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A new switching strategy design for uncertain switched linear systems based on min-projection strategy in guaranteed cost control problem

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An effective way to deal with uncertainties in control of uncertain switched linear systems is so-called guaranteed cost control (GCC). Existing works on the GCC of these systems only provide asymptotical stability analysis. This paper focuses on the GCC to provide exponential stability in these systems. To this end, we design a new switching strategy and a state-feedback controller to exponentially stabilize the GCC of continuous-time switched linear systems with norm-bounded uncertainties. First, a new switching strategy is designed based on the common Lyapunov function technique and min-projection strategy. Second, a sufficient condition on the existence of proposed switching strategy is presented in the form of linear matrix inequalities. It is proved that the system to be exponentially stable under applying this switching strategy and guaranteed cost value and convergence rate of states are calculated. Finally, some numerical examples and comparisons are performed.

Keywords: uncertain switched linear systems; guaranteed cost control; common Lyapunov function; linear matrix inequality; exponential stability.

1. Introduction

The switched linear system is an important class of hybrid systems with a switching signal that allows selecting among subsystems to reach some control objectives. Recently, several researchers have been concerned the control of switched linear systems (see Dacarlo *et al.*, 2000; Daafuz *et al.*, 2002; Zhai *et al.*, 2003; Lin & Antsaklis, 2009). The basic problems in stability and design of switched systems are given by Liberzon & Morse (1999). Many real-world processes such as switched circuits, switching power converters, computer controlled systems and communication networks can be modelled as switched systems (see Deaecto *et al.*, 2010; Solmaz, 2011; Xu *et al.*, 2011, 2012). In designing controller for a real plant, it is necessary to design a control system which is not only being stable but also possesses a strong robust performance. An effective way to deal with uncertainties is so-called guaranteed cost control (GCC) approach proposed by Chang & Peng (1972). This approach provides an upper bound on the given performance index in the presence of uncertainties. Many results are emerged on this topic (see Petersen & McFarlane, 1994; Yu & Chu, 1999; Yu & Gao, 2001; Shi *et al.*, 2003; Park & Jung, 2004; Chen & Liu, 2005; Zhang *et al.*, 2007; Wang *et al.*, 2012; Nian *et al.*, 2013). Based on

GCC approach, some significant results for controlling uncertain switched systems are presented (see Chen *et al.*, 2006; Wang & Zhao, 2006, 2007; Ying & Guangren, 2007; Wen *et al.*, 2008; Wu & Wang, 2009; Zhang *et al.*, 2009a,b; Wang *et al.*, 2010; Tissir, 2011; Zhao, 2011; Tian *et al.*, 2012). Some of them study the GCC of uncertain switched linear systems (see Chen *et al.*, 2006; Wang & Zhao, 2006; Ying & Guangren, 2007; Wen *et al.*, 2008; Zhang *et al.*, 2009a,b; Wang *et al.*, 2010; Zhao, 2011; Tian *et al.*, 2012). In these studies, switching strategy and state-feedback controller are designed by using common Lyapunov function (CLF) or multiple Lyapunov functions techniques. Also, switching strategy is designed based on selecting a subsystem which has the lowest Lyapunov function value. Furthermore, these switching strategies provide asymptotic stability of the overall system. Especially, when the CLF technique is used to design switching strategy, some complex linear matrix inequalities (LMIs) are constructed that should be solved.

Recently, some studies have been performed on the exponential stability analysis and GCC design for delayed switched systems (see Wang & Zhao, 2007; Hien & Phat, 2009a,b; Hien *et al.*, 2009; Wu & Wang, 2009; Tissir, 2011). In Wu & Wang (2009), the exponential stability is guaranteed after presenting a sufficient condition in terms of a set of LMIs using the average dwell time approach and piecewise Lyapunov function method. In another literature, a sufficient condition for exponential stability and weighted guaranteed cost performance is developed in terms of LMIs for the delayed systems with actuator failures (see Wang & Zhao, 2007). In more recent work Tissir (2011), new delay-dependent conditions is presented based on CLF method, to guarantee both the exponential stability and an upper bound of some performance index. Moreover, in Hien & Phat (2009a), (2009b) and Hien *et al.* (2009) some significant works on exponential stability and stabilization of a class of uncertain linear time-delay systems without considering guaranteed cost function have been studied. See also some important recent results (see Cao & Wang, 2004; Ren & Cao, 2006; Xia & Cao, 2008; Cao *et al.*, 2013; Hu *et al.*, 2013; Liu *et al.*, 2014). The main objective of this paper is to exponentially stabilize and GCC design for uncertain switched linear systems. To this end, a new switching strategy is proposed based on the min-projection strategy (see Pettersson & Lennartson, 2001) and the CLF technique. It is proved that under applying this new switching strategy, the origin of the closed-loop system is exponentially stable. Also, a sufficient condition on the existence of state-feedback guaranteed cost controller is presented in the form of LMIs. It is proved that applying the switching strategy and guaranteed cost controller, guarantees the exponential stability of the closed-loop system. Finally, the upper bound of the cost is obtained and the convergence rate is calculated. The contributions of the paper are as the following:

(i) Presenting a new switching strategy to guarantee the exponential stability of the uncertain switched linear systems.

(ii) Proving that the proposed CLF satisfies the exponential stability theorem of switched systems.

(iii) Evaluating the convergence rate of the states.

This paper is organized as follows. In the next section, problem formulation and preliminaries are given. Section 3 proposes a new switching strategy, which guarantees the exponential stability of the overall system. Also, sufficient condition on the existence of guaranteed cost controller is presented. Sections 4 and 5 are dedicated to illustrative examples and conclusion, respectively.

NOTATION 1.1 The following notations will be used throughout this paper. m is some arbitrary positive integer, $\underline{m} = \{1, 2, \ldots, m\}$ is the set of subsystem indices, $\|\cdot\|$ denotes the standard Euclidean norm in R^n and $\lambda(A)$ stands for eigenvalues of matrix A.

