Opposition of Magnetohydrodynamic and Al₂O₃–water nanofluid flow around a vertex facing triangular obstacle

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Abstract
This article is an attempt to enhance the heat transfer rate around an obstacle by nanofluids and control the instabilities of the unsteady flow by Magnetohydrodynamics. The finite volume method (FVM) is used to simulate the Al₂O₃–water nanofluid flow around a triangular obstacle. An external magnetic field is used to study the effects of magnetic field on fluid flow and heat transfer. The range of Stuart number and solid volume fraction of nanoparticles are 0–10 and 0–0.05, respectively. Also, the Reynolds number is fixed at Re = 100. The effects of above parameters on the flow and heat transfer characteristics are investigated. The obtained results indicate that a stronger magnetic field is needed for vanishing the recirculation wake and stabilizing the flow in nanofluid comparing with the clear fluid. Also, the effect of magnetic field on reduction of heat transfer increases with increase in solid volume fractions.

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1. Introduction
The investigation of fluid flow and convective heat transfer over an equilateral triangular obstacle has many industrial and engineering applications. Some of these applications are cooling of electronic components, flow meters, cooling towers, sedimentation, melting, agricultural products, vaporization, oil and gas pipelines, combustion, and heat exchangers [1]. In all of these applications, enhancement of heat transfer rate and control of wake destructive behavior are very important. As we know, control of wake behind a structure or bluff body leads to a reduction in the unsteady forces and vibrations on it. Adding nanoparticles to a base fluid is a new method for improving their heat transfer properties. Researchers have used this method widely in past two decades [2–6].

For example, Sheikholeslami et al. [7] performed a study on nanofluid natural convection heat transfer in an enclosure. They observed that the Nusselt number increases with increase in solid volume fraction of nanoparticle. In another research, Sheikholeslami et al. [8] investigated the effects of four different types of nanoparticles on heat transfer and fluid flow around a permeable stretching wall. These nanoparticles were Copper (Cu), Silver (Ag), Alumina (Al₂O₃) and Titanium Oxide (TiO₂). In addition, they considered water as a base fluid. They found that the effects of nanoparticles on the heat transfer and fluid flow characteristics are noticeable in the Ag–water solution than in the other solutions. Ellahi et al. [9] investigated the natural convection boundary layer nanofluid flow along an inverted cone.

Some researchers studied the Magnetohydrodynamics [10–13]. For example, Bovand et al. [14] controlled the wake behind a porous circular obstacle by Magnetohydrodynamics. The value of disappearance Stuart number decreases with increase in Darcy number. Note that the disappearance Stuart number was known as the intensity of the required magnetic field for disappearing the re-circulating wake. As a complement work, the effect of magnetic field on instabilities of heat transfer from an obstacle in a channel has been investigated by Rashidi and Abolfazli [15]. They reported that the thickness of thermal boundary layer increases with increase in Stuart number.

Srikanth et al. [16] studied the effects of magnetic field on the fluid flow and heat transfer around a solid circular cylinder wrapped with a porous ring. They utilized the least square technique [17–20] to suggest empirical equations for average Nusselt number that the effects of Darcy numbers and magnetic field were taken into account for these equations.

Some researchers studied the heat transfer and fluid flow around a triangular obstacle. The flow and heat transfer across a long equilateral triangular obstacle have been investigated numerically by Srikanth et al. [16]. They found that the bottom and top surfaces have the maximum crowding of the isotherm contours as compared with the rear part of the obstacle. Prashanna et al. [21] focused on 2-D laminar flow past a triangular obstacle. It was observed that the heat transfer rate increases with increase in the degree of shear-thinning at fixed values of the Prandtl and Reynolds numbers. Some researchers focused on nanofluid flow in the present of magnetic field [22–29]. For example, Ul Haq et al. [30] studied the Magnetohydrodynamics (MHD) flow of the nanofluid
over a sensor surface. They used several nanoparticles in their study (Cu, TiO₂ and Al₂O₃). They found that the velocity profiles and the boundary layer thickness decrease with increase in solid volume fractions. Flow analysis for a non-Newtonian blood in porous arteries in the presence of magnetic field is investigated by Ghasemi et al. [31]. It was observed that the velocity profiles decrease with increase in the MHD parameter. MHD stagnation point nanofluid flow over a stretching sheet has been studied by Ul Haq et al. [32]. Their results indicated that increase in Hartmann number (the ratio of the electromagnetic force to viscous force) gives the resistive type flow within the boundary layer. MHD nanofluid flow in non-parallel walls has been investigated by Hatami et al. [33]. They found that the skin friction coefficient decreases with increase in Hartmann number. Malvandi and Ganji [34] studied the effects of nanoparticle migration on mixed convection of Al₂O₃–H₂O nanofluid in vertical channel. It was found that the peak of the velocity profile in the core region of the channel decreases with increase in Hartmann number.

