+T_3+T_2+T_4=mg$ $\tau_1=\tau_3,\quad \tau_2=-\tau_4$
Buoyancy	$\varphi=\theta=\psi=\dot{\varphi}=\dot{\theta}=\dot{\psi}=x=y=\dot{x}=\dot{y}=\dot{z}=0$	$T_1=T_2=T_3=T_4,\quad \tau_1=\tau_3=-\tau_2=-\tau_4$ $T_1+T_3+T_2+T_4=mg$

Table 2. Parameter initialization used in the dynamical equations

Parameter	Definition	Value
l	Quadrotor arm length	$0.232m$
b	Propeller lift coefficient	$3.13\times10^{-5}N.s^2$
d	Propeller resistance coefficient	$7.5\times10^{-7}m.s^2$
m	Total quadrotor mass	$0.52kg$
Ix	Moment of inertia about X axis	$6.228\times10^{-3}kg.m^2$
Iy	Moment of inertia about Y axis	$6.228\times10^{-3}kg.m^2$
Iz	Moment of inertia about Z axis	$1.121\times10^{-2}kg.m^2$
Jr	Moment of propeller inertia about Z axis	$6\times10^{-5}kg.m^2$
tad	Coefficient of drag force exerted on the body in translational motion	$0.5kg.s^2$
rad	Coefficient of drag force exerted on the body in rotational motion	$0.02kg.m.s^2$

The engine used in this study was a DC brushless model (T. 09/2215 E, M, A, X 6). Technical characteristic of this model are shown in Table 3:

Table 3. Technical characteristics of the engine T. 09/2215 E, M, A, X 6

Characteristic	Value
Voltage (v)	12
Maximum power (W)	130
Weight (gr)	55
relative propulsion force (g / g)	13.64

Using the equations of the system as well technical characteristics of quadrotor and engines, we will need engine model around operating points given in Table 4:

Table 4. operating speed of engines

Operating point	1	2	3	4	5	6	7
Angular speed (rpm)	1895	1900	1905	1910	1915	1920	2090

According to Table 4, linear model of engine around seven operating point is required.
In what follows, these models are derived.

ENGINE MODELING

As noted earlier, DC Brushless motor was used in this study. According to reference (Dvorak, 2012) and based on recursive least squares model, this engine was evaluated and the results are shown in Table 5 (Eq.1 to 7).

Table 5. Functions identified for engine model about operating points

Linear function identified around operating points in point …						
1	2	3	4	5	6	7
$G(s)\approx\frac{625}{s+19.02}$ (1)	$G(s)\approx\frac{625}{s+19.1}$ (2)	$G(s)\approx\frac{635}{s+19.12}$ (3)	$G(s)\approx\frac{639}{s+19.2}$ (4)	$G(s)\approx\frac{649}{s+19.21}$ (5)	$G(s)\approx\frac{660}{s+19.24}$ (6)	$G(s)\approx\frac{670}{s+19.3}$ (7)

MODELING THE BODY

This section deals with nonlinear modeling of quadrotor. This model is used in the following sections to be simulated in the Simulink environment of MATLAB software and design controller. The assumptions used in developing the models include assumptions concerning the dynamic modeling of the body and the modeling of propulsion system. Assumptions concerning the dynamic modeling of the body are as follows:

A) Quadrotor is a rigid vehicle.

B) Quadrotor has the intended symmetrical structure (the inertia matrix is diagonal).

C) Estimating the aerodynamic resistance forces, which are generated by translational and rotational movements of the vehicle, exerted on the body of the quadrotor with a linear function of rotational and translational speed (Madani and Benallegue, 2006).

And assumptions used in modeling the propulsion system include:

A) Estimating the resistance torque acting on the propeller as a proportional function of the square of the propeller's velocity (Meriam and Kraige, 2007).

B) Estimating the thrust force as a proportional function of the square of the propellers' velocity (Dvorak, 2012).

COORDINATE SYSTEM

Quadrotor modeling requires defining various coordinate systems, each of which have different applications. For example, to determine the movement equations, a coordinate system fixed on the quadrotor is required; forces and torques are measured in body coordinate system and GPS finds positions in the fixed land system. Therefore, the knowledge of coordinate systems and their applications is a perquisite. The necessary coordinate systems have been defined in Table 6. Fig.6 show the body and inertial coordinate system

Using definitions presented in Table 6, the transfer equation from body coordinate system to inertial coordinate system was achieved according to Eq. 8