2. Problem formulation and preliminaries

An uncertain switched linear system can be considered as

$$\begin{aligned} \dot{x} &= (A_{\sigma(x,t)} + \Delta A_{\sigma(x,t)})x(t) + (B_{\sigma(x,t)} + \Delta B_{\sigma(x,t)})u(t), \\ x(t_0) &= x_0 \in R^n, \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^q$ is the continuous control input vector and $\sigma(x,t) \in \underline{m}$ is the piecewise constant discrete switching signal that determines the discrete mode. $A_i \in R^{n\times n}$ and $B_i \in R^{n\times q}$, $i \in \underline{m}$ are the dynamics of each subsystem with appropriate dimensions. ΔA_i and ΔB_i, $i \in \underline{m}$ are uncertainties satisfying the following assumption.

ASSUMPTION 2.1 The time-varying parameter uncertainties ΔA_i and ΔB_i, of the system (2.1) have the following form:

$$[\Delta A_i, \Delta B_i] = N_i F_i [C_i, D_i], \quad i \in \underline{m}, \tag{2.2}$$

where C_i, D_i and N_i are known matrices with appropriate dimensions and $F_i, i \in \underline{m}$ are unknown time-varying matrices with Lebesgue measurable elements satisfying

$$F_i^\top F_i \leqslant I, \quad i \in \underline{m}. \tag{2.3}$$

Consider the cost function for the uncertain switched system (2.1) as follows:

$$J = \int_0^\infty (x^\top Q x + u^\top R u)\,\mathrm{d}t, \tag{2.4}$$

where $Q \in R^{n\times n}$ and $R \in R^{q\times q}$ are symmetric positive definite weighted matrices. The objective of this paper is to find the state-feedback controller $u = K_i x(t)$, where $K_i \in R^{q\times n}, i \in \underline{m}$ such that, the uncertain switched systems (2.1) be exponentially stable and the cost function (2.4) satisfies $J \leqslant J^*$ where J^* is a guaranteed cost, which is defined in Definition 2.1.

Before presenting our main results, we introduce some necessary definitions, lemmas and theorems.

DEFINITION 2.1 (see Wang & Zhao, 2006) For the uncertain switched system (2.1), if there exist a state-feedback control $u^*(t)$ for each subsystem, a switching law $\sigma^*(t)$ and a positive scalar J^* such that for all admissible uncertainties, the closed-loop system be asymptotically stable and the value of the cost function (2.4) satisfies $J \leqslant J^*$, then, J^* and u^* are said to be guaranteed cost value (GCV) and guaranteed cost control law (GCCL).

DEFINITION 2.2 (see Hien & Phat, 2009a,b) The system (2.1) is said to be exponentially stable under applying switching signal $\sigma(t)$, if the solution $x(t)$ of the system (2.1) satisfies

$$\|x(t)\| \leqslant k_1 e^{-k_2 t}\|x(0)\|, \tag{2.5}$$

where $k_1 > 0$ and $k_2 > 0$, and $x(0)$ is the initial state.

LEMMA 2.1 (Schur complement in Boyd *et al.*, 1994) For any matrices S, P and Q so that $Q > 0$, then

$$\begin{bmatrix} P & S \\ S^\top & -Q \end{bmatrix} < 0 \Leftrightarrow P + SQ^{-1}S^\top < 0. \tag{2.6}$$

LEMMA 2.2 (see Hien & Phat, 2009a,b) Let D, E and F be real matrices with appropriate dimensions. If $F^\top F \leqslant I$, then for any scalar $\varepsilon > 0$, the following inequality holds:

$$DFE + E^\top F^\top D^\top \leqslant \varepsilon^{-1} DD^\top + \varepsilon E^\top E. \tag{2.7}$$

LEMMA 2.3 (Rayleigh's inequality in Mayer, 2000) Suppose that $A \in R^{n\times n}$ is a real symmetric positive definite matrix, then

$$\lambda_{\min}(A)\|x\|^2 \leqslant x^\top Ax \leqslant \lambda_{\max}(A)\|x\|^2, \tag{2.8}$$

where $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the smallest and largest eigenvalues of A, respectively.

THEOREM 2.1 Consider the uncertain switched system (2.1) with $u(t) = K_i x(t)$ in the form of $\dot{x} = f_i(x)$, $i \in \underline{m}$, where $f_i(x) = (A_i + B_iK_i + N_iF_i(C_i + D_iK_i))x(t)$ and $W(x) = \{f_i(x),\ \ i \in \underline{m}\}$, where F_i, $i \in \underline{m}$ satisfies (2.3). If there exist positive-definite matrix P and positive scalar γ such that for all states x at least some $f_i(x) \in W(x)$ satisfies

$$2x^\top Pf_i(x) \leqslant -\gamma\|x\|^2, \tag{2.9}$$

then, under the switching strategy (2.10) the switched system (2.1) to be exponentially stable.

$$\sigma(t) = \arg\min_{i\in\underline{m}} x^\top Pf_i(x). \tag{2.10}$$

Proof. This theorem is an extension of min-projection switching strategy in uncertain switched systems (see Pettersson & Lennartson, 2001). It can be proved by using the Lyapunov function $V(x) = x^\top Px$, whose time derivative satisfies $\dot{V}(x) \leqslant -\gamma x^\top x, \gamma > 0$ for all trajectories. If the conditions in Theorem 2.1 are satisfied, the trajectory converges to the origin, according to (2.5). To prove exponential stability, it must be shown that there exist two positive numbers $k_1 = \sqrt{\lambda_{\max}/\lambda_{\min}}$ and $k_2 = \gamma/2\lambda_{\max}$ such that (2.5) holds, where $\lambda_{\min}$ and $\lambda_{\max}$ denote the smallest and largest eigenvalues of positive-definite matrix P, respectively. The largest positive constant k_2 which may be utilized in (2.5) is called the rate of convergence (Pettersson & Lennartson, 2001, for more details). □

3. Main results

In this section, first the new switching strategy is presented. Then, a sufficient condition for the existence of guaranteed cost controllers $u = K_i x(t)$ is derived. Also, the controller gains K_i, $i \in \underline{m}$ and symmetric positive-definite matrix P are calculated via solving a set of LMIs.