The previous results in the literature show that using the triangular obstacle increases the heat transfer compared with that in the square and circular obstacles. Therefore, this shape is selected as a case study in this paper. We will try to enhance the heat transfer rate around a triangular obstacle by nanofluids and control the instabilities of the unsteady flow by Magnetohydrodynamics. However, it is possible that the simultaneous use of two methods affect the performance of them. Therefore, the results discuss for the effects of nanofluid and magnetic field on the heat transfer and flow parameters. The literature review showed this is for first time that the instabilities of the unsteady nanofluid flow controls by Magnetohydrodynamics.

2. Problem statement

The computational domain and physical model for this research are presented in Fig. 1. Consider a nanofluid flow with uniform inlet velocity (U∞) and temperature (T∞) past a triangular obstacle with constant wall temperature (Tw). The obstacle with side length S is mounted against the flow. A streamwise magnetic field is exerted (magnetic field in x direction). The following assumptions are considered in order to make the problem amenable to the numerical simulations:

- The flow is unsteady, two-dimensional, incompressible, viscous and laminar. Also, the wall temperature of the obstacle is higher than that of the fluid temperature.
- The width and length of the computational domain are set to 60S and 160S for minimizing the effects of outer boundary [35].
- The Joule heating is neglected in the energy equation based on the results of Srikanth et al. [16].

3. Problem formulation

3.1. Governing equation

Governing equations (i.e. energy, momentum and continuity equations) are solved to simulate this problem. These equations can be expressed in the following manner:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

Momentum equations:

$$\rho_{\text{eff}} \left( \frac{Du}{Dt} \right) = -\frac{\partial p}{\partial x} + \mu_{\text{eff}} \left( \nabla^2 u \right), \tag{2}$$

$$\rho_{\text{eff}} \left( \frac{Dv}{Dt} \right) = -\frac{\partial p}{\partial y} + \mu_{\text{eff}} \left( \nabla^2 v \right) -\sigma_{\text{eff}} B^2 v. \tag{3}$$

Energy equation:

$$\rho_{\text{eff}} C_{\text{eff}} \left( \frac{DT}{Dt} \right) = k_{\text{eff}} \left( \nabla^2 T \right), \tag{4}$$

where $\mu_{\text{eff}}, \rho_{\text{eff}}, k_{\text{eff}}$ and $C_{\text{eff}}$ are effective viscosity, effective density, effective conductivity and effective specific heat, respectively. The effective density is provided as follows:

$$\rho_{\text{eff}} = (1-\phi)\rho_f + \phi\rho_p, \tag{5}$$

where $\phi$ is the volume fraction and subscripts $p$ and $f$ denote the particle and fluid respectively.
Furthermore, the effective specific heat can be obtained by [36]:

\[
C_{\text{eff}} = \frac{(1-\phi)C_f + \phi C_p}{\rho_f} \rho_f \rho_p C_p
\]  