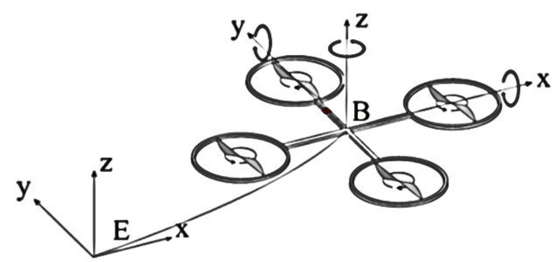


Figure 6. Representation of body and inertial coordinate systems (the system attached to quadrotor is the body coordinate system and the system on the left is inertial coordinate system [3]

$$R_b^i = R_b^v = R_{v_1}^v \times R_{v_2}^{v_1} \times R_{v_b}^{v_2} \quad (8)$$

Body Modeling Using Newton-Euler's Method

In mathematical modeling of the body, the famous Newton – Euler's formula is used. For the translational and rotational motion, Newton – Euler's formula is defined according to Eq. 9 and Eq. 10:

$$\vec{F} = \frac{d(m\vec{v})}{dt} \quad (9)$$

$$\vec{M} = \frac{d(\vec{H})}{dt} \quad (10)$$

Where $\vec{F}$ is the sum of external forces and $\vec{M}$ is the sum of external torques acting on the vehicle.

Linear velocity of the vehicle $\vec{V}$ and $\vec{H}$ is the angular momentum of the vehicle. The main forces acting on quadrotor are mass and lift forces. These equations can be written in the body coordinate system and inertia coordinate systems. To derive equations form body coordinate system, forces need to be displayed in body coordinate system. In Eq. 11, the gravity force is displayed in body coordinate system and in Eq. 12, the total force acting on the body in the body coordinate system are measured.

Table 6. Coordinate systems

Coordinate system	Position	Transition matrix

$F^i:[i^i, j^i, k^i]$	Fixed system and origin point, based on the desired point on the ground	
$F^v:[i^v, j^v, k^v]$	Axis is along system F^i and origin point, based on the gravity center of the vehicle	
$F^{v_1}:[i^{v_1}, j^{v_1}, k^{v_1}]$	Rotated F^v system with angle ψ around k^v axis	$R_V^{V_1}(\Psi)=\begin{pmatrix} C\psi & S\psi & 0 \\ -S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}^*$
$F^{v_2}:[i^{v_2}, j^{v_2}, k^{v_2}]$	Rotated F^{v_1} system with angel θ around j^{v_1} axis	$R_{V_1}^{V_2}(\theta)=\begin{pmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{pmatrix}$
$F^b:[i^b, j^b, k^b]$	Rotated F^{v_2} system with angle ϕ around i^{v_2} axis	$R_{V_2}^{b}(\phi)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & C\phi & S\phi \\ 0 & -S\phi & C\phi \end{pmatrix}$

$$\vec{g}_{body} = R_{\Psi\theta\Phi} \cdot \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} = \begin{bmatrix} +g.s_\theta \\ -g.c_\theta s_\Phi \\ -g.c_\theta c_\Phi \end{bmatrix} \tag{11}$$

$$\vec{F}_{Body} = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} + m. \begin{bmatrix} +g.s_\theta \\ -g.c_\theta s_\Phi \\ -g.c_\theta c_\Phi \end{bmatrix} = m. \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \tag{12}$$

In the above formula, components of linear acceleration are in the direction of body coordinate system and T is the total "lift" force. Lift force and total "lift" force are calculated according to Eq. 13 and Eq. 14:.

$$\bar{T}_i = b\,\omega_i^2, \qquad i = 1 to 4 \tag{13}$$

$$T = \sum_{i=1}^{4} T_i \tag{14}$$

The Eq. 13, b is the aerodynamic coefficient of the thrust force. To obtain transitional movement equations in inertia coordinate system, the total lift force vector needs to be transferred from body coordinate system to inertial coordinate system. As such, we will have (Eq.15) :

$$\vec{F}_{inertial} = \begin{bmatrix} (c_\Psi s_\theta c_\Phi + s_\Psi s_\Phi).T \\ (s_\Psi s_\theta c_\Phi - c_\Psi s_\Phi).T \\ -mg + (c_\theta c_\Phi).T \end{bmatrix} = m. \begin{bmatrix} a_X \\ a_Y \\ a_Z \end{bmatrix} \tag{15}$$

Where a_X , a_Y and a_Z are the components of linear momentum in the direction of inertia coordinate system.
To derive the equations of rotational motion, Newton- Euler's equation is used for rotational movement. In Eq. 10, total angular momentum is made of two angular momentums, one generated by body rotation and the other generated by the propeller's rotation. Eq.16 and eq.17:

$$\vec{H}_{Body} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} . \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{16}$$

$$\vec{H}_{Blade} = \begin{bmatrix} 0 \\ 0 \\ J_r \Omega_{blades} \end{bmatrix} \tag{17}$$