THEOREM 3.1 For given positive scalars δ_i, $i \in \underline{m}$, if there exist a symmetric positive-definite matrix P, positive scalars ε_i and matrices K_i, $i \in \underline{m}$, with proper dimensions, such that the following inequality holds:

$$\sum_{i=1}^{m} \delta_i[PAc_i + Ac_i{}^\top P + \varepsilon_i PN_iN_i^\top P + \varepsilon_i^{-1}E_iE_i{}^\top + Q + K_i^\top RK_i] < 0, \tag{3.1}$$

where $Ac_i = A_i + B_iK_i$ and $E_i = C_i + D_iK_i$, $i \in \underline{m}$, then the system (2.1) is exponentially stable under the following stabilizing switching strategy:

$$\sigma(x,t) = \operatorname*{argmin}_{i\in\underline{m}}\{x^\top Z_i x\}, \tag{3.2}$$

where

$$Z_i = PAc_i + Ac_i^\top P + \varepsilon_i PN_iN_i^\top P + \varepsilon_i^{-1} E_iE_i^\top + Q + K_i^\top RK_i \tag{3.3}$$

and GCCL is $u = K_ix(t)$. Moreover, the GCV is given by $J^* = x^\top(0)Px(0)$.

Proof. First, let us define the following sectoral sets of R^n:

$$L(Z_i) = \{x \in R^n : x^\top Z_i x < 0\}, \quad i \in \underline{m}.$$

From (3.1) and (3.3), we have $\sum_{i=1}^m \delta_i Z_i < 0$ which yields

$$\bigcup_{i=1}^{m} L(Z_i) = R^n \backslash \{0\}.$$

Thus, for any $\forall x \in \mathbb{R}^n, x \neq 0$, there is an index $i \in \underline{m}$ such that $x^\top Z_i x < 0$.

Second, we propose $V(x) = x^\top Px$ as a CLF for the uncertain switched linear system (2.1) and we show that by selecting $\sigma(x,t) = \operatorname{argmin}_{i \in \underline{m}}\{x^\top Z_i x\}$ as a switching strategy, for any $t \in R$ there exist an index $i \in \underline{m}$ such that

$$\dot{V}(x) < -x^\top Qx - x^\top K_i^\top RK_ix = -x^\top(Q + K_i^\top RK_i)x.$$

To do this, substituting $u = K_ix(t)$ into (2.1) and combining (2.2), we have

$$\dot{x} = (A_i + B_iK_i + N_iF_i(C_i + D_iK_i))x(t) \tag{3.4}$$

and

$$\dot{V}(x) = x^\top[P(A_i + B_iK_i) + (A_i + B_iK_i)^\top P + (C_i + D_iK_i)F_iN_i^\top P + PN_iF_i^\top (C_i + D_iK_i)^\top]x. \tag{3.5}$$

Applying Lemma 2.2, we have

$$(C_i + D_iK_i)F_iN_i^\top P + PN_iF_i^\top(C_i + D_iK_i)^\top \leqslant \varepsilon_i PN_iN_i^\top P + \varepsilon_i^{-1}(C_i + D_iK_i)(C_i + D_iK_i)^\top. \tag{3.6}$$

Now, by adding $P(A_i + B_iK_i) + (A_i + B_iK_i)^\top P + Q + K_i^\top RK_i$ to the both sides of (3.6) and pre and post multiplying with $x^\top$ and x, respectively, we have

$$\begin{aligned} x^\top &\begin{bmatrix} P(A_i + B_iK_i) + (A_i + B_iK_i)^\top P + (C_i + D_iK_i)F_iN_i^\top P + \\ PN_iF_i^\top(C_i + D_iK_i)^\top + Q + K_i^\top RK_i \end{bmatrix} x \\ &\leqslant x^\top \begin{bmatrix} P(A_i + B_iK_i) + (A_i + B_iK_i)^\top P + \varepsilon_i PN_iN_i^\top P + \\ \varepsilon_i^{-1}(C_i + D_iK_i)(C_i + D_iK_i)^\top + Q + K_i^\top RK_i \end{bmatrix} x. \end{aligned} \tag{3.7}$$

From (3.5) and (3.7), we have

$$\dot{V}(x) + x^\top(t)(Q + K_i^\top RK_i)x(t) \leqslant x^\top(t)Z_ix(t).$$

Since for any $\forall x \in R^n, x \neq 0$ there is an index $i \in \underline{m}$ such that $x^\top Z_i x < 0$, thus, by selecting $\sigma(x,t) = \operatorname{argmin}_{i \in \underline{m}}\{x^\top Z_i x\}$ as a switching strategy and for any $t \in R$, there exists an index $i \in \underline{m}$ such that $x^\top Z_i x < 0$ and consequently results

$$\dot{V}(x) < -x^\top Qx - x^\top K_i^\top RK_i x = -x^\top (Q + K_i^\top RK_i)x. \tag{3.8}$$

Since Q and R are positive-definite matrices, then $G_i = Q + K_i^\top RK_i$ is positive-definite matrix for any $i \in \underline{m}$. Then by using Lemma 2.3, the following inequality holds:

$$-\lambda_{\max}(G_i)\|x\|^2 \leqslant -x^\top G_i x \leqslant -\lambda_{\min}(G_i)\|x\|^2 \quad \forall x \in R^n,\ i \in \underline{m}. \tag{3.9}$$

Now, by choosing

$$\gamma = \lambda_{\min}(G) = \min_{i\in\underline{m}}(\lambda_{\min}(G_i)) \tag{3.10}$$

and based on Theorem 2.1, conditions of exponential stability holds and the system (2.1) is exponentially stable under switching strategy (3.2) which is an extension of min-projection strategy (2.10) and state-feedback controller $u = K_i x(t)$. □

REMARK 3.1 Based on Theorem 2.1 and since $\dot{V}(x)$ is equal to $2x^\top Pf_i(x)$, then, by selecting $k_1 = \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}$, $k_2 = \lambda_{\min}(G_i)/2\lambda_{\max}(P)$, $i \in \underline{m}$, the trajectory of system (2.1) converges according to

$$\|x(t)\| \leqslant \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} e^{-\lambda_{\min}(G_i)/2\lambda_{\max}(P)t}\|x(0)\|. \tag{3.11}$$

To find the GCV, let T be an arbitrary positive integer, taking the sum both sides of (3.8) from $t=0$ to $t = T$, we obtain

$$\int_0^\top \dot{V}(x)\,\mathrm{d}t \leqslant -\int_0^\top (x^\top Qx + x^\top K_i^\top RK_i x)\,\mathrm{d}t. \tag{3.12}$$

Let $T \to \infty$ then

$$\int_0^\infty (x^\top Qx + x^\top K_i^\top RK_i x)\,\mathrm{d}t < -(V(\infty) - V(x(0)) = V(x(0))) = x_0^\top Px_0. \tag{3.13}$$

It follows from (3.13) that the cost function $J \leqslant J^* = x(0)^\top Px(0)$. Then proof is completed.