Masoumi et al. [37] presented a theoretical model for the effective dynamic viscosity. According to this model, the effective dynamic viscosity depends on temperature \(T\), nanoparticle density \(\rho_p\), nanoparticle volume fraction \(\phi\), nanoparticle diameter \(d_p = 30\ \text{nm}\) and the base-fluid physical properties. This model can be given as:

\[
\mu_{\text{eff}} = \mu_f + \frac{\rho_p V_B d_p^2}{2 N_0},
\]  

where \(N\) is a function of the volume fractions, base fluid viscosity and diameter of the nanoparticles. This parameter can be expressed as:

\[
N = \frac{1}{\mu_f \rho_f \rho_p} \left[ (n_1 d_p + n_2) \phi + (n_3 d_p + n_4) \right],
\]  

\[
\begin{align*}
   n_1 &= -0.00000000000000113 \ \text{kg/m}^2 \ \text{s} \\
   n_2 &= -0.00002771 \ \text{kg/m}^2 \ \text{s} \\
   n_3 &= 0.00000009 \ \text{kg/m}^2 \ \text{s} \\
   n_4 &= -0.000000393 \ \text{kg/m}^2 \ \text{s}
\end{align*}
\]

Furthermore, \(V_B\) is the Brownian velocity of the nanoparticles that depends on temperature, diameter and density of the nanoparticles. Also, \(\delta\) is the distance between particles that depends on diameter and volume fractions of the nanoparticles. These parameters can be calculated by [37]:

\[
V_B = \frac{1}{d_p} \sqrt{\frac{18K_B T}{\pi \rho_p d_p}}
\]

\[
\delta = \sqrt{\frac{\pi}{6d_p}}.
\]

Also, \(d_p\) is the nanoparticle diameter \((= 30\ \text{nm})\) and \(K_B\) is Boltzmann constant \((= 1.3807 \times 10^{-23} \text{J/K})\). A model for the thermal conductivity is developed by Chon et al. [38]. This model depends on the mean diameter of the nanoparticles and Brownian motion, which can be
given as:

\[
\frac{k_{\text{eff}}}{k_f} = 1 + 64.7 \times \phi^{0.746} \left( \frac{d_f}{d_p} \right)^{0.360} \left( \frac{k_s}{k_f} \right)^{0.7476} \times Pr^{-0.9955} \times Re^{-1.2321},
\]

where \(d_f\), \(k_f\) and \(k_s\) are molecular diameter of the base fluid (=0.3 nm), fluid and solid thermal conductivities, respectively. The Reynolds and Prandtl numbers in Eq. (11) are determined from:

\[
Re^* = \frac{\rho_f K_BT}{3\eta_f},
\]

\[
Pr = \frac{\mu}{\rho_f \alpha_f}.
\]

where \(l_{BF}\) is the mean free path of water (=0.17 nm) and \(\mu\) can be estimated by [38]:

\[
\mu = 2.414 \times 10^{-5} \times 10^{3.64T}.
\]

The effective electrical conductivity of nanofluid is calculated with the following equation [39]:

\[
\frac{\sigma_{\text{eff}}}{\sigma_f} = 1 + \frac{3}{\left( \frac{\sigma_p}{\sigma_f} + 2 \right) - \left( \frac{\sigma_p}{\sigma_f} - 1 \right)} \phi.
\]

Local Nusselt number \((Nu)\) based on the obstacle size is introduced as the following form:

\[
Nu = \frac{hS}{k_{\text{eff}}},
\]

Also, the surface-averaged Nusselt number \((Nu)\) is defined as follows:

\[
\overline{Nu} = \frac{1}{A} \int_A Nu dA.
\]

Furthermore, the time-averaged Nusselt number \((\overline{Nu})\) can be obtained by:

\[
\overline{Nu} = \frac{1}{t_p} \int_0^{t_p} Nu dt.
\]

where \(A\) and \(t_p\) are the obstacle surface and the period of time integration, respectively. Hence, Stuart number (the ratio of the electromagnetic force to the inertia force) is defined as follows:

\[
Stu = \frac{\sigma B^2 S}{\rho U_\infty}.
\]

### 3.2. The boundary and initial conditions

The boundary conditions for this problem are given according to the geometry of Fig. 1. These boundary conditions are as follow:

The uniform flow and temperature is used at the inlet section of the computational domain:

\[
u = 0, \quad v = 0, \quad T = T_\infty.
\]

For the obstacle surface, no slip boundary condition with a constant surface temperature is considered as:

\[
u = 0, \quad v = 0, \quad T = T_w.
\]

Also, zero gradient boundary conditions are used at the outlet of the computational domain. These boundary conditions are defined as:

\[
u = 0, \quad v = 0, \quad T = T_w.
\]

### Table 1

The effect of grid size on the wake length and Nusselt number for the case of \(Re = 100, \phi = 0.05\) and \(Stu = 2\).