Where $\vec{H}_{Body}$ is angular momentum of the body, $\vec{H}_{Blade}$ is the angular momentum caused by rotation of the propeller in body coordinate system and Ix, Iy and Iz are the moments of inertia of the vehicles. Because of body symmetry, the products of inertia became zero. Ω_{blades} is the difference between clockwise and counterclockwise propeller's rotation. As a result, the angular momentum in body coordinate system is calculated according to Eq. 18.

$$\vec{H} = \begin{bmatrix} I_x \omega_x \\ I_y \omega_y \\ I_z \omega_z + J_r \Omega_{blades} \end{bmatrix} \tag{18}$$

Now, the total drag force in inertia coordinate system needs to be computed, which requires general Eq. 19 [17]. This equation is used to calculate torque in inertial coordinate system based on angular moments in body coordinate system as well as the rotation velocity of body coordinate system.

$$\vec{M} = \left.\frac{d(\vec{H})}{dt}\right)_i = \frac{d(\vec{H})}{dt} + \Omega \times \vec{H} \quad (19)$$

In the above equation, $\vec{M}$ is external torque vector acting on inertia coordinate system and H is angular momentum in body coordinate system that rotates with angular velocity of $\vec{\Omega} = [\omega_x \quad \omega_y \quad \omega_z]$. Drawing on Eq. 18 and Eq. 19, the following equation is derived: (20).

$$\left.\frac{d(\vec{H})}{dt}\right)_i = \begin{bmatrix} I_x\dot{\omega}_x + \omega_y\omega_z\left(I_z - I_y\right) + J_r\Omega\omega_y \\ I_y\dot{\omega}_y + \omega_x\omega_z\left(I_x - I_z\right) + J_r\Omega\omega_x \\ I_z\dot{\omega}_z + \omega_x\omega_y\left(I_y - I_z\right) \end{bmatrix} \quad (20)$$

In Eq. 20, the angular velocity of the body can be calculated using Euler's angular velocity as follows (21) :

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ 0 \\ 0 \end{bmatrix} + R_{V_2}^{b}(\varphi)\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R_{V_2}^{b}(\varphi)R_{V_1}^{V_2}(\theta)\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \quad (21)$$

Which results in (22):

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & sin\varphi\ tan\theta & cos\varphi\ tan\theta \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi\ sec\theta & cos\varphi\ sec\theta \end{bmatrix}^{-1} \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (22)$$

To find the left side of Eq.19, the external torques acting on the system should be calculated. Such torques is the result of lift or moments differences caused by rotations of the propeller. External torques in the body coordinate system is calculated according to Eq. 23.

$$\vec{M}_b = \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{bmatrix} \left(T_1 - T_3\right).l \\ \left(T_4 - T_2\right).l \\ \left(-\tau_1 + \tau_2 - \tau_3 + \tau_4\right) \end{bmatrix} \quad (23)$$

In the above equation, τ_x , τ_y and τ_z are respectively roll, pitch and yaw torques; l is the distance between the center of body mass and the axis of propeller rotation and τ_i is the resistant torque, which is generated in the opposite direction of propeller rotation and is exerted on the propeller by air. This torque is defined as follows (24) :

$$\tau_i = \mathrm{d}\omega_i^2 \quad \mathrm{i} = 1 \text{ to } 4 \quad (24)$$

In Eq. 24, d is the aerodynamic coefficient of resistant torque. Now, the external torques acting on inertia coordinate system can be achieved (25) :

$$\vec{M} = R_{\Psi\theta\Phi}\vec{M}_b \quad (25)$$

AERODYNAMIC RESISTANCE

What is meant by aerodynamic resistances, which are added to the equations in this section, are forces and torques exerted on the vehicle by the air. Given the small cross section of the quadrotor and its low velocity (1 to 2 meters per second), the effect of resistant forces will be negligible. However, the following two equations are used to add these two to the set of equations (Eq. 26 and Eq. 27).

$$\vec{F}_{ad} = -t_{ad}\vec{V} \quad (26)$$

$$\vec{M}_{ad} = r_{ad}\Omega \quad (27)$$

In the above equations, r_{ad} and t_{ad} are the coefficients of total drag force acting on a body in translational movement and total drag force acting on the body in rotational movement. These two equations yield force in inertia coordinate system [13].