REMARK 3.2 It is noted that for implementation of switching strategy (3.2), we need to find the matrix P, the state-feedback gains K_i and the scalars $\varepsilon_i > 0$. In the following theorem, by applying Schur complement (Lemma 2.1) we show that (3.1) is equal to LMIs given by (3.14). Therefore, solving (3.14) results matrices P, K_i and scalars ε_i and then, stabilizing switching strategy (3.2) can be implemented.

THEOREM 3.2 For given positive scalars δ_i, $i \in \underline{m}$, assume that there exist positive scalars ε_i, invertible symmetric positive-definite matrix X and matrices Y_i, $i \in \underline{m}$, such that the following inequality holds for any admissible uncertainty satisfies (2.3):

$$\begin{bmatrix} S_i & M & X & Y' \\ M^\top & -\mathrm{diag}(\varepsilon_1 I, \ldots, \varepsilon_m I) & 0 & 0 \\ X & 0 & -Q^{-1} & 0 \\ Y'^\top & 0 & 0 & -\mathrm{diag}(\underbrace{R, \ldots, R_m})^{-1} \end{bmatrix} < 0, \qquad X > 0, \tag{3.14}$$

where

$$S_i = X\left(\sum_{i=1}^{m} \delta_i A_i\right) + \left(\sum_{i=1}^{m} \delta_i A_i\right)^\top X + \left(\sum_{i=1}^{m} \delta_i B_i Y_i\right) + \left(\sum_{i=1}^{m} \delta_i B_i Y_i\right)^\top + \left(\sum_{i=1}^{m} \varepsilon_i N_i N_i^\top\right),$$
$$M = [\delta_1^{1/2}(D_1 Y_1 + C_1 X)^\top, \ldots, \delta_m^{1/2}(D_m Y_m + C_m X)^\top], \tag{3.15}$$
$$Y' = [\delta_1{}^{1/2} Y_1, \ldots, \delta_m{}^{1/2} Y_m], \quad i \in \underline{m}$$

then, condition (3.1) holds and switching strategy (3.2) can be implemented.

Proof. Let $P = X^{-1}$and $K_i = Y_i X^{-1},\ i \in \underline{m}$. By using complement Schur Lemma 2.1, it is concluded (3.14) is equivalent to

$$\sum_{i=1}^{m} \delta_i [PAc_i + Ac_i{}^\top P + \varepsilon_i P N_i N_i^\top P + \varepsilon_i^{-1} E_i E_i{}^\top + Q + K_i^\top R K_i] < 0. \tag{3.16}$$

Consequently, condition (3.1) of Theorem 3.1 holds. Thus by solving (3.14), the switching strategy (3.2) can be implemented and then, system (2.1) is exponentially stable under the strategy (3.2) as concluded in Theorem 3.1. The proof is now completed. □

THEOREM 3.3 Assume that for given positive scalars $\delta_i, i \in \underline{m}$ there exist positive scalars $\varepsilon_i, i \in \underline{m}$, symmetric positive-definite matrix X and matrices $Y_i, i \in \underline{m}$, such that LMIs (3.14) holds. Then, the system (2.1) is exponentially stable under the switching strategy (3.2), where $P = X^{-1}$ and $K_i = Y_i X^{-1},\ i \in \underline{m}$. Moreover, the cost function (2.4) is bounded by $J^* = x^\top(0) P x(0)$.

Proof. The proof of Theorem 3.3 is straightforward from Theorems 2.1, 3.1 and 3.2. □

REMARK 3.3 In summary, to find the GCV J^*, the GCCL $u(t)$ and the switching rule $\sigma(x,t)$, first by selecting arbitrary some positive scalars $\delta_i,\ i \in \underline{m}$, we solve LMIs (3.14) in Theorem 3.2. Then, positive scalars $\varepsilon_i,\ i \in \underline{m}$, invertible symmetric positive-definite matrix X and matrices $Y_i,\ i \in \underline{m}$ are determined and obtained, respectively. Now, $Z_i,\ i \in \underline{m}$ which is defied in (3.3) is known and switching strategy $\sigma(x,t) = \mathrm{argmin}_{i \in \underline{m}}\{x^\top Z_i x\}$ can be derived which selects the subsystem that has the minimum $x^\top Z_i x$.

Then, the GCCL and GCV can be defined as

$$u^*(t) = K_i x(t) = Y_i X^{-1} x(t),$$
$$J^* = x^\top(0) X^{-1} x(0).$$

It is noted that switching strategy (3.2) depends on the states and is state-dependent switching law (surely, each state x is a function of time t).

REMARK 3.4 Our results in this paper can be extended to the following uncertain time-delay switched linear systems:

$$\begin{aligned}\dot{x} &= (A_{\sigma(x,t)} + \Delta A_{\sigma(x,t)})x(t) + (A_{d\sigma(x,t)} + \Delta A_{d\sigma(x,t)})x(t-d) + (B_{\sigma(x,t)} + \Delta B_{\sigma(x,t)})u(t),\\ u(t) &= K_i x(t) + L_i x(t-h),\\ x(t) &= \phi(t), t \in [-H, 0],\\ H &= \max(d, h), d > 0, h > 0.\end{aligned}$$

Where the time-varying parameter uncertainties ΔA_i, ΔA_{di} and ΔB_i of the system have the following form:

$$[\Delta A_i, \Delta A_{di}, \Delta B_i] = N_i F_i [C_i, C_{di}, D_i],$$
$$F_i^\top F_i \leqslant I, \quad i \in \underline{m}$$

(N_i, C_i, C_{d_i} and D_i are known matrices). For this extension, we can use

$$u(t) = K_i x(t) + L_i x(t-h)$$

as a state-feedback controller and a suitable CLF. K_i and L_i are determined by constructing and solving a set of LMIs.