<table>
<thead>
<tr>
<th>No.</th>
<th>Re</th>
<th>Grid size</th>
<th>(L_A)</th>
<th>Percentage difference</th>
<th>(Nu)</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>25 × 65</td>
<td>0.994</td>
<td>1.4</td>
<td>11.9</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>100 × 130</td>
<td>1.008</td>
<td>0.9</td>
<td>12.06</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>200 × 520</td>
<td>1.02</td>
<td>–</td>
<td>12.18</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>500 × 520</td>
<td>1.02</td>
<td>–</td>
<td>12.18</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 3. Ratio of pressure drop for alumina/water nanofluid versus Reynolds numbers, \(\phi = 0.01\).

Fig. 4. Wake length versus Stuart numbers for different solid volume fractions at \(Re = 100\).
follow:
\[
\frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0.
\]

For upside and downside of the domain, frictionless and adiabatic boundaries are used as the velocity and thermal boundary conditions.

### Table 2
The critical Stuart number for maximum and zero wake length.

<table>
<thead>
<tr>
<th></th>
<th>Maximum $L_r$</th>
<th>Zero $L_r$ (no wake)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0$</td>
<td>$St_{ur} = 0.1803$</td>
<td>$St_{ur} = 5.088$</td>
</tr>
<tr>
<td>$\phi = 0.01$</td>
<td>$St_{ur} = 0.1952$</td>
<td>$St_{ur} = 5.287$</td>
</tr>
<tr>
<td>$\phi = 0.03$</td>
<td>$St_{ur} = 0.2090$</td>
<td>$St_{ur} = 5.420$</td>
</tr>
<tr>
<td>$\phi = 0.05$</td>
<td>$St_{ur} = 0.2300$</td>
<td>$St_{ur} = 5.531$</td>
</tr>
</tbody>
</table>

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**Fig. 5.** Contours of streamline at different Stuart numbers for $Re = 100$ and $\phi = 0.03$ (Solid line: pure fluid; Dash line: nanofluids).
These conditions are as below:

$$\nu = 0, \frac{\partial u}{\partial y} = 0, \frac{\partial T}{\partial y} = 0.$$ \hspace{1 cm} (23)

It is assumed that there is no flow inside the domain at the initial time \((t = 0)\).

4. Computational model

4.1. Numerical method

In the present computation, a pressure base finite volume approach and staggered grid arrangement have been used for discretizing the governing Eqs. (1)–(4). Note that in staggered grid arrangement, the velocity components and pressure are stored at cell faces and cell center, respectively. Also, SIMPLE algorithm has been used to couple the velocity and pressure \cite{40}. The time, convective and diffusion terms have been discretized using the first order implicit, third-order accurate QUICK and Green–Gauss methods, respectively. Converged solutions are acceptable when the summation of residuals to be lower than \(10^{-7}\) for all equations.

4.1.1. Grid-independent study

Mesh distributions in the entire computational domain are shown in Fig. 2. Also, a closer view near the obstacle is added to this figure. A two-dimensional quad mesh is used with a more refined grid generated around the obstacle surface that the velocity and temperature gradients vary rapidly. A mesh testing procedure was performed to guarantee that the results are grid independent. Four mesh combinations were selected for the case of \(Re = 100, \phi = 0.05\) and \(Stu = 2\). This test is performed by calculating the wake length and Nusselt number. The results are presented at Table 1. As shown in this table, the difference in the wake length and Nusselt number between cases 3 and 4 are 0.3\% and 0.2\%, respectively. Therefore, the grid of case 4 ensures a grid-independent solution.