DEFINING NEW CONTROL INPUTS

In determining the system inputs, i.e., control inputs, it is necessary to understand the correct structure of the system and control targets. Actual system inputs are the type of inputs that are fed to the system during operation. As to the quadrotor, actual inputs are voltage systems. In many cases, control targets need to define other inputs, which are known as virtual inputs.

In this study, the following categories of inputs are introduced as input virtual to the system.

$U_1 = V_1 - V_3$: Rotation around X horizontal axis

$U_2 = V_4 - V_2$: Rotation around Y horizontal axis

$U_3 = V_1 - V_2 + V_3 - V_4$: Rotation around Z axis

$U_4 = V_1 + V_2 + V_3 + V_4$:   Movement along Z axis

As it can be seen, the new inputs are a mixture of system actual inputs. If system actual inputs are used to control system, they cannot provide a detailed understanding of the system behavior. For example, it is not possible to determine the kind of movement generated by an increase or decrease in engine voltage (which is one of the system actual inputs), without observing other system inputs. However, since the voltage corresponding to the external force and torque are generated by the engine and the propeller, an increase or decrease in the first input of the new control inputs system is equal to an increases or decreases in the difference of two opposite engines. Given the description provided in Section 2, zero, positive or negative values of the difference between opposite engines would lead to the stability, clockwise and counterclockwise rotation around the axis perpendicular to the connecting axis of two engines. Thus, the new set of inputs provides a better description of subsequent behavior of the system.

Dynamic Simulation Of The System In Simulink Environment Of MATLAB Software

The propulsion system has been simulated in the Simulink environment of MATLAB software.

In Fig. 6m it has been shown as a sub-system of propulsion system-total

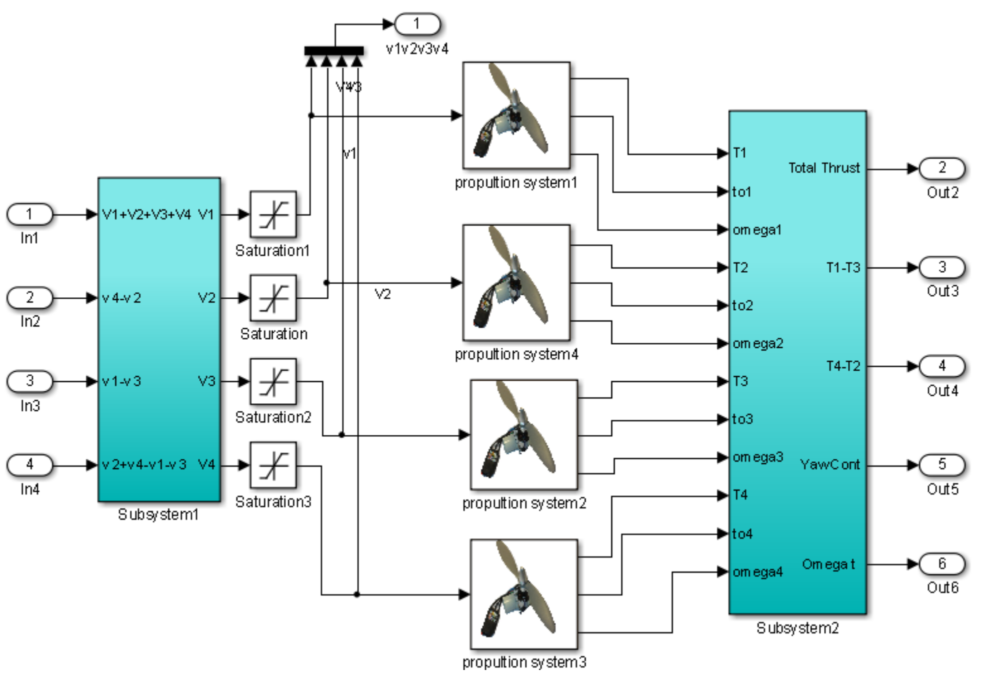


Figure 7. Sub-System Of Propulsion System-Total

In Fig. 6, propulsion system block contains the motor and propeller. In Fig. 7, a part of body modeling, which has been modeled under the title of Rigid Body in Simulink environment, has been shown.