4. Illustrative examples

In order to demonstrate the efficiency of the proposed method, we present the following examples. Without loss of generality, we choose positive scalars δ_i, $i \in \underline{m}$ arbitrary in the examples such that $\sum_{i=1}^m \delta_i = 1$. As a discussion, an optimization algorithm can be used to find optimal value for these scalars such that $\sum_{i=1}^m \delta_i = 1$. As an optimization method, GCV $J^* = x_0^\top P x_0$ can be chosen as an index and optimal value of these scalars can be found based on reaching to lower value of this index. In the first step, positive scalars δ_i, $i \in \underline{m}$ are selected arbitrary and then, LMIs (3.14) is solved and matrix P is obtained. In the second step, $J^* = x_0^\top P x_0$ is calculated and this algorithm is repeated to reach stop condition and finally lowest J^*. Stop condition can be defined based on difference of two consecutive J^*.

EXAMPLE 4.1 Consider the uncertain switched linear system in (2.1) with two subsystems.

$$\begin{aligned}\dot{x} &= (A_{\sigma(x,t)} + \Delta A_{\sigma(x,t)})x(t) + (B_{\sigma(x,t)} + \Delta B_{\sigma(x,t)})u,\\ x_0 &= x(0) = [-1 \quad 2]^\top, \quad \sigma = \{1, 2\}.\end{aligned} \tag{4.1}$$

The system parameters are described as follows:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -5 & 0 \\ -3 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.1 & 0 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 1 & 0 \\ 2 & -2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0.2 \\ 0.4 & 0 \end{bmatrix}, \\
N_i &= \begin{bmatrix} 0 & 0.3 \\ 0.5 & 0 \end{bmatrix}, \quad D_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad F_i(t) = \sin(t), \quad i = 1, 2.
\end{aligned} \tag{4.2}
$$

Also, consider the following symmetric positive-definite weighted matrices:

$$
Q = R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \tag{4.3}
$$

Note that the two subsystems of system (4.1) are unstable.

The aim is to find guaranteed cost controller $u = K_i x(t),\ i \in \{1, 2\}$, switching signal $\sigma(x, t)$ and guaranteed cost $J^* = x_0^\top P x_0$ of the switched system (4.1) with the cost function weighted matrices (4.3). This problem was discussed by Wang & Zhao (2006). Solving the obtained LMIs problem and the optimization problem, they found an upper bound of the cost (2.4) as $J^* = 4.7373$ without solving optimization problem and $J^* = 2.4710$ after solving optimization problem. Here, the results of our proposed method (we call it proposed method) are compared with the results in Wang & Zhao (2006) (we call it Method 1). We perform the following steps for designing the switching signal and guaranteed state-feedback controller in the presented method.

Step 1: Scalars δ_1 and δ_2 are selected as

$$
\delta_1 = 0.5, \quad \delta_2 = 0.5.
$$

Step 2: Solving LMIs (3.14), we obtain

$$
\begin{aligned}
&\varepsilon_1 = 1.0160, \quad \varepsilon_2 = 2.0000, \quad X = \begin{bmatrix} 3.7788 & -1.3822 \\ -1.3822 & 3.6562 \end{bmatrix}, \\
&Y_1 = \begin{bmatrix} -3.0469 & 3.5737 \\ 1.8837 & -0.3342 \end{bmatrix}, \\
&Y_2 = \begin{bmatrix} 0.9990 & 0 \\ -0.0052 & 4.6904 \end{bmatrix}
\end{aligned}
$$

and thus

$$
P = X^{-1} = \begin{bmatrix} 0.3054 & 0.0015 \\ 0.0015 & 0.3854 \end{bmatrix}.
$$

Step 3: Guaranteed cost controller gains are

$$
K_1 = Y_1 X^{-1} = \begin{bmatrix} 0.0585 & 0.6255 \\ 0.3297 & 0.5333 \end{bmatrix}, \quad K_2 = Y_2 X^{-1} = \begin{bmatrix} 1.0010 & 0 \\ 0.0011 & 0.2132 \end{bmatrix}. \tag{4.4}
$$

Step 4: The GCV is $J^* = x_0^\top P x_0 = 1.8410$.

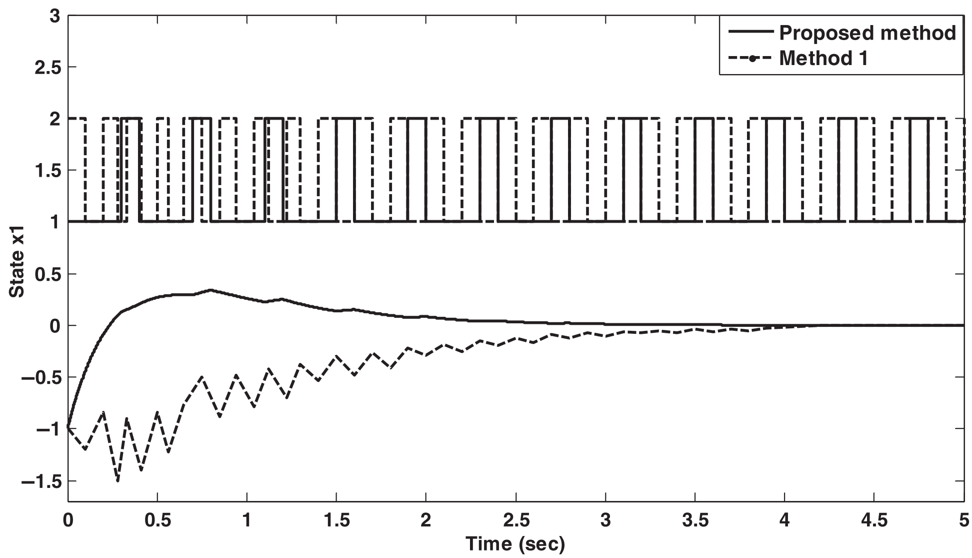

FIG. 1. System state x_1 and switching signals of two methods for system (4.1).