4.2. Validation

In order to assess the accuracy of the numerical method, present model are compared with some experimental results in the open literature. Comparison is done for alumina/water nanofluid flowing through a circular tube with constant surface temperature condition. The comparison of the numerical pressure drop ratio with the experimental one is presented in Fig. 3. The results are presented for fully developed region and \(\phi = 0.01\). Also, the ratio of pressure drop is defined as the ratio of pressure drop in nanofluid to that of base fluid. As shown in this figure, the numerical results agree well with the experimental pressure drop ratio of Heyhat et al. \cite{41} with error \(-7\%\). It can be seen that the pressure drop ratio decreases with Reynolds number for \(Re < 600\).

5. Results and discussion

In this research, forced convection heat transfer of nanofluid in the presence of constant stream wise magnetic field is investigated. The working fluid is alumina/water nanofluid. Simulations are performed for various values of Stuart number \((0 < Stu < 10)\) and solid volume fraction \((0 < \phi < 0.05)\). Also, Reynolds number is set to 100 in all calculations.

It is interesting to note that a resistive type force (Lorentz force) produces by exerting a magnetic field in a moving conductive fluid. This force acts on the flow past the obstacle in opposite of the flow direction. The effects of this force are obvious into the Navier Stokes equation (see term \(\sigma_j \mu B^2\)). Also, Lorentz force opposes the transverse velocity component \(v\) for a streamwise magnetic field.

Fig. 4 discloses that how the Stuart number and solid volume fractions affect on the wake length. As shown in this figure, the wake length increases rapidly by increasing Stuart number for weak streamwise magnetic fields. For example, this augmentation is 110\% of its hydrodynamic value \((Stu = 0)\) for \(\phi = 0.05\). We expect that the flow regime to be still unsteady by exerting weak magnetic fields. It is obvious that the inertia forces are dominate at such regimes and Lorentz force acts weakly in opposite of flow direction. Therefore, a narrower vortex street forms behind the obstacle and the wake length increases with respect to the hydrodynamic case. For stronger magnetic fields, the flow stabilizes and changes its distribution from the time-dependent behavior with vortex shedding to the steady state because the damping effect of the Lorentz forces dominates by using stronger magnetic fields. Further increase of Lorentz force leads to decrease in wake length by further increase in Stuart number. Also, the recirculating wake vanishes for \(Stu > 4\). This is due to the formation of a very slow-moving region after the obstacle. It is worth mentioning that the wake length increases by increase in solid volume fraction of nanoparticles. It is due to the fact that the flow separation occurs earlier in nanofluid. The inertial forces in the flow field increase in nanofluid comparing with clear fluid and these forces have a significant effect on the separation.

Table 2 presents the value of critical Stuart number that wake length has a maximum value (the peak in Fig. 4) and disappearance Stuart number that wake length has a zero value. Note that the flow regime is steady for critical Stuart number that wake length has a maximum value. As indicated in this table, both critical and disappearance Stuart numbers increase with increase in solid volume fraction of nanoparticles. Therefore, a stronger magnetic field is needed for vanishing the recirculating wake and stabilizing the flow in nanofluid comparing with clear fluid. For example, the magnetic field for vanishing the recirculating wake at \(\phi = 0.05\) is 9\% stronger than that of \(\phi = 0\) (clear fluid case).

Contours of streamline at different Stuart numbers for \(Re = 100\) and \(\phi = 0.03\) are shown in Fig. 5. In this figure, solid line refers to pure fluid without nanoparticles and dash line refers to nanofluids. It is observed that the flow is time-dependent and the vortices shed periodically behind the obstacle at \(Stu = 0\). In this paper, \(Stu = 0\) represents the hydrodynamic case without magnetic field. The unsteadiness increases in nanofluid comparing with clear fluid for \(Stu = 0\) that the flow regime is unsteady due to the effects of inertial forces that is increased in nanofluid. For the weak magnetic field \((Stu = 0.15)\), the fluctuations of shedding decrease comparing with \(Stu = 0\). The flow stabilizes and the distribution changes from the time-dependent distribution to the steady state one with two wakes for \(Stu = 2\). These wakes are symmetric because the obstacle is symmetrical about the centerline of the
direction of the flow. The center of this wake shifts away from the obstacle wall and the strength of the vorticity increases by adding nanoparticles to the base fluid because the flow separates earlier in nanofluid comparing with pure fluid. The inertial forces in the flow field increases by adding nanoparticles to the base fluid and these forces have considerable effect on separation phenomenon. Therefore, separation point shifts upstream by adding the nanoparticles to the base fluid and this leads to increase in wake length. These wakes vanish with further increase in magnetic field strength ($Stu = 10$). This is due to the decrease in inertia force at higher Stuart number. Also, the separation phenomenon delays with increase in Stuart number. Note that the effects of nanoparticles on streamlines are small at this Stuart number.