In this subsystem, inputs, forces and the total drag force are exerted on the center of mass of the vehicle. Total drag forces, before entering the body dynamic calculations sub-system, which has been written based on Newton - Euler's law, is transferred from body coordinate system to inertia coordinate system. Fig. 8 shows dynamic model sub-system. In this sub-system, propulsion system and the transition of forces and moments from body coordinate system to inertia coordinate system are added to the Rigid Body sub-system

ISSN:2250-0138(Online)
ISSN : 0976-2876 (Print)

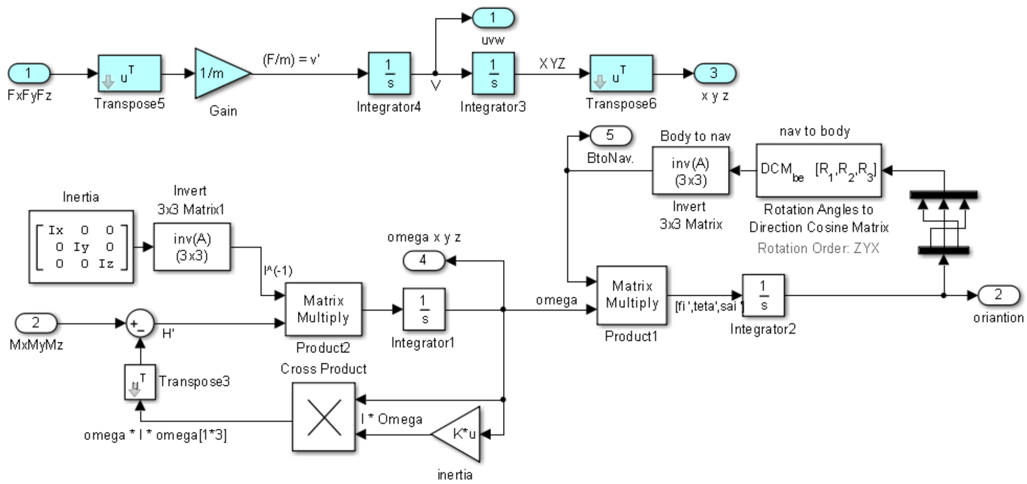


Figure 8. **Rigid Body Subsystem: simulator of dynamic equations - the relationship between forces and the location of the vehicle**

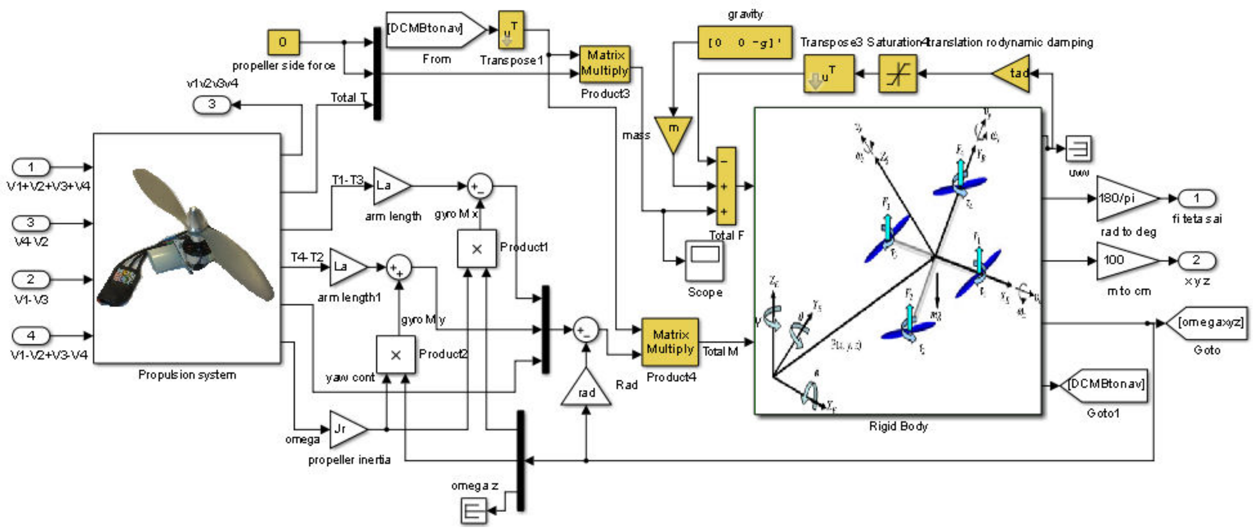


Figure 9.