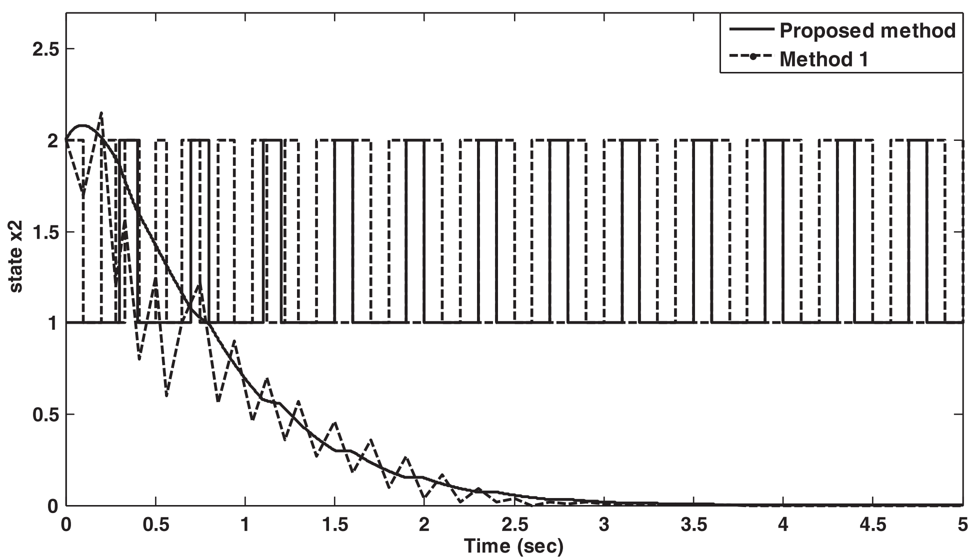

FIG. 2. System state x_2 and switching signals of two methods for system (4.1).

Step 5: The switching strategy is (3.2) for obtained scalars and matrices in Steps 2 and 3.

Step 6: k_1 and k_2 in Theorem 2.1 are obtained as $k_1 = 1.1235$, $k_2 = 0.6732$.

Figures 1 and 2 depict the state x_1, the state x_2 and switching signals of two methods, starting the system from an initial condition $x_0 = x(0) = [-1\ 2]^\top$. It can be seen that theoretical results in Theorems 3.1

and 3.2 coincide with the results in these figures. These results confirm that the proposed switching strategy leads to an exponential stable uncertain switched system as theoretical results suggested. Comparing states in Figs 1 and 2 shows that, the states in proposed method are smoother than states in Method 1 and tend to zero faster. It is shown that the number of switching in our method is less than Method 1 and this is another advantage of the proposed switching strategy over (see Wang & Zhao, 2006). Finally, comparing the GCV obtaining from two methods shows that, GCV J^*in proposed method is less than that obtained by Method 1 in Wang & Zhao (2006). $J^*_{\text{Proposed method}} = 1.8410 < J^*_{\text{Method 1}} = 2.4710$.

EXAMPLE 4.2 Consider the uncertain switched linear system in (2.1) including four subsystems as following:

$$\begin{aligned} \dot{x} &= (A_{\sigma(x,t)} + \Delta A_{\sigma(x,t)})x(t) + (B_{\sigma(x,t)} + \Delta B_{\sigma(x,t)})u, \\ x_0 &= x(0) = [4 \ -6]^\top, \quad \sigma(x,t) = \{1,2,3,4\}. \end{aligned} \tag{4.5}$$

The system parameters are described as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.3 & 0.1 \\ -0.1 & -4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.5 & 0.7 \\ -2 & -1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.1 & 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1.6 & 0.2 \\ -0.2 & 2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0.2 \\ 0.4 & 0 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -1.5 & -0.1 \\ 0.1 & 3.1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.5 & 0.8 \\ -3 & -1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0 & 0.2 \\ 0.4 & 0 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} -3.3 & -0.2 \\ 0.1 & 2 \end{bmatrix}, \quad B_4 = \begin{bmatrix} -0.5 & -0.5 \\ -2.5 & 0.8 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 0 & 0.2 \\ 0.4 & 0 \end{bmatrix}, \\ N_i &= \begin{bmatrix} 0 & 0.3 \\ 0.5 & 0 \end{bmatrix}, \quad D_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad i = 1,2,3,4. \end{aligned} \tag{4.6}$$

Also, consider the following symmetric positive-definite weighted matrices:

$$Q = R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \tag{4.7}$$

Note that all subsystems of system (4.5) are unstable. Letting that the initial state is $x_0 = x(0) = [-6\,4]^\top$ and the time-varying uncertain function $F_i(t),\ i \in \underline{m} = \{1,2,3,4\}$ as shown in Fig. 5 is a random number between -1 and 1.

Here, the results of presented method are compared with the results in Wang & Zhao (2006). First, we simulate the Method 1. We choose the following scalars in designing switching signal and optimal state-feedback controllers for four subsystems as are selected for two subsystems in Wang & Zhao (2006).

$$\begin{aligned} &E_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad H_i = 0.5, \quad i \in \underline{m} = 1,2,3,4, \\ &\beta_{12} = \beta_{13} = \beta_{14} = 1.5, \quad \beta_{21} = \beta_{23} = \beta_{24} = 1, \\ &\beta_{31} = \beta_{32} = \beta_{34} = 2, \quad \beta_{41} = \beta_{42} = \beta_{43} = 1, \end{aligned}$$

$$\begin{aligned}
&\beta_{11}=10, \quad \beta_{22}=10, \quad \beta_{33}=5, \quad \beta_{44}=3, \\
&\lambda_1=\lambda_2=\lambda_3=\lambda_4=0.5, \\
&\varepsilon_{11}=\varepsilon_{12}=\varepsilon_{13}=\varepsilon_{14}=1, \\
&\varepsilon_{21}=\varepsilon_{22}=\varepsilon_{23}=\varepsilon_{24}=1.
\end{aligned} \tag{4.8}$$