The variation of the drag coefficient with Stuart number for the Reynolds number of 100 at different solid volume fractions is presented in Fig. 6. This figure shows that when Stuart number is small, the drag coefficient decreases with increase in Stuart number. After a critical value of Stuart numbers, the drag coefficient increases with increase in Stuart number. As shown in Table 2, the wake length has a maximum at these values of Stuart number. Therefore, the maximum wake length and minimum drag coefficient are occurred at same Stuart numbers. Rashidi et al. (2015b) shown that these values are close to steadiness Stuart number. It was defined as a value of the Stuart number that was required to change the flow behavior from the time-dependent to the steady-state one. For $Stu > Stu_{cr}$, the viscous forces are dominate.

![Contours of isotherm at different Stuart numbers for Re = 100 and φ = 0.03 (Solid line: pure fluid; Dash line: nanofluids).](image-url)
The random motion of nanoparticles suspended in base fluid heat energy. These augmentations are in the vicinity of 17.2% and surrounding nanoparticles and movement of nanoparticles that transport heat energy. These mechanisms are the micro-convection of Brownian motion of nanoparticles contributes to the heat transfer increment by two ways. These mechanisms are the micro-convection of fluid surrounding nanoparticles and movement of nanoparticles that transport heat energy. These augmentations are in the vicinity of 17.2% and 5.17% for \( \text{Stu} = 0.001 \) and 10, respectively for \( \varphi = 0.05 \). Therefore, the effect of nanoparticles on augmentation of heat transfer decreases with increase in Stuart numbers.

Proposed Nusselt number as a function of Stuart number and solid volume fraction is presented in this research. This equation is:

\[
Nu = 10.61 \times (1 + \varphi)^{2.98} + 2.15e^{\left(\frac{1 - \text{Stu}}{100}\right)}
\]  

(24)

Note that the constants are obtained by the least square method in this equation.

**6. Conclusion**

In this research, MHD force convection for \( \text{Al}_2\text{O}_3 \)-water nanofluid past a triangular obstacle is investigated numerically using finite volume method (FVM). Effects of the nanoparticle volume fractions, Reynolds and Stuart numbers on heat transfer and flow behaviors are examined. The main results of this research are listed as follows:

- The wake length increases rapidly by increasing Stuart number for weak streamwise magnetic fields. For example, this augmentation is 110% of its hydrodynamic value (\( \text{Stu} = 0 \)) for \( \varphi = 0.05 \).
- A stronger magnetic field is needed for vanishing the recirculating wake and stabilizing the flow in nanofluid comparing with clear fluid. For example, the magnetic field for vanishing the recirculating wake at \( \varphi = 0.05 \) is 9% stronger than that of \( \varphi = 0 \) (clear fluid case).
- The drag coefficient decreases with increase in Stuart number when Stuart number is small.
- The time-averaged Nusselt number reduces rapidly with increase in Stuart number for weak streamwise magnetic fields (\( \text{Stu} < \text{Stu}_{cr} \)). These reductions are in the vicinity of 9.3% and 18.7% for \( \varphi = 0 \) and 0.05, respectively.
- Note that the effects of magnetic field on heat transfer is negligible for higher Stuart numbers (\( \text{Stu} > \text{Stu}_{cr} \)).
- The effect of magnetic field on reduction of heat transfer increases with increase in solid volume fractions.

As a result, MHD method has a positive effect upon the suppression of vortex shedding and a negative impact on the drag coefficient enhancement and heat transfer reduction. Therefore, optimization analysis is necessary in order to calculate the optimum conditions for intended goals. The author hope that the results from this research provided useful guidelines to the reader and can be used as initial data for optimization analysis in order to provide the optimum conditions.

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**References**