Figure 10. Dynamic model subsystem

MODEL EVALUATION

This section examines the reliability of the dynamic mo the designed simulator. Verification is conducted on the using the following five tests:

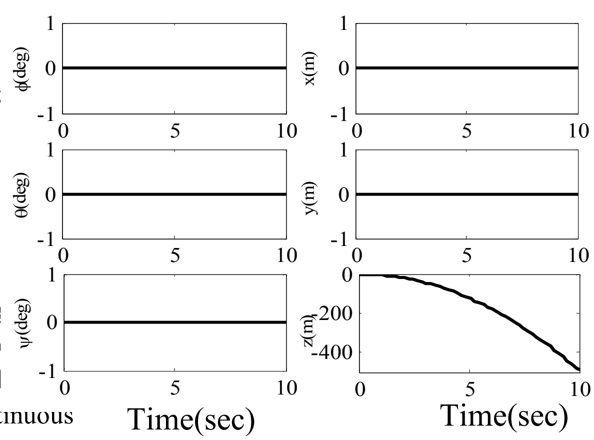


Figure 11. system response in the test in which all inputs are zero

TEST ONE

In this test, all inputs are equal to zero. Since all volta zero and only weight force is exerted, vehicle is expe stay on the ground. Given the fact that ground proviso l been taken into account, this is equivalent to a continuous decrease of the height. Fig. 9 shows the system outputs in terms of four zero inputs.As it can be sees, the system performance is as expected.

---

[1]**Corresponding author**

TEST TWO

In this test, all system inputs except for the lift force are estimated to be zero. The total force is equal to mg given to the system. The lack of difference between opposite engines halts the rotary movement around the horizontal axes and vertical axes. Further, quadrotor is expected to stay in place without any variation in its height. The output torque of engines should also halt any movement. The system response to the test is shown in Fig. 10.

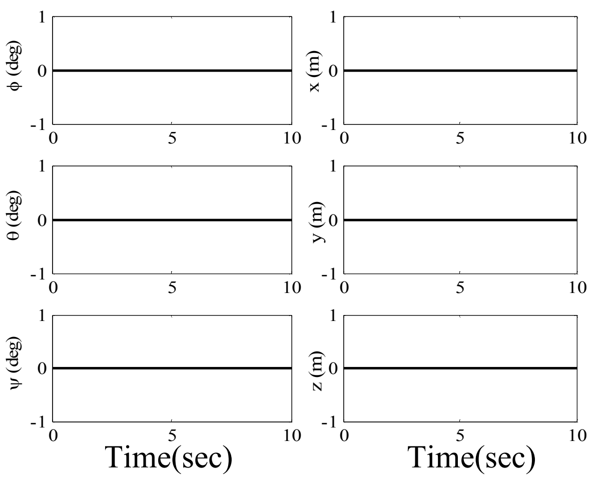


Figure 12. System response in a test in which all inputs except for lift force are equal to zero and total lift force equals the mass

TEST THREE

In this test, total lift force equals the mass, the difference between the two opposite engines (the first and the third engine) equals zero, the difference between the first and the fourth engines is a positive value and the sum of resisting torques equals zero. It is expected that the vehicle begins rotating about one of its horizontal axes, and moves along another horizontal axis. Further, since the rotation of vehicle around horizontal axis has a reducing effect on the vertical component of total lift, the height of vehicle is decreased. The results of these tests are shown in Fig.11, which indicate the compatibility of the system performance with the expectations of the tests.

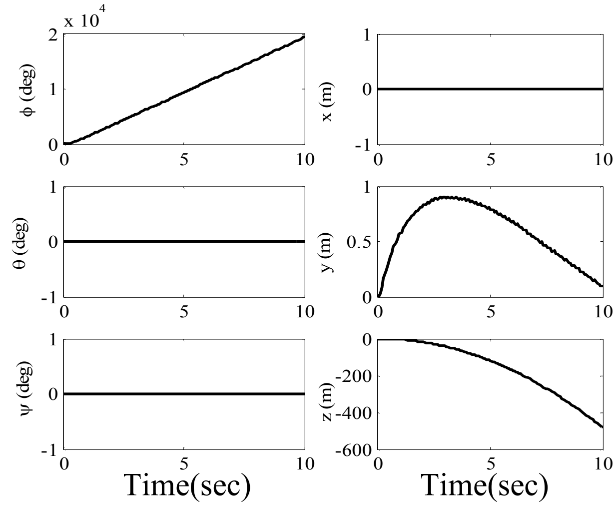


Figure 13. Fig. 11: System response in the test in which total lift force equals mass, the difference between the first and the third engines equal a positive value and other inputs are equal to zero

TEST FOUR

In this test, total lift force is equal to the mass; the sum of resistant torques is equal to a positive value in the system and other inputs are zero. In this test, it is expected that the system begins rotating about z axis. The system response to the test is shown in Fig. 12. As it can be seen, test four also was able to fulfill the intended expectations.