Also, we apply Wang & Zhao (2006, Theorems 1 and 2) to design the optimal state-feedback controllers. By solving optimization problem (optimization problem (14) in Wang & Zhao (2006), the optimal positive-definite matrices for each subsystem is obtained as

$$\begin{aligned}
P_{1\text{opt}} &= \begin{bmatrix} 1.4446 & 0.2206 \\ 0.2206 & 1.5096 \end{bmatrix}, \quad P_{2\text{opt}} = \begin{bmatrix} 1.4287 & 0.1835 \\ 0.1835 & 1.4571 \end{bmatrix}, \\
P_{3\text{opt}} &= \begin{bmatrix} 1.4185 & 0.1594 \\ 0.1594 & 1.4214 \end{bmatrix}, \quad P_{4\text{opt}} = \begin{bmatrix} 1.3988 & 0.1212 \\ 0.1212 & 1.4452 \end{bmatrix}
\end{aligned} \tag{4.9}$$

and the optimal gain of state-feedback controllers are designed as

$$\begin{aligned}
\tilde{K}_1 &= \begin{bmatrix} 0.5817 & 1.5648 \\ -0.3953 & 0.6776 \end{bmatrix}, \quad \tilde{K}_2 = \begin{bmatrix} 0.6226 & -0.6368 \\ -1.3369 & 0.5450 \end{bmatrix}, \\
\tilde{K}_3 &= \begin{bmatrix} -0.1156 & 2.0922 \\ -0.4877 & 0.6469 \end{bmatrix}, \quad \tilde{K}_4 = \begin{bmatrix} 0.5012 & 1.8368 \\ 0.3012 & -0.5478 \end{bmatrix}
\end{aligned} \tag{4.10}$$

and the upper bound of performance is $J^* = 65.5707$ after solving optimization problem ((14) in Wang & Zhao, 2006). Also, we perform the following steps for designing proposed switching strategy and guaranteed state-feedback controller.

Step 1: Scalars δ_1, δ_2, δ_3 and δ_4 are selected as

$$\delta_1 = 0.45, \quad \delta_2 = 0.05, \quad \delta_3 = 0.45, \quad \delta_4 = 0.05.$$

Step 2: Solving LMIs (3.14), we obtain

$$\begin{aligned}
&\varepsilon_1 = 1.4502, \quad \varepsilon_2 = 1.1344, \quad \varepsilon_3 = 1.3218, \quad \varepsilon_4 = 1.1573, \\
&P = X^{-1} = \begin{bmatrix} 1.4075 & 0.1781 \\ 0.1781 & 1.1300 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 2.7160 & 0.4126 \\ 0.30374 & 1.7577 \end{bmatrix}, \\
&Y_2 = \begin{bmatrix} 2.37016 & -3.4686 \\ -5.9250 & 0.6622 \end{bmatrix}, \quad Y_3 = \begin{bmatrix} -0.4392 & 0.0174 \\ 0.7372 & 1.6184 \end{bmatrix}, \\
&Y_4 = \begin{bmatrix} 3.5193 & 2.9335 \\ 2.7316 & -1.6375 \end{bmatrix}.
\end{aligned}$$

And thus

$$X = \begin{bmatrix} 0.7249 & -0.1142 \\ -0.1142 & 0.9030 \end{bmatrix}.$$

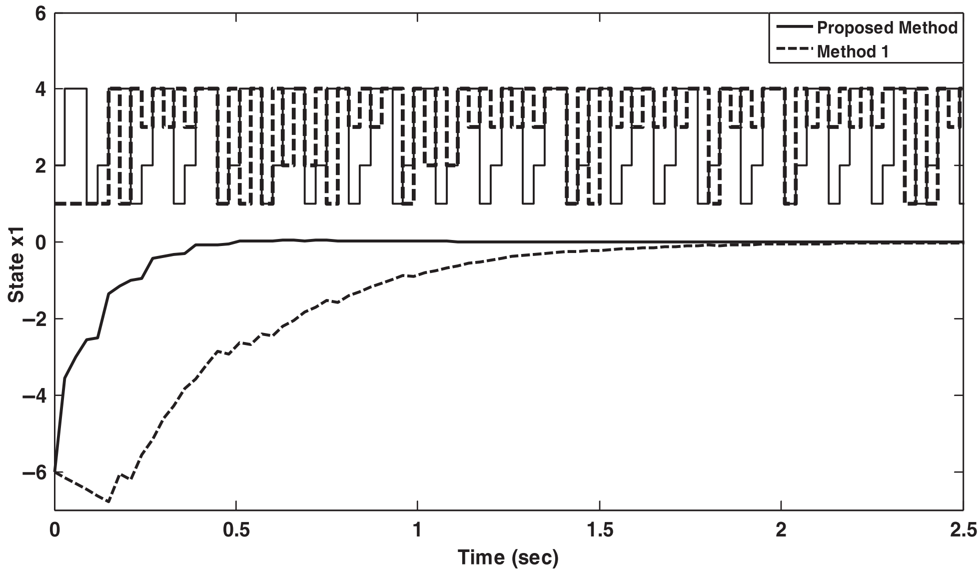

FIG. 3. System state x_1 and switching signals of two methods for system (4.5).

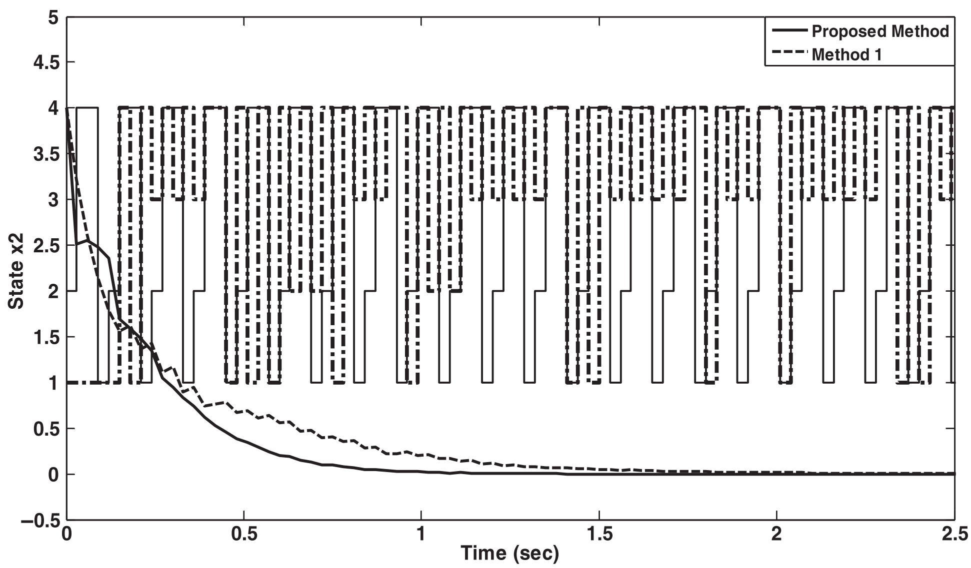

FIG. 4. System state x_2 and switching signals of two methods for system (4.5).