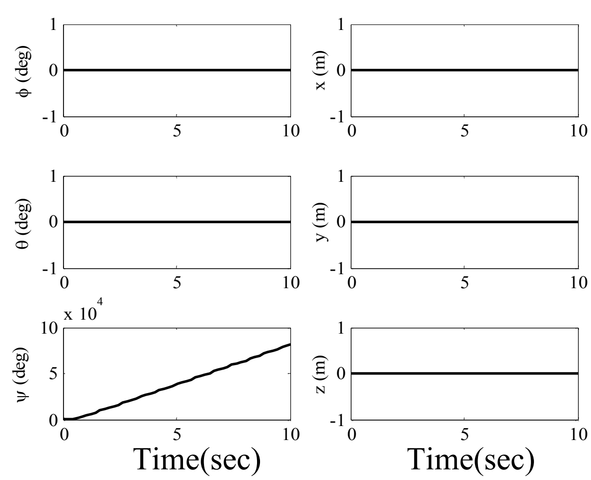


Figure 14. System response to the test in which total lifts force equals the mass, resistant torques is equal to a positive value and other inputs are zero

TEST FIVE

In this test, by feeding positive input into the total velocity of engines, and feeding positive input into the force difference between the first and the third engines, it is expected that propeller rotation about z axis and the body rotation around its horizontal axis generate rotation around the other axis. In fact, in this test, the gyroscopic movement of the system is investigated. Fig.13 shows the system output in this test.

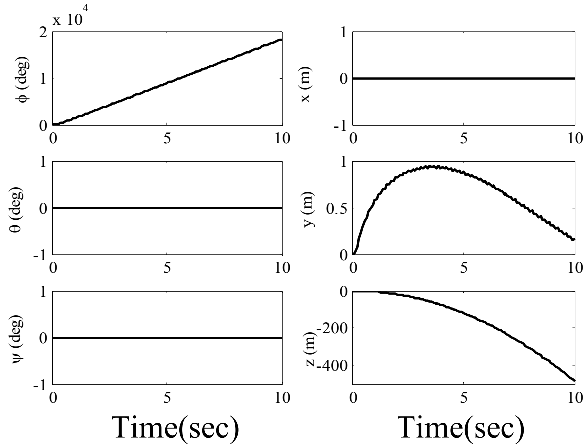


Figure 15. gyroscopic effect of the propeller

As it can be observed, gyroscopic phenomenon has not affected outputs. The torque generated by the gyroscope of propeller is equal to 3.005×10^{-5}. Since the inertia of propeller is insignificant, the gyroscopic thrust force would be paltry, making it unable to generate a significant movement.

ANGLES CONTROL

In this section, angles are controlled in the buoyancy phase and PD controller is used. Control laws used in Eq. 28 to 30 are shown below.

$$U_1 = Kp_1(\varphi_d - \varphi) + Kd_1(\dot{\varphi}_d - \dot{\varphi}) \tag{28}$$

$$U_2 = Kp_2(\theta_d - \theta) + Kd_2(\dot{\theta}_d - \dot{\theta}) \tag{29}$$

$$U_3 = Kp_3(\psi_d - \psi) + Kd_3(\dot{\psi}_d - \dot{\psi}) \tag{30}$$

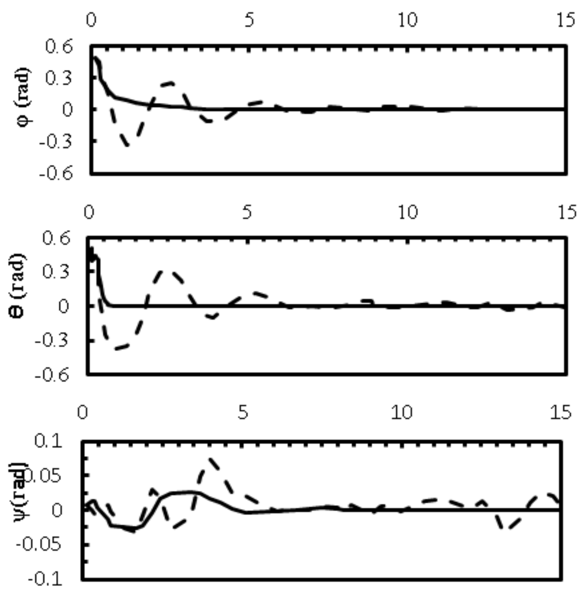
Controller's coefficients were obtained by trial and error. These coefficients are shown in Table 7:

Table 7. Coefficients of angles controllers

Coefficient	Value	Coefficient	Value
Kp_1	0.4	Kd_1	0.055
Kp_2	0.25	Kd_2	0.06
Kp_3	-4	Kd_3	-2

To evaluate the efficacy and accuracy of the presented control laws, they were used to control the stability of quadrotor in Hover movement condition. In this case, quadrotor, which is floated 1 meter above the ground, is released with initial conditions. Quadrotor is expected to reach equilibrium state ($\varphi = \theta = \psi = 0 rad.$) in an optimal time.

Fig.14 shows the variation of quadrotor's external angles after applying the above rules and conditions. Further, the findings of Mian andWang (Mian and Wang, 2007), which is shown by dashed line, have been shown for the comparison of the results. In their study, Mian and Wang examined a quadrotor control (0.6 m and 0.6 kg) using feedback linearization method.



T
ime response of roll, pitch and yaw angels of the quadrotor with the initial conditions (dashed line in graph: Mian and Wang's response; solid graph, response to the designed controller)

As it can be seen, in this study the effective time of roll and pitch responses for the quadrotor was about 0.7s, which compared to the study of Mian and Wang (6 s) was 85% faster. Also, yaw angels were controlled using PD controller.

CONCLUSION

This paper presented a nonlinear model for quadrotor STOL vehicle. The model consists of two parts: a dynamic model and the propulsion system model. In body modeling, Newton-Euler's method was used. Moreover, the propulsion system in each phase was modeled separately using recursive least squares method. To do so, first the movement phase was defined and operating points were identified. The model switches on propulsion system in each movement phase. An advantage of this model was convenience of obtaining nonlinear model. Such modeling helps researchers include propulsion system in their study model. After that, it was simulated in the Simulink environment of MATLAB software. Finally, the design and implementation of six tests on the model confirmed the accuracy of the obtained model. In the final section, three angle controllers for controlling design angels were designed and applied to the system. A comparison of this controller with the one proposed by Mian and Wang shows that PD controllers were able to detect system inputs 85 % faster.

REFERENCES

Hoffman, G., Waslander, S., and Tomlin, C.; 2008. Quadrotor Helicopter Trajectory Tracking Control".Proc. of the AIAA Guidance, Navigation, and Control Conference,: 7220 – 7235.

Altug E., Ostrowski, J., P. AND Mahony R.; 2002. Control of a quadrotor helicopter using visual feedback, proceedings of the 2002 IEEE, International conference on Robotics and Automation, Washington DC, May.

Castillo P., Lozano R. and Dzul A.; 2005. stabilization of a mini rotorcraft with four rotors, IEEE control system magazine, December, **25(**6): 45-55

PID vs LQ.; 2008. Control Techniques Applied to an Indoor Micro Quadrotor,Bouabdallah, S., Noth, A., Siegwart, R.

Dynamic modeling and nonlinear control strategy for an underactuated quadrotor rotorcraft, Mian, A., A., Wang, D.; 2007. Journal of Zhejiang University Science A(ISSN 1673-56X)

A new robust adaptive-fuzzy control method applied to quadrotor helicopter stabilization, Coza, C., Macnab, C.J.B.; 2006. IEEE.

Robust Low Altitude Behavior Control of a Quadrotor Rotorcraft Through Sliding Modes, Efe, M. O.; 2007. proceedings of the 15th Mediterranean conference on control & Automation Athena-Greece,

Technical report, Draganflyer X-pro modeling and control.; 2006, Alborg university, Department of control engineering

Modeling.; 2011. Identification and control of a quadrotor aircraft, De Lllis, M., Oliveira, C.

Claudia, M., Luminita, C.; 2010. Modeling and control of autonomous quadrotor",Msc thesis, university of Alborg.

Huang, H., Hoffman, G., M., Waslander, S., Tomlin, C., J.; 2009. "Aerodynamics and control of autonomous quadrotor helicopters in aggressive maneuvering", . ICRA '09. IEEE International Conference on Robotics and Automation, Kobe, May 12-17.

Pipatpaibul, P., Ouyang, P. R.; 2011. Quadrotor UAV control: Online learning approach", ASME 2011 international design engineering technical conferences & computers and information in engineering conference, Washington DC, USA.

Mahmudi, S., Norouzi, S., Javareshkian, H., Amini, S.; 2013. Experimental modeling of propulsion system and control of model by a PID tuned by GA" (in Persian), 21th international conference on mechanical engineering (ISME2013), Khaje Nasir university, Tehran, Iran, 2013.

J. Dvorak.; 2012. Micro quadrotor: design, Modeling, Identification and control, BSc Thesis, university of Parague.

Madani, T., Benallegue, A.; 2006. Control of a quad rotor Mini- Helicopter via full state backstepping technique", 45th IEEE conference on decision and control, San Diego, USA,.

Meriam, J. L., Kraige, L. G.; 2007. Engineering Mechanics Dynamics", 6th edition, John Wiley and sons.