Step 3: Guaranteed cost controller gains are

$$K_1 = Y_1 X^{-1} = \begin{bmatrix} 1.9217 & 0.0622 \\ 0.0193 & 1.5525 \end{bmatrix}, \quad K_2 = Y_2 X^{-1} = \begin{bmatrix} 2.1143 & -3.4027 \\ -4.3708 & -1.2748 \end{bmatrix},$$
$$K_3 = Y_3 X^{-1} = \begin{bmatrix} -0.3203 & 0.0658 \\ 0.3495 & 1.3772 \end{bmatrix}, \quad K_4 = Y_4 X^{-1} = \begin{bmatrix} 2.2161 & 2.2468 \\ 2.1673 & -1.7906 \end{bmatrix}. \tag{4.11}$$

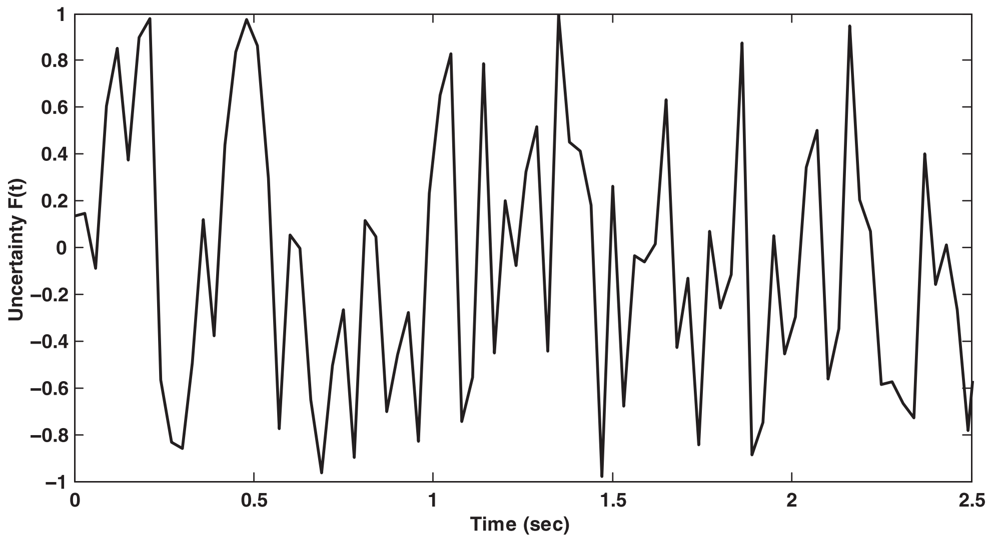

FIG. 5. The time-varying uncertainty $F_i(t),\ i \in \underline{m} = \{1, 2, 3, 4\}$.

TABLE 1 *Comparing two methods for some different initial conditions in Example 4.2*

$x_0 = [x_1\ x_2]^\top$	$J^*_{\text{Proposed method}}$	$J^*_{\text{Method 1}}$
$[0.25\ -0.1]^\top$	0.3834	0.3989
$[-1\ 1]^\top$	2.4040	2.4995
$[-2\ 4]^\top$	20.1036	25.5318
$[-4\ 3]^\top$	26.8300	31.3929
$[6\ -2]^\top$	45.1466	52.2534
$[5\ -8]^\top$	94.0307	112.4056

Step 4: The GCV is $J^* = x_0^\top P x_0 = 60.2038$.

Step 5: The switching strategy is (3.2) for obtained scalars and matrices in Steps 2 and 3.

Step 6: k_1 and k_2 in Theorem 2.1 are obtained as $k_1 = 1.1970$, $k_2 = 0.1852$.

Figures 3 and 4 show the state x_1 and the state x_2, starting the system from a specific initial condition. Also, the time-varying uncertain function $F_i(t),\ i \in \underline{m} = \{1, 2, 3, 4\}$ as shown in Fig. 5 is a random number between -1, 1. Comparing states, it can be seen the states in proposed method are smoother than states in Method 1 and tend to zero faster. It is shown that the number of switching in our method is less than Method 1. Finally, comparing the guaranteed cost obtaining from two methods shows that, guaranteed cost J^*in the proposed method is less than obtained guaranteed cost by Method 1 (Wang & Zhao, 2006), $J^*_{\text{Proposed method}} = 60.2038 < J^*_{\text{Method 1}} = 65.5707$.

Table 1 shows comparison between guaranteed costs of two methods for some different initial conditions.

It is important to say that there are many scalars which must be selected by designer for designing switching and GCCL in Wang & Zhao (2006) (see Wang & Zhao, 2006, (34) in Example 2), while, in the proposed method, the number of these scalars ($\delta_i, i \in \underline{m}$) is only equal with the number of the subsystems and surely is less than the Method 1. Compared with Wang & Zhao (2006), we proposed a new switching strategy which guarantees exponential stability (versus asymptotical stability) of this class of systems.

5. Conclusion

Switching strategy must be chosen consciously, because it may lead to asymptotic or exponential stability. The main objective of this paper is to propose a new switching strategy to provide exponential stability (versus asymptotical stability) in GCC design for a class of uncertain switched linear systems. Our switching design method is based on CLF technique and extension of min-projection strategy. Furthermore, the sufficient condition on the existence of guaranteed state-feedback control is presented in the form of LMIs. Finally, convergence rate of states is determined.

